AN IMPROVED CODE FOR NOZZLE TYPE STEADY STATE MPD THRUSTERS

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Abstract

In order to predict the overall performance of continuously running, self-field MPD thrusters, a semi-two-dimensional model calculation and computer program has been developed. The extended Ohm's law is used to calculate the current contour lines, and a one-dimensional, two-component expansion flow model is employed to obtain velocity, temperature and pressure distributions for calculating the gas properties, which are again used in Ohm's law. Outside the nozzle the expansion flow was considered to be isothermal for the ions and adiabatic in the case of electrons. Outside the nozzle the expansion flow was considered hyperbolic and adiabatic for both components. The results of this first step were presented at the 19th IEPC.

In a second step this first model has been improved. Within the nozzle of the thruster the radial heat transport and the coupling of electrons and heavy particles are taken into account. The results of the code are compared with measurements in a vacuum tank which always has a residual gas pressure. This ambient gas, which also is at a high temperature (T, ≈ 10000K) is included in the model with an estimated free stream boundary.

The differential equation is solved by means of a finite difference method for the geometry of the nozzle type plasma thruster DT2-IRS, which has been investigated experimentally in a steady state as well as in a pulsed mode at the same institute. The results of these calculations are compared with the experiments.

Nomenclature

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<td>electric field strength</td>
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Subscripts

- $c$ chamber
- $e$ electron
- $s$ surface
- $i$ ion
- $T$ thrust
- $r$ radial
- $z$ axial

1 Introduction

For several years in the USA and Europe, nuclear space power sources have been under development and will be available in the near future. This, along with new optimization studies has again raised interest in continuously running, self-field MPD thrusters [1,2].

Despite their simplicity in design and power conditioning and their high reliability MPD thrusters are handicapped even today by the shortcomings of low efficiency and performance limits caused by instabilities which restrict the current level at a given propellant flow rate. [3,4,5] Therefore, parallel to the development of these thrusters, effort has been made to understand the physics of MPD discharges and to develop numerical codes with the goal of optimizing these thruster designs. This is no simple task, because the physics involved are extremely complex. The main difficulties are: no thermal equilibrium of the plasma, real gas effects, only partly known electrode effects, a not negligible friction due to the low Reynold's numbers caused by low densities and high temperatures, a magnetic Reynolds number too high to separate flow and discharge, etc. These are only the most important physical problems; added to them are the numerical difficulties: different partial differential equation systems, namely elliptical and hyperbolic ones that have to be coupled, nonlinearities, steep gradients especially at the electrodes, difficult geometries, etc. This list is by no means a complete one.

Due to this complexity, great simplifications had to be made for all codes developed to investigate the MPD thrusters, so that they solve at least parts of the whole problem.

The electrode instabilities have been studied mainly analytically [3,6], and the oscillations are being investigated by discussing the dispersion relation [7,8].

For investigating the flow and discharge fields and their interaction and effect on discharge power, thrust and efficiency of an MPD thruster, numerical codes have been developed. These codes are one-dimensional [9], quasi-two-dimensional [10,3] and simplified two-dimensional ones [11,12,13,14,15].
In this paper, a partly two-dimensional, time independent code for continuously running, nozzle type MPD thrusters is presented. The simplified two-dimensional electromagnetic calculation follows from Ohm's law, similar to [12], and the flow is modeled by a one-dimensional, two-component expansion flow.

2 Modeling

For continuously running self-field MPD thrusters in the power range of 100 to 500 kW, a nozzle type configuration leads to the best performance, because in this power range the thermal part of the thrust is in the same order of magnitude as the electromagnetic part. Such a nozzle type thruster DT2-IRS (Fig. 1) is being developed and tested at the Institut für Raumfahrttechnik at the University of Stuttgart [4,16]. In order to predict the overall performances for such thrusters, this MPD code has been developed and the results have been compared with the experimental data. In this approach, independent codes for the discharge, the flow field and the electron temperature distribution have been installed and then solved iteratively [17]. The relatively low magnetic Reynold's number in the region of magnetic acceleration justifies these decoupled calculations.

