MAGNETIC INDUCTION THRUSTER WITH LITHIUM PROPELLANT

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ABSTRACT

A performance comparison is made between a lithium and an aluminum reaction mass expelled from a magnetic induction thruster. The lithium reaction mass was expected to produce higher exit velocities due to its lower density. In addition, examination of the reaction mass material properties, geometry dependence, mass dependence and dynamic properties are conducted. In the magnetic induction thruster, an induced current is produced in a ring shaped reaction mass. The induced current is opposite to the drive coil current, thus creating opposing magnetic fields that provide the accelerating force to expel the reaction mass. A computer code was used to simulate the magnetic induction thruster by numerically integrating the governing ordinary differential equations by a 4th order Runge-Kutta method. The use of lithium as the reaction mass produced an increase in the exit velocity. The desired material properties of the reaction mass are the lowest possible density and resistivity and highest possible specific heat. The geometry of the reaction mass should be a large radius on the order of the drive coil with a very small cross-sectional area relative to the radius. Finally, large masses of the reaction mass are preferred since the higher mass produces a larger net thrust, but at smaller specific impulses.

NOMENCLATURE

C - Capacitance
Cp - Specific heat
F - Force
I - Current
L - Self inductance
m - Mass
M - Mutual inductance
N - Number of coils of driver circuit
R - Resistance
t - Time
T - Temperature
Vc - Voltage of capacitor

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There are several advantages of the magnetic induction thruster. The first advantage is that there are no electrode losses. During the acceleration process, resistive heating in the reaction mass occurs. However, the heat is not transferred to the drive coil or to the surrounding chamber, due to the quick acceleration of the reaction mass.

Another advantage is that high efficiencies in converting electrical energy to kinetic energy can be achieved. In previous investigations, argon gas has been used as the reaction mass. When using gases there are energy losses due to the initial ohmic heating. The initial heating produces a plasma at which time the gas becomes electrically conducting and can create an opposing magnetic field. Because of the heating losses, the maximum efficiency of the gas thruster is 50 percent. As a result of using a material which is already electrically conducting in its natural state, up to 90 percent efficiencies can be achieved.

A third advantage is that many different materials can be used as propellant in this thruster. The major criterion is that the material must be electrically conductive. The material should have a low resistivity. Given a low resistivity, greater currents can be induced in the reaction mass and thus a greater opposing magnetic field. Another is that the material must have a low density. A low density material develops higher accelerations than a high density material for the same applied force. The extra acceleration then produces higher exit velocities. Another property of importance is the specific heat of the material. Higher specific heats are desirable. Higher specific heats retard the heating of the reaction mass and thus the resistance is lower for a longer period of time. A final consideration of the reaction mass criteria is its environmental impact in space.

Previous investigations by Mongeau et al. focused on aluminum as the material of the reaction mass. However, the changes in the thermal properties of the material and the reaction mass resistive heating were not taken into account in the analysis of Mongeau.

In the Zang and Micci investigation, reaction masses of aluminum and copper were compared. It was reported that the aluminum produced higher exit velocities and therefore specific impulses due to its lower density than copper. The thermal analysis predicted velocity values near the experimental values obtained by Mongeau and Bondalev and Ivanov.

The Zang and Micci conclusions prompted this study. In this investigation a performance comparison is conducted between an aluminum reaction mass and a lithium reaction mass since lithium is the lowest density metal.

MODEL

This research used the same numerical code to simulate the magnetic induction thruster as Zang and Micci. Ordinary differential equations which are derived below were integrated numerically by a 4th order Runge-Kutta method. The governing equations are obtained by modelling the magnetic induction thruster as two electrical circuits shown in Figure 2.

The drive coil circuit shown has the inductance \( L_0 \) and \( R_0 \) associated with the capacitance \( C \). \( L_1 \) is the self inductance of the drive coil and the current \( I_1 \) is produced by the capacitor discharge.

The reaction mass circuit has the induced current \( I_2 \) with the associated self inductance \( L_2 \). \( R_2 \) is the resistance of the reaction mass calculated from the resistivity of the material. The two circuits are coupled together by the mutual inductance \( M_{12} \).

