Influence of Phase-Energy Correlation on Electron Cross-Field Mobility in a Hall Thruster

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The correlation between electron kinetic energy and the gyrophase at which collisions with neutrals occur modifies the value of the cross-field electron mobility in a Hall thruster. It is shown that the mobility is up to 40% higher than predicted with the fluid description in most regions of the Hall Thruster.

Nomenclature

$\widetilde{\mu}_\perp$	fluid cross-field mobility, $\widetilde{\mu}_\perp \equiv \lvert q_e\rvert \nu/(m_e\omega_c^2)$	$\sigma(\mathcal{E})$	collision cross-section
$\mu_\perp$	cross-field mobility	v	electron velocity
q_e	electron charge, $q_e = -1.602 \times 10^{-19}$ C	$\mathcal{E}$	electron kinetic energy, $\mathcal{E} \equiv m_e v^2/2$
m_e	electron mass, $m_e = 9.109 \times 10^{-31}$ kg	T_e	electron temperature
ν	collision frequency	$v_\perp$	electron gyration velocity
ω_c	cyclotron frequency	$\mathcal{E}_0$	average kinetic energy of electrons
β	Hall parameter, $\beta \equiv \omega_c/\nu$	$\langle\mathcal{E}\rangle$	average kinetic energy of collided electrons
x	coordinate in the axial direction	x_{gc}	guiding center position
y	coordinate in the azimuthal direction	$\langle x_c\rangle$	average collision position
B	magnetic field	$V_{\mathbf{E}\times\mathbf{B}}$	$\mathbf{E}\times\mathbf{B}$ drift velocity
E	electric field	$\mathcal{E}_{E\times B}$	drift energy, $\mathcal{E}_{E\times B} \equiv m_e V_{\mathbf{E}\times\mathbf{B}}^2/2$

ϕ	phase of electron rotation at which collision occurs
Δx	average axial displacement of the guiding center during collision (random walk)
Δx_1	average axial distance between the guiding center and the collision point
Δx_2	average axial distance between the collision point and the new guiding center
$Q(\mathcal{E})$	function proportional to collision probability, $Q(\mathcal{E}) \equiv \sigma(\mathcal{E})v$
$D(\mathcal{E})$	distribution function of an electron gyrating in crossed electric and magnetic fields
$\check{\mathcal{E}}$	minimum energy of gyrating electron, $\check{\mathcal{E}} \equiv m_e\left(v_\perp - E/B\right)^2/2$
$\hat{\mathcal{E}}$	maximum energy of gyrating electron, $\hat{\mathcal{E}} \equiv m_e\left(v_\perp + E/B\right)^2/2$

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Introduction

The classical mobility of electrons in the direction perpendicular to the magnetic field and parallel to the electric field is conventionally adopted to be [1]:

$$\widetilde{\mu}_{\perp} = \frac{|q_{\mathrm{e}}|}{m_{\mathrm{e}}} \frac{\nu}{\nu^2 + \omega_{\mathrm{c}}^2} \overset{\beta \gg 1}{=\!=} \frac{|q_{\mathrm{e}}|\nu}{m_{\mathrm{e}}\omega_{\mathrm{c}}^2} \,. \tag{1}$$

Here $q_{\mathrm{e}}/m_{\mathrm{e}}$ is the electron charge-to-mass ratio, $\beta = \omega_{\mathrm{c}}/\nu$ is the Hall parameter, ω_{c} is the cyclotron frequency and ν is the total momentum-transfer collision frequency.

It must be noted, however, that Eq. (1) is derived from the fluid description of plasma and does not take into account kinetic effects that stem from the finite Larmor radius of electron gyration. The authors have shown recently [2] that the correlation between the phase of electron cyclotron motion and the electron kinetic energy modifies the value of mobility. This means that in the presence of crossed electric and magnetic fields, electron mobility departs from Eq. (1).

In this paper, the authors estimate the influence of the phase-energy correlation on the electron cross-field mobility in a Hall effect thruster.