Basic assumptions for all codes are:

* steady state conditions
* rotational symmetry
* fully singly ionized, quasi-neutral plasma

2.1 Flow field code

The flow field is taken as a one-dimensional, frictionless expansion flow, assuming adiabatic behavior for the ions and isothermal for the electrons within the whole thruster. This assumption is justified, since primarily the electron component is heated by Ohmic energy input, and since within the nozzle expansion flow the energy exchange between the lightweight electrons and the heavy ions is fairly weak. Therefore, reaction collisions such as recombinations, etc., are neglected. This means on the other hand the model approach distinguishes between electron and ion temperatures and assumes a frozen flow within the thruster. Outside the thruster both components are assumed to be adiabatic. The results of the codes are compared with data from experiments performed within a vacuum tank which always has a residual gas pressure. Therefore, a free stream boundary is implemented corresponding to experiments with a similar thruster [18]. Outside the free stream boundary, see fig. 2, the ambient gas mass density is, according to the experiments, set to \( 1 \cdot 10^{-4} \text{ m}^{-3} \). At the free stream boundary a linear transition of the mass density over a distance of 30mm is implemented to avoid numerical problems by the solution of equation 16, caused by discontinuities.

![Figure 1: DT2-IRS thruster](image)

![Figure 2: thruster with the calculation areas. All length coordinates are in [mm]](image)

Additional assumptions for the flow field code:

* one-dimensional frozen flow
* ambipolar expansion flow
* no friction
* Within the thruster:
  * electrons: isothermal
  * ions: adiabatic
* within the nozzle:
  * only radial energy transfer
* outside the thruster:
  * electrons and ions adiabatic

Basic equations:

* Continuity Equation:
  \[ \rho v A = \dot{m} \] (1)
* Bernoulli Equation:
  \[ \nu \frac{dv}{dt} + \frac{1}{\nu} \frac{dp}{dx} = 0 \] (2)
2.2 Discharge code

In order to calculate the current distribution of the arc discharge, a two-dimensional computer code has been developed.

Additional assumptions for the discharge code:
- no azimuthal current
- electric field normal to electrode surfaces

Basic equations:
- Maxwell's equations:
  \[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]
  \[ \nabla \times \mathbf{E} = 0 \]
  \[ \nabla \cdot \mathbf{B} = 0 \]
  \[ \nabla \cdot \mathbf{E} = 0 \]
- Ohm's law:
  \[ \mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{\omega_r}{B} (\mathbf{J} \times \mathbf{B}) \]

Rewriting Ohm's law by means of Maxwell's equations, one obtains a vector equation for the magnetic induction vector \( \mathbf{B} \) in the form
\[ 0 = \frac{1}{\mu_0} (\nabla \times \frac{1}{\sigma} (\nabla \times \mathbf{B}) - (\nabla \times (\mathbf{v} \times \mathbf{B})) + \frac{1}{\mu_0} (\nabla \times (\beta (\nabla \times \mathbf{B}) \times \mathbf{B})) \]
with
\[ \beta = \frac{\omega_r}{B \sigma} = \frac{1}{\epsilon_n} \]

From this equation follows with a stream function \( \Psi = r B_s \) and with respect to the rotational symmetry and the zero azimuthal current the elliptical, partial differential equation of 2nd order
\[ 0 = \frac{\partial^2 \Psi}{\partial r^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{\partial \Psi}{\partial r} \left( \frac{1}{r} + \frac{1}{\sigma} \frac{\partial \beta}{\partial r} - \sigma \frac{\partial \beta}{\partial z} - \frac{\partial \Psi}{\partial \beta} - \frac{\sigma \partial \Psi}{\partial z} \right) \]
\[ - \frac{\partial \Psi}{\partial z} \left( \frac{1}{\sigma} \frac{\partial \beta}{\partial r} + \frac{\sigma \partial \beta}{\partial z} + 2 \sigma \frac{\partial \Psi}{\partial \beta} + \mu_0 \sigma \nu_s \right) \]
\[ - \sigma \mu_0 \Psi \left( \frac{\partial \nu_s}{\partial r} + \frac{\partial \nu_s}{\partial z} - \frac{\nu_s}{r} \right) \]