Governing equations for the two currents, \( I_1 \) and \( I_2 \), are found by summing the voltage drops about the two circuits. For the driver circuit, the equation obtained is

\[
V_0 = R_0 I_1 + \left( L_0 \frac{dI_1}{dt} \right) + \frac{d}{dt} \left( M_{12} I_1 \right) \tag{1}
\]

where \( V_0 \) is the voltage drop across the capacitor. The first term on the right hand side of the equation is the voltage drop caused by the internal resistance of the capacitor and driver circuit wire. In this experiment, the driver circuit is assumed to be made of copper and \( R_0 \) is assumed throughout the experiment to be 0.005 ohms. The second term is the potential drop caused by the transient current and the self inductance of the circuit wire \( L_0 \) and the coil \( L_1 \). \( L_0 \) is assumed to be 0.25 micro-Henries throughout the experiment. The final term represents the potential drop due to the mutual inductance with the reaction mass coil.

For the reaction mass circuit the governing equation is

\[
0 = \frac{d^2 I_2}{dt^2} + \frac{d}{dt} \left( M_{12} I_1 \right) \tag{2}
\]

As for the driver circuit, the first term is due
to the resistance inherent in the reaction mass material. The second potential drop derives from the self inductance in the reaction mass coil due to the induced transient current $I_2$. The final term comes from the mutual inductance with the driver coil.

Solving Equations 1 and 2 simultaneously with the equation for the instantaneous capacitor voltage

$$\frac{dV}{dt} = \frac{I_1}{C}$$

will give the values of $I_1$ and $I_2$ as a function of time.

The mutual inductance, $M$, is proportional to the number of turns of the reaction mass and the magnetic flux and inversely proportional to the current of the drive coil or

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

where $N_2$ is the number of coil turns in the reaction mass and $\Phi_{21}$ is the magnetic flux. Thus the mutual inductance is solved from the geometry of the driver and reaction mass coils.

The equation for finding the instantaneous force produced by the opposing magnetic fields is directly proportional to the circuit currents and the derivative of the mutual inductance with respect to $x$.

$$F = I_1 I_2 \frac{dM}{dt}$$

During the acceleration of the reaction mass an energy loss is encountered. This loss is due to resistive heating of the reaction mass. As the temperature changes in the reaction mass material, the thermodynamic properties of specific resistivity also change. The changes in the thermodynamic properties of the material are modelled by first solving for the instantaneous temperature given by

$$\frac{dT}{dt} = \frac{R_1 I_1^2}{C M}$$

where $R_1$ is the resistance of the reaction mass coil at a constant temperature by active or passive means and that assumption was continued in this study. The equations were solved as a function of time by a 4th order Runge-Kutta method.

RESULTS AND DISCUSSION

Temperature-Dependent Material Properties

The specific heat and resistivity of lithium and aluminum were compared. The importance of these properties are shown in Equation 6. In determining the instantaneous temperature of the reaction mass, the material's resistivity and the specific heat are used. A comparison of resistivity as a function of temperature of aluminum and lithium is shown in Figure 3. The values were experimentally obtained and are tabulated in Reference 6 for aluminum and Reference 7 for lithium. The discontinuities in the curves correspond to the sharp increases in resistivity when the material changes from solid to liquid. As can be seen in the figure, the resistivity of lithium is greater than aluminum. Lithium resistivity was found to be an average of 3.09 times greater than aluminum.

If equation 5 were solved for the reaction mass current and all the values other than resistance were the same for lithium and aluminum, the lithium reaction mass should have a lower exit velocity. The lower velocity would be due to a smaller current induced in the lithium reaction mass coil because of lithium's higher resistance than aluminum. The smaller current would generate a smaller opposing magnetic field and thus a smaller accelerating force.

The values of specific heat for lithium and aluminum found in References 8 and 9 respectively, are graphed as a function of temperature in Figure 4. The spikes in the curves correspond to the energy needed for phase change at the respective melting points. The specific heat for lithium was found to be an average of 4.08 times greater than aluminum. Examining Equation 6 again, this time assuming the variables are identical except for the specific heats, the lithium reaction mass should have a greater exit velocity than the aluminum.