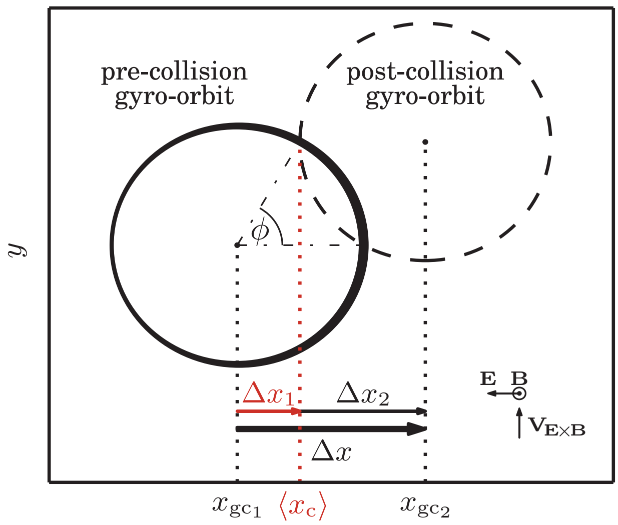

Figure 1. Average pre- and post-collision electron gyro-orbits in the drifting $\mathbf{E}\times\mathbf{B}$ frame. The random walk is divided into two substeps $\Delta x = \Delta x_1 + \Delta x_2$.

I. Phase-energy correlation

In a guiding center description, the motion of electrons in the direction perpendicular to the magnetic field is a sum of discrete displacements of their guiding centers, triggered by particle collisions. The collisional motion of electron guiding centers resembles a random walk. Figure 1 depicts an average trajectory of an electron during a collision with an atom. The phase of the collision is not equiprobable due to the varying kinetic energy of the electron and the fact that collision cross-section depends on kinetic energy. The varying thickness of the pre-collision orbit illustrates the variation of the collision probability. The random walk Δx is divided into two substeps in order to show that Δx_1 depends on the shape of the collision cross-section function, while Δx_2 does not.

Let us define function:

$$Q(\mathcal{E}) \equiv \sigma(\mathcal{E})v \tag{2}$$

which provides an unnormalized measure of the collision probability, where $\mathcal{E}$ is the electron energy, $\sigma(\mathcal{E})$ is the collision cross section and v is the electron velocity. Daniłko and Barral [2] have shown that in the small drift limit ($\mathbf{E}\times\mathbf{B}\to 0$), the mobility of electrons in a partially-ionized Maxwellian plasma in the presence of strong magnetic field is:

$$\mu_\perp\big|_{\frac{E}{B}\to 0} = \left(1+\frac{2}{3}\frac{\displaystyle\int_0^\infty \frac{dQ}{d\mathcal{E}}\mathcal{E}^{3/2}\mathrm{e}^{-\frac{\mathcal{E}}{T_e}}\mathrm{d}\mathcal{E}}{\displaystyle\int_0^\infty Q(\mathcal{E})\mathcal{E}^{1/2}\mathrm{e}^{-\frac{\mathcal{E}}{T_e}}\mathrm{d}\mathcal{E}}\right)\frac{|q|\nu}{m\omega_c^2}. \tag{3}$$

Equation (3) provides an upper bound to the effect of phase-energy correlation on electron mobility. For cases when the small drift approximation is not valid, the correct value of the cross-field mobility may be obtained with the use of a Monte-Carlo simulation.

It can be inferred from Eq. (3) that if $dQ/d\mathcal{E}=0$ than μ is identical to $\widetilde{\mu}_\perp$ of Eq. (1). However, for $dQ/d\mathcal{E}>0$, μ is greater than $\widetilde{\mu}_\perp$ and for $dQ/d\mathcal{E}<0$, it is smaller.

The essence of the phase-energy correlation is best illustrated with an electron population gyrating in two dimensions with the same average kinetic energy $\mathcal{E}_0$. The distribution function of the monoenergetic electrons gyrating in crossed electric and magnetic fields is [3]:

$$D(\mathcal{E}) = \begin{cases} \dfrac{1}{\pi\sqrt{\mathcal{E}-\check{\mathcal{E}}}\sqrt{\hat{\mathcal{E}}-\mathcal{E}}} & ,\mathcal{E}\in(\check{\mathcal{E}},\hat{\mathcal{E}}) \\ 0 & ,\mathcal{E}\notin(\check{\mathcal{E}},\hat{\mathcal{E}}) \end{cases}, \tag{4}$$

where $\check{\mathcal{E}}$ and $\hat{\mathcal{E}}$ are the minimum and maximum energies of electrons involved in an $\mathbf{E}\times\mathbf{B}$ drift:

$$\check{\mathcal{E}} = \frac{m_e}{2}\left(v_\perp-\frac{E}{B}\right)^2, \quad \hat{\mathcal{E}} = \frac{m_e}{2}\left(v_\perp+\frac{E}{B}\right)^2. \tag{5}$$

Here $v_\perp$ is the velocity of electron rotation around its guiding center in the plane perpendicular to $\mathbf{B}$. Velocity parallel to the magnetic field is assumed to be 0.