The function \( \Psi(r,z) = \text{const} \) represents now a current contour line, since \( B = B_z \) is proportional to \( \frac{I(r)}{r^2} \), where \( I(r) \) is the electric current carried through a cross sectional area of \( \pi r^2 \). The proper boundary conditions for \( \Psi \) follow from the geometry of the thruster walls and electrodes. At the insulator and the inflow boundary \( \Psi \) is set to \( -\frac{\mu_0}{\sigma} I \). For the electrodes one assumes that the electric field is normal to the surfaces, therefore; \( \mathbf{E} \cdot \mathbf{r} = 0 \), where \( \mathbf{r} \) is the tangential vector of the electrodes surfaces in \( r-z \) plane. At the other boundary sections \( \Psi \) is set to 0.
2.3 Electron temperature code

The electron temperature has a strong effect on the electrical conductivity and the electron density, which again influences the discharge pattern. Therefore, a two-dimensional code for the electron temperature distribution, corresponding to the two-dimensional discharge code, was written.

The electron temperature distribution is determined by the energy equation for the electron component.

\[
\frac{\partial T_e}{\partial t} = n_e n_i a_e (T_e - T_i) - \lambda_e \nabla \cdot T_e - \frac{k_e}{2} \nabla T_e 
\]  

(17)

The energy input due to ohmic heating equals the sum of losses as the energy transfer from the electron gas to the ion gas and the heat flux due to conduction and convection. The heat transfer coefficient \(a_e\), and the thermal conductivity \(\lambda_e\) depend on the electron temperature:

heat transfer coefficient:

\[
a_e = \frac{8 \sqrt{2}}{\pi} Q_{et} \sqrt{\frac{1}{\mu_e k T_{et}}} \frac{k}{m_e + m_i} 
\]  

(18)

thermal conductivity:

\[
\lambda_e = 2.08 \frac{k_f}{\sqrt{m_i}} \frac{T_e}{\sqrt{Q_{et}}} \approx \lambda_{et} T_e^4 
\]  

(19)

where \(Q_{et}\) is the Gvosdover cross section [19].

\[
Q_{et} = \frac{\pi}{4} \left( \frac{\pi e^2}{4 \pi e k T_{et}} \right)^\frac{1}{2} \ln \left( 1 + 144 \pi^2 \frac{e^2 T_e^5}{n_e e^2 (z(z + 1))} \right) 
\]  

(20)

With respect to the rotational symmetry, equations 17 - 20 result in the following elliptical, partial differential equation of 2nd order:

\[
0 = \frac{\partial T_e}{\partial t} - n_e n_i a_e (T_e - T_i) 
+ \left( \frac{\partial T_e}{\partial \tau} + \frac{\partial T_e}{\partial \tau^2} \right) \lambda_e T_e^4 
+ \left( \frac{\partial T_e}{\partial \tau^2} \right)^2 \frac{5}{2} \lambda_e T_e^4 
+ \frac{5 k_e}{2} \left( \frac{\partial T_e}{\partial \tau^2} \right) 
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+ \frac{5 k_e}{2} \left( \frac{\partial T_e}{\partial \tau^2} \right) 

\]  

(21)

The boundary condition for \(T_e\), along the free stream boundary and the calculation field begin and end is set to a value of 10000 K in accordance with the measurements [18]. Outside the free stream boundary the electron temperature is set to a constant value of 7000 K [18]. To avoid a discontinuity at the free stream boundary, a linear transition in radial direction is used. By using a value of 10000 K for the anode and the insulator one obtains reasonable results for the electron temperature inside the nozzle. The cathode is treated as a thermal insulator; therefore \(\nabla T_e \cdot n = 0\), where \(n\) is the normal vector of the cathode surface. Due to the axial symmetry, \(\frac{\partial T_e}{\partial \tau} = 0\) on the axis.