The combined effect of the resistivity and specific heat is that the lithium reaction mass should have a higher exit velocity than the aluminum reaction mass because the specific heat to resistivity ratio is greater for lithium than aluminum. Inserting the ratio into Equation 6 produces a smaller change in temperature with time. The smaller temperature increase would result in a smaller resistance increase and thus a greater induced current and opposing magnetic field.

Coil Radii

The dependent variable of interest is the exit velocity. The integration process for calculating the reaction mass velocity terminated when either one of two conditions were reached. The first condition is the boiling point of the reaction mass material and the second value is
when the force on the reaction mass becomes zero. The obtained exit velocities are graphed as a function of the reaction mass initial position relative to the drive coil.

Figure 5 shows the exit velocity as a function of the initial position for various radii. Lithium and aluminum results are plotted on the same graph for comparison. As can be seen, the lithium reaction mass velocity is greater than the aluminum for the same coil radius. This result was predicted in the previous discussion of the reaction mass thermodynamic properties. The lithium velocities were an average of 1.06 times greater than aluminum.

It can be seen from Figure 5 that the exit velocity increases with radius. This is to be expected from the magnetic flux term in the definition of the mutual inductance. Recall the magnetic flux is a function of the effective area of the reaction mass ring which increases with the coil radius. The graph plots four different radii from 10 mm to 20 mm for each material.

As the initial distance between the drive coil and reaction mass coil was increased, the exit velocity decreased to a fairly constant value. This trend is in agreement with the Zang and Micci investigation. The exit velocity is zero when the initial separation is zero. This result is explained by the mutual inductance term in Equation 4. When the initial position is zero the magnetic field produced by the drive coil is symmetric about the reaction mass, resulting in a net axial magnetic force of zero. Therefore, an initial separation is needed to produce an exit velocity.

Exhaust Material Mass

Figure 6 shows the exit velocity as a function of the initial position for various exhaust material masses. The mass was increased from one to eight times $M_0$, where $M_0$ equals 0.0501 gm. The graph depicts 4 different mass results where $4 M_0$ was the mass of the reaction mass for the radius dependence examination.

The results as a function of initial separation were similar to results found for the varying radii. When the mass increased from $M_0$ to $8 M_0$, the velocity decreased by 36.5% for the lithium reaction mass and 38.2% for aluminum. The accelerating force is now required to repel more mass which results in a decrease in the acceleration and thus velocity. Again the lithium reaction mass produced higher exit velocities. For these cases, the lithium was again an average of 1.06 times higher than the aluminum.

Reaction Mass Current

To provide further insight into the performance characteristics of the two reaction masses, the induced current was examined. The currents in the aluminum and lithium reaction masses were plotted as a function of time. The current was examined for both the different radii and masses and the initial separation distance was held constant at 0.2 mm.

Figure 7 plots the calculated values of current for different radii. When the radius
mass ring must decrease. Since the resistance of a conductor is a function of the cross-sectional area, the current decreases.

Due to the decrease in current, the opposing magnetic fields apply a reduced force. Thus, the reaction ring is not repelled as quickly, so the ring is within the effective area of the drive coil for a longer period of time. This can be seen in the widening of the curves as the radius increases and the peak values occurring at later times.

The mass dependence of the current was also examined. The radius of the reaction mass ring remained constant at 12 mm, slightly smaller than the driver loop radius of 12.84 mm. Figure 8 shows that the current of the lithium reaction ring is greater than the aluminum. This explains the greater exit velocities found for the lithium reaction ring in the graph of the various masses.

Figure 8 also shows that the current increased with mass. This is explained by the increase of the reaction ring cross-sectional area to compensate for the extra mass. Again, the current curves widened over time and the peak values occurred at later times. However, the reaction ring is in the effective range of the drive coil longer because the increase in current was not enough to compensate for the increased mass.