Due to the presence of the electric field, the kinetic energy of electrons varies with x. This makes the phase at which collision occurs not equiprobable. As a result, the distribution function of collided electrons is proportional to $D(\mathcal{E})\cdot Q(\mathcal{E})$ rather than to $D(\mathcal{E})$. Figure 2 shows that the average energy of collided electrons $\langle\mathcal{E}\rangle$ depends on the shape of the function $Q(\mathcal{E})$. It departs from $\mathcal{E}_0$ whenever $dQ/d\mathcal{E}\neq 0$. Due to the direct correspondence between the kinetic energy and the position of an electron, the average collision position $\langle x_c\rangle$ does not coincide with the guiding center position x_{gc} for $Q(\mathcal{E})\neq const$.

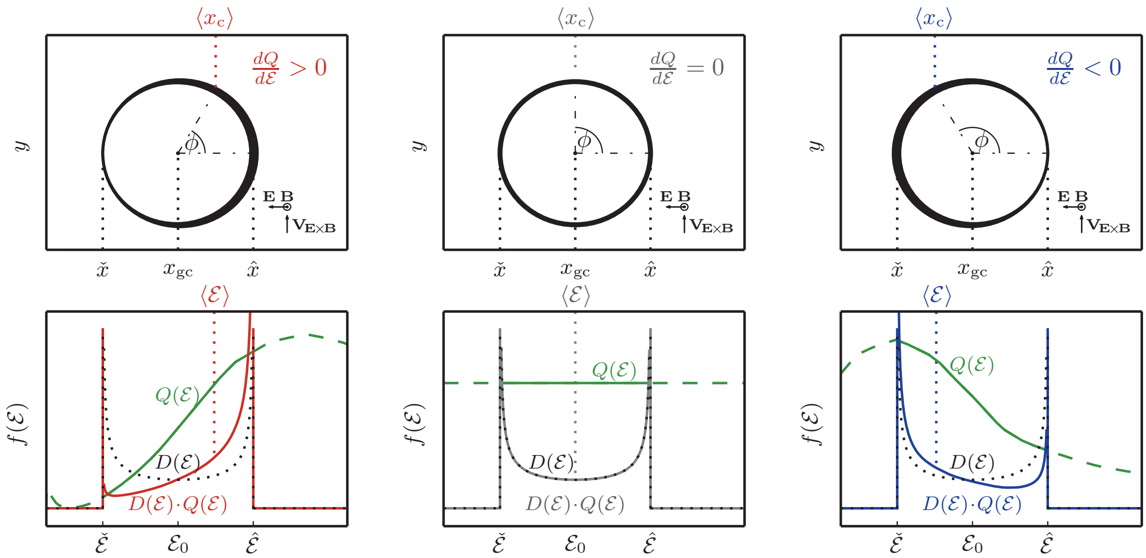

Figure 2. The top row presents electron orbits with variable thickness, illustrating the variation of the collision probability for $Q'(\mathcal{E})>0$ (left), $Q'(\mathcal{E})=0$ (middle) and $Q'(\mathcal{E})<0$ (right). The corresponding probability functions: $Q(\mathcal{E})$, $D(\mathcal{E})$ and $D(\mathcal{E})\cdot Q(\mathcal{E})$ are plotted below each orbit.

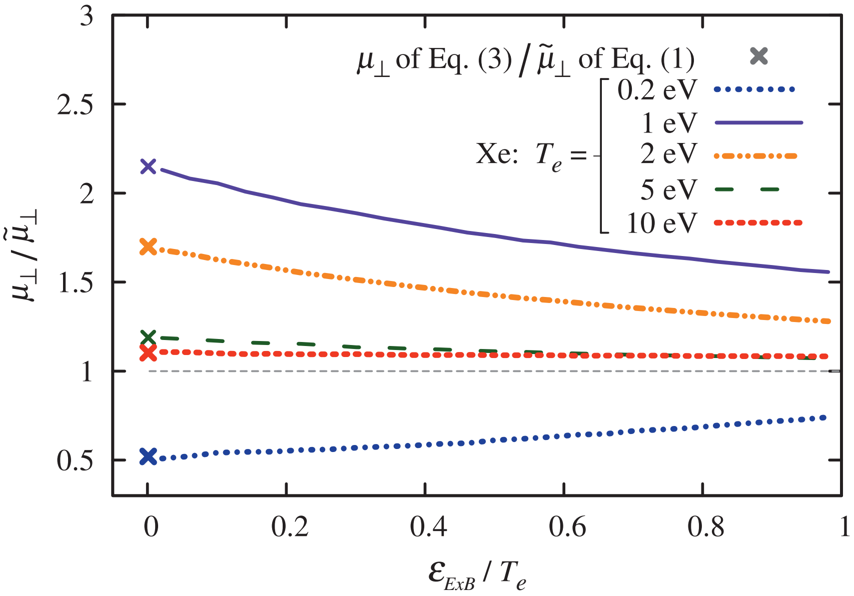

Figure 3. The ratio of mobility $\mu_\perp$ accounting for phase-energy correlation to the fluid mobility $\widetilde{\mu}_\perp$ of Eq. (1). Numerical results (lines) are compared to analytic solutions of Eq. (3) ($\times$ points).

