ABSTRACT
A non-linear problem concerning the propagation of a blast wave, satisfying the moving boundary conditions at the shock front of a violent explosion in plasma of varying density, is solved. Such a problem arises in connection with the applications of the shock waves in astrophysics, blast-wave propagation in the ionosphere, problems resulting from laser-ray focusing, explosive detonation, or electrical discharge in high-temperature plasma under laboratory conditions, as well as for explosive magnetohydrodynamic generators. In this paper, the plasma is assumed to be an ideal gas with infinite electrical conductivity and the source of explosion is a supersonic expansion. The basic equations with cylindrical symmetry in the presence of magnetic field and solar gravity in his work on the propagation of a spherical blast wave.

INTRODUCTION
Solar flares, which are bright eruptions at the surface of the sun, release energy at 17 minute intervals. This sudden release of energy, which is comparable to the Krakatoa explosion or a 100 megaton hydrogen bomb, sends out blast waves which travel both across the face of the sun and out into the outwardly flowing solar wind of interplanetary space. We consider the problem of a blast wave propagating into the outwardly flowing solar wind as it approaches the outer surface of the sun.

The propagation of a blast wave has been extensively studied analytically, numerically, and experimentally by a good number of researchers. Lin, one of them, carried out the pioneer studies of this phenomenon for cylindrical blast waves and showed that a similarity solution exists for the flow behind a strong cylindrical blast wave. Parker neglected the effects of magnetic field and solar gravity in his work on the propagation of a spherical blast wave. Parker’s results were further extended by Hundhausen and Gentry. They included the effect of solar gravity but not the magnetic field. Pai and Greenspan accounted for the effect of magnetic field. They also established that a corresponding MHD problem, not considered by Lin, admits a similarity solution behind a blast wave produced by the sudden release of energy along a line of infinite extent in an ideal plasma. This plasma is an ideal gas of uniform density distribution with infinite electrical conductivity and permeated by the azimuthal magnetic field of a line current.

For this kind of flow, Greenspan, as well as Greifinger and Cole pointed out two limiting cases. In the first case, the quiescent gas has zero conductivity. The magnetic field across the shock is continuous and the boundary conditions at the shock are ordinary hydrodynamic strong shock conditions. This case has been fully investigated by Greenspan.

In the second limiting case, the magnetic field is discontinuous across the shock, and the boundary conditions at the shock are those appropriate to the MHD shock. This case is a most general MHD case, because the magnetic field does not only have effect at the shock but also behind it. This case was first discussed by Pai, and solutions were obtained by neglecting the magnetic pressure in front of the shock. Although it is true that for small values of magnetic pressure, the error introduced at the shock is small, this produces a profound effect at the axis and also changes the energy content of the gas. In both cases, the effect of the inhomogeneity exponent (i.e., the varying density parameter) has been neglected. In this paper, the magnetic pressure in front of the shock is accounted for, and the effect of varying density is investigated on a propagating blast wave in an ideal plasma of solar gas.

BASIC EQUATIONS
The basic equations with cylindrical symmetry in the presence of an azimuthal magnetic field and devoid of dissipative mechanism are

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{pu}{r} = 0, \]

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + h = 0, \]

\[ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + 2h \frac{\partial u}{\partial r} = 0, \]

\[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \gamma p \frac{\partial u}{\partial r} + \frac{p}{r} = 0. \]

In these equations, \( p \) is the gas pressure, \( \rho \) is the gas density, \( u \) is the radial velocity, \( h \) is the magnetic pressure, and \( \gamma \) is the specific-heat ratio. All quantities are functions of radius \( r \) and time \( t \).

The shock conditions are

\[ u_s = \frac{2V}{\gamma + 1} (1 - \frac{a^2}{V^2}), \]

\[ p_s = \frac{2\rho_0 V^2}{\gamma + 1} \left( \frac{\gamma - 1}{2} \right)^2 \frac{(\gamma - 1) V^2}{(\gamma - 1)V^2 + 2a^2}, \]

\[ \rho_s = \frac{\gamma + 1}{\gamma - 1} \rho_0 \left( 1 + \frac{a^2}{(\gamma - 1)V^2} \right), \]

\[ h_s = \frac{1}{2} C_0 \left( \frac{\gamma - 1}{\gamma - 1} \right)^2 \rho_0 V^2 \left( 1 + \frac{a^2}{(\gamma - 1)V^2} \right), \]