3 Solution of the equation systems

The three models described in subsection 2.1-2.3 were solved in the following sequence:

1. Flow field code
2. Discharge code
3. Electron temperature code

This sequence is repeated until the calculated datas, such as electron temperature distribution and current distribution have reached numerical equilibrium. Usually eight iterations are necessary to get good results.

The expansion relations 6 and 7 were solved with the Newton method. The two nonlinear, elliptical, partial differential equations 16 and 21 are solved with the finite differential method. Figure 2 shows the thruster with the free stream boundary which devides the field into two areas. Area 1 and 2 is used as calculation area for \(\Psi\), whereas only area 1 is used for \(T_e\). These areas are discretized with an aquisistant finite differential grid of 1mm height (radial) and 2mm width (axial). The partial derivatives, 1nd and 2nd order are replaced with the following differential quotients of any selective function \(f(x)\)

\[
\frac{\partial f}{\partial \tau} = \frac{f_{x+1} - f_{x-1}}{2\Delta \tau} 
\]  

(22)

\[
\frac{\partial^2 f}{\partial \tau^2} = \frac{f_{x+1} - 2f_x + f_{x-1}}{\Delta \tau^2} 
\]  

(23)

With this discretization, one obtains a nonlinear system of equations, which is iteratively solved with a modified Gauss-Seidel algorithm.

4 Discussion of Results

A main goal for developing this numerical code was to obtain better insight into the current distribution within a continuously running MPD thruster. With quasi-steady pulsed thrusters it is possible to measure the current contour lines, but because of the high heat load of the probes it is not yet possible to compare these with experimental data on the continuously running thruster. This means there is a need for a calculation code for the steady state thruster development.

The magnetic Reynold's number indicates if the discharge influences the flow field significantly. Therefore to evaluate the only weak coupling of flow and discharge within our iterative solution of the three codes, the magnetic Reynold's number \(R_m = \mu_0 c v L\) has been calculated. In Fig. 3 its distribution within the discharge region has been plotted, where as reference length the throat diameter was chosen. Since the main influence of the electromagnetic forces on the flow field occurs in the supersonic part, where the magnetic Reynold's
number is fairly low, the decoupling of the flow field and the electromagnetic field model seems justified.

Figure 3: Magnetic Reynold's number distribution for 3 kA and a mass flow of 0.8 g/s argon. As characteristic length the nozzle diameter was chosen.

In Fig. 4 the calculated current contour lines are shown between 1 and 4 kA with a mass flow of 0.8 g/s argon.

Figure 4: Calculated current contour lines for currents between 1 and 4 kA and a mass flow rate of 0.8 g/s argon.

A comparison between calculated and measured current patterns (Fig. 5) shows fairly good agreement. These measurements are carried out with a thruster geometry similar to DT2-IRS (Fig. 1) but in a quasi-stationary mode [20].

Figure 5: Calculated (a) and measured (b) current contour lines of the MPD discharge within the thruster DT2-IRS (Fig. 1) with a current of 1.3 kA and a mass flow rate of 0.8 g/s argon.

The electron temperature distribution within and outside the thruster at a current level of 3 kA is shown in Fig. 6.

Figure 6: Electron temperature distribution within the discharge at a current level of 3 kA and a mass flow rate of 0.8 g/s argon.

The axial temperature profile is compared with experiments [18] in Fig. 7.

Figure 7: Calculated and measured electron temperature profile at a current level of 3 kA and a mass flow rate of 0.8 g/s argon.
The coincidence is fairly good. The reason for the lower calculated temperatures upstream is that the temperature boundary condition on the upstream flow boundary of 10000 K at a distance of 185 mm from the thruster end is too low compared with the experiment.

The potential lines within the discharge at 3 kA are plotted in Fig. 8. An integration across these lines yields the discharge voltage.