II. Monte-Carlo simulation

The expression in Eq. (3) is limited to the $V_{\mathbf{E}\times\mathbf{B}} \to 0$ case. To get the mobility beyond that limitation, a numerical Monte-Carlo simulation was performed using the null collision method [4]. The macroscopic parameters of the model used in this study are not computed self-consistently, but are imposed to be constant. This yields that the results may be compared with experimental data provided that there exists a steady-state solution corresponding to the imposed conditions. Spatial variation of all macroscopic parameters is also ignored, which reduces the generalized Ohm's law to:

$$V_x = -\mu_\perp E. \tag{6}$$

The ratio of mobility of Eq. (3) to the fluid expression of Eq. (1) for the xenon plasma is plotted in Fig. 3 along with the numerical results. The Monte-Carlo simulation is in good agreement with the expression of Eq. (3) in the $V_{\mathbf{E}\times\mathbf{B}} \to 0$ limit, when the effect of phase-energy correlation on cross-field mobility seems to be the largest.

III. Mobility in a Hall Thruster

In order to assess the influence of the phase-energy correlation on electron mobility in a Hall Thruster, we compare the results of the Monte-Carlo simulation with experimental data obtained by Meezan et al. [5]. The simple Monte-Carlo model described in Sec. II is not capable of covering the whole thruster in one go, which is why a separate simulation was run for any given position along the channel. The results of the simulation, presented in Fig. 5, suggest that the classical mobility is enhanced by the phase-energy correlation in most regions of the thruster. Careful examination of the data indicate a $10-40\%$ increase of the classical mobility.

IV. Conclusion

The phase-energy correlation increases the classical electron cross-field mobility in a Hall thruster by up to 40%. There was no region of diminished mobility observed for the experimental values used. Since the Monte-Carlo model used in this study did not account for the anomalous transport, the results of the simulation are far from the experimental values. However, it is worth noting that in the case of strong departure

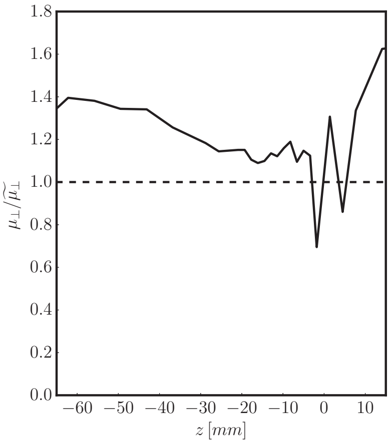

Figure 4. Numerical results of the ratio of mobility $\mu_\perp$ accounting for the phase-energy correlation to fluid mobility $\widetilde{\mu_\perp}$ of Eq. (1).

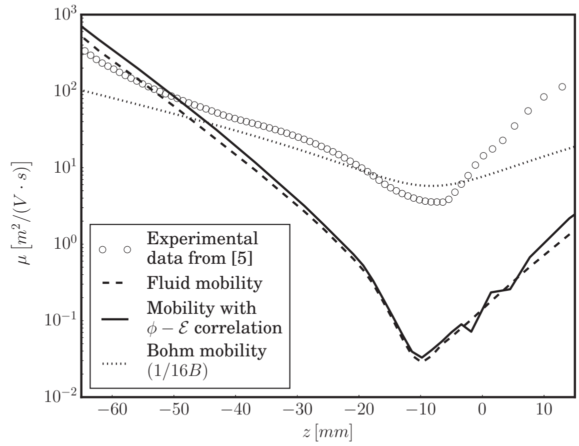

Figure 5. The comparison of mobility accounting for phase-energy correlation (solid line) with fluid mobility (dashed line) and experimental values (circles). Bohm mobility (dotted line) is plotted for reference.

from the Maxwellian distribution, the influence of the phase-energy correlation on electron mobility may be much higher. It was shown by Daniłko and Barral [2] that for low-temperature monoenergetic population of electrons, the classical mobility may actually be several times higher than predicted by the fluid description. The influence of the phase-energy correlation is prominent enough to account for it when calculating classical mobility of electrons in magnetized plasma devices such as Hall effect thrusters.

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