where \( V \) is the shock velocity, \( a \) is the sound velocity of the undisturbed medium, \( C_0 = \frac{2\rho_0}{\gamma + 1} \) is the cowling number, and the suffixes \( s \) and \( o \) denote evaluation of the parameters just behind and just ahead of the shock respectively. Following Sedov, we
seek a self-similar solution of the system of Eqs. (1) to (4) which must satisfy the boundary conditions (5) to (8) at the shock; this solution is of the form

\[ u = V f(\eta), \quad p = \rho_0 f(\eta), \quad q = \rho_0 V^2 \pi(\eta), \quad h = \rho_0 V^2 H(\eta), \]  

where \( \eta = r/R \) is the similarity variable which is zero at the source of explosion and one at the shock front. Since the initial explosion energy is very large, the shock speed \( V \) is not known a priori. However, if we substitute these values of \( p \) and \( q \) into Eq. (16) and (17), the following relations can be derived

\[ \frac{\partial E}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (ru) = 0, \quad E = \frac{1}{2} \rho u^2 + \frac{p}{\gamma - 1} + h, \]  

where \( I = E + p + h \). Using the similarity transformation (9) (in (21), and using (15) and (17), the following relations can be derived

\[ \pi + \frac{(\gamma - 1)H(\eta - 2f)}{(\eta - \gamma f)} + \frac{g(\eta - f)(\gamma - 1)}{2(2f - \eta - \gamma f)} = C(r - 2(\gamma - 1)) \]  

where \( C \) is a constant of integration, which can be determined by using Eq. (12).

The Eqs. (19), (22), and (23) provide us with the values of \( H(\eta), \pi(\eta), \) and \( g(\eta) \) in terms of \( \eta \) and \( f \), but the analytical expression for \( f \) is not known in prior. However, if we substitute these values of \( H, \pi, \) and \( g \) in Eq. (20) and integrate numerically, we get integral curves for the velocity profiles behind a blast wave. The results of the numerical integration are shown in Fig. 1 and Fig. 2.

The integral curves in Fig. 1 show the effect of an inhomogeneity exponent \( \alpha \) (varying density parameter) on the flow distributions behind a blast wave. The gas pressure and velocity of the shock vary accordingly, decreasing from their strong shock values to zero over the range of \( \alpha \). The magnetic pressure at the shock, on the other hand, exhibits an asymptotic behavior at a value of \( \eta \) between .18 to .22. The existence of this behavior depends only on the jump conditions at the shock. The density distributions show that the major portion of the mass of the fluid contained in the blast wave region is concentrated near the wave front, provided the inhomogeneity exponent \( \alpha \) is less than some critical values. On the other hand, if \( \alpha \) exceeds the critical value, then the mass of the fluid is concentrated around a contact discontinuity appearing at the critical point where the corresponding velocity profile intersects the line \( \eta = f \).

At this critical point, the density asymptotically approaches infinity and the pressure tends to zero. Because of these abnormal changes in flow variables, a discontinuity appears at the critical point. A large concentration of material takes place at this point and a huge amount of internal energy is converted into kinetic energy. At the same time there is no flux through the new discontinuity. This shows that the surface of discontinuity expands and moves with the fluid with a huge amount of kinetic energy. This results in a cavity formation. Fig. 3 shows the formation of a cavity through integral curves. This is the most interesting in which the density shows an increasing trend from the shock front, whereas the pressure decreases sharply.

The effect of \( \alpha \) is to slow down the rate of decrease of the gas pressure coefficient \( \pi(\eta) \) and the velocity profiles. Fig. 2 shows the effect of the magnetic pressure on the flow distributions behind a blast wave. The magnetic pressure results in the growth of density and accelerates the decaying process of the velocity profiles \( f(\eta) \) and that of the gas pressure.
Fig. 1 Effect of $\alpha$ (inhomogeneity index) on the velocity profiles, gas pressure, density and magnetic pressure distributions in an adiabatic flow behind a blast wave for $C_0 = 0.01$ and $\gamma = 5/3$.

Fig. 2 Effect of Magnetic Field on the velocity profiles, pressure and density distributions in an adiabatic flow behind a blast wave for $\alpha = 1.6$ and $\gamma = 5/3$.

Fig. 3 Cavity formation for $\alpha = 1.6$, $\gamma = 2.0$ and $C_0 = 0.0$.

REFERENCES