![Figure 8: Potential lines within the discharge at a current level of 3 kA and a mass flow rate of 0.8 g/s argon. Cathode potential equal zero.](image)

This calculated discharge voltage is compared with the measured voltage in the current range between 1 and 5 kA in Fig. 9.

![Figure 9: Calculated and measured discharge voltage. The measurements were performed with DT2-IRS.[16]](image)

With the measurements, in contrast to the calculation, the electrode fall voltages are included. In the case of the cathode this voltage drop equals about the work function of the cathode material, which means in the case of thoriated tungsten between 1 and 3 volts, and is nearly independent of the discharge current. The anode fall voltage depends strongly on the current; starting with several negative volts at currents up to about 3 kA, the anode voltage rises steeply in the onset region [9]. This explains the different behavior of the two curves at high currents and the fairly agreement for currents between 1.5 and 3 kA.

The density map (Fig. 10) demonstrates the expansion flow with the flow modeling described in section 2.1, where the pinch effect was taken into account.

![Figure 10: Density map for an argon discharge at 3 kA with a mass flow of 0.8 g/s.](image)

According to the flow conditions and the known electromagnetic force configuration, the thrust can be calculated. The thrust of an MPD thruster is the sum of all gas dynamic surface forces and the electromagnetic volume forces. Hence it is

\[ \mathcal{T} = \int_A \left( \rho \frac{\partial v}{\partial t} + \mathbf{F} \right) \cdot d\mathbf{A} - \int_V \mathbf{j} \times \mathbf{B} \, dV \quad (24) \]

where \( A \) represents the surface of all internal walls and \( V \) is the current carrying volume. Using the well-known method to match the gasdynamic effect by using a thrust coefficient \( c_T \), where \( c_T \) is defined as

\[ c_T = \frac{1}{P_e A_t} \int_A \left( \rho \frac{\partial v}{\partial t} + \mathbf{F} \right) \cdot d\mathbf{A} \quad (25) \]

the thrust yields then

\[ \mathcal{T} = c_T P_e A_t - \int_V \mathbf{j} \times \mathbf{B} \, dV \quad (26) \]

This calculated thrust is compared with experimental data in Fig. 11. They show excellent agreement for a thrust coefficient chosen \( c_T = 1 \).

![Figure 11: Comparison of calculated and measured thrust for a mass flow of 0.8 g/s argon. Experiments with DT2-IRS.](image)

To improve the flow data at the upper flow boundary, which were calculated from the nozzle throat model, the calculated chamber pressure is compared with measurements...
carried out with DT2-IRS (Fig. 12). The very low deviation which is always less than 10 per cent is an indication of the validity of this model.

Since the pinch effect is a two-dimensional phenomenon and is applied here numerically on a one-dimensional flow model, numerical instabilities occur with currents higher than 3.5 kA at a mass flow of 0.8 g/s. These difficulties can only be overcome with a two-dimensional flow model. The results for 4 kA and higher were therefore calculated without respect to the pinch effect. To evaluate the influence of the pinch effect on the discharge, a calculation was carried out with and without respect to the pinch effect at a current of 3 kA and a mass flow of 0.8 g/s argon for comparison with the results of the codes described above. No significant influence could be detected on the electron temperature distribution, the discharge voltage or the thrust. The influence on the current distribution is demonstrated in Fig. 13. The main difference is that with respect to the pinch effect the current is at the anode more concentrated at the nozzle end.

5 Conclusions

The presented numerical analysis of an MPD thruster was applied to a rather complex nozzle type geometry. Despite the shortcomings of the code, namely the decoupled computation of the electromagnetic and flow equations and the one-dimensional flow model, it yields valuable results:

- Calculated integral values, such as thrust, discharge voltage and discharge chamber pressure fit the measured ones very well.
- The current patterns of the discharge and the electron temperature coincide with the measured data fairly well.

Further steps to improve the method will be a refinement of the flow field code to two-dimensionality and the inclusion of chemical reactions to overcome the restriction of a fully ionized gas assumption.

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