**Reaction Mass Force**

Figure 9 shows the force as a function of time for different radii. From the graph, lithium has a greater force applied to it than the aluminum. This follows from the greater current induced in the lithium ring.

As mentioned earlier, the desirability of having the current and thus the acceleration over a longer period of time is that it creates a greater total force. An increased force results in an increased thrust.

The force produced on the various masses is shown in Figure 10. The force increased with increasing mass. This increase in force shows that despite lower acceleration and thus exit velocity, higher amounts of mass are desirable for increased thrust.
Reaction Mass Acceleration

The magnitude of the current in the reaction mass determines the magnitude of the opposing magnetic field generated by the reaction mass. The opposing magnetic fields provide the force to accelerate the reaction mass away from the drive coil. Figures 11 and 12 show the curves of acceleration as a function of time.

Figure 11 plots the lithium and aluminum reaction mass acceleration for different radii. As expected from the reaction ring current values, the lithium ring accelerates faster than the aluminum. Also expected was the decline in acceleration with increasing radius. This decline in acceleration coincides with the decline in current for the increasing radius.

Figure 12 shows accelerations of the different masses as a function of time. The trends of the curves in Figure 12 differ from the corresponding graph of the current, Figure 8, in which the current increased with mass. Even though the current increased with the mass, the increasing opposing magnetic fields could not compensate for the extra mass. As shown in the figures for the acceleration, the peaks decreased and the time of acceleration increased with mass.

Specific Impulse Versus Thrust

As a final comparison the specific impulse calculated from the exit velocity was graphed as a function of thrust. To calculate the thrust values a pulse rate of 10 Hz was assumed. The pulse rate was taken from a study of the commutation of the capacitor discharge conducted by Mongeau.

The radius analysis is shown in Figure 13. The graph of specific impulse as a function of thrust shows a linear relationship. The linear relationship is as expected since both the specific impulse and thrust are function of exit velocity. However, what is interesting is the identical slopes for all different radii and both reaction mass materials. Each time the radius increased the curve extended. The linear extensions indicate that any reaction mass geometry would fit that curve.

From the graph, a specific impulse of 850 seconds can be achieved at a thrust level of 17 Newtons. This is a large amount of thrust for a thruster which is on the order of 40 mm in diameter. Also, lithium reaction mass outperformed the aluminum reaction mass, something which was anticipated from the previous results.

Figure 14 is a plot of the specific impulse as a function of thrust for various masses. Again, there was a linear relationship between specific impulse and thrust, as seen in the preceding figure. However, the slope of specific impulse to thrust varied with the mass. As the mass was increased, the thrust also increased, but the specific impulse decreased. This trend can be explained by the decreasing acceleration and the increasing force for extra mass as discussed previously.

Fig. 10 Induced axial force for various masses of lithium and aluminum as a function of time. The increase in force is due to increase in current.

Fig. 11 Comparison of lithium and aluminum reaction mass acceleration for various radii. Acceleration decreases with radius due to decreased force, but the acceleration occurs over an increasing amount of time.

Fig. 12 Comparison of lithium and aluminum reaction mass acceleration for various masses. The increase of current with mass does not produce enough force to compensate the increase in mass, thus the acceleration decreases.

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REFERENCES


CONCLUSIONS

The lithium reaction mass obtained a 6% increase in exit velocity and thus specific impulse over the aluminum. It was found that the performance is greatly influenced by the density of the reaction mass; i.e. the desired density should be as low as possible and there should be a large specific heat. The specific heat prevents the reaction mass from being heated too quickly increasing the resistance. In examining the induced current, acceleration, and force on the reaction mass, it was found that the desired geometry of the reaction mass is large radius on the order of the drive coil with a small cross-section relative to the radius. The lithium reaction mass at a repetition rate of 10 Hz obtained a maximum value of thrust of 25 newtons at a specific impulse of 650 seconds. The lithium reaction mass obtained a maximum value of specific impulse for the same repetition rate of 1000 seconds at a thrust level of 4.5 newtons. Finally, the mass of the reaction mass can be large since it produces higher thrust, but in turn decreases the specific impulse.