ABSTRACT

An investigation on the performance of a low power DC arcjet thruster was conducted using the numerical simulation code, in which the inlet flow was treated as subsonic and the nonequilibrium ionization-recombination was considered. The nozzle geometry effect on the performance characteristics was also investigated using a simplified calculation model which was developed from the calculation code described above. The results obtained from the numerical simulation and the experiment for two different nozzle geometries, i.e. conical and bell-shaped nozzles, were qualitatively in good agreement. As a consequence, it was revealed by both the numerical simulation and the experiment that the performance of the bell-shaped nozzle was superior to the conical nozzle.

1. INTRODUCTION

Arcjet thrusters can be used in various space-propulsion missions, such as the North-South station keeping and the orbit transfer from LEO to GEO of spacecraft. The thrust of the low power arcjet is mainly produced by the thermal expansion of the propellant. Therefore, in order to design a practical thruster, it is necessary to acquire a knowledge of the unusual flow field of the arcjet, such as the strong non-uniform radial temperature distribution at the constrictor and the ionization and recombination reactions taking place in the discharge section.

Numerical simulations can be a useful tool for understanding the thruster characteristics as it is easy to change the parameters, such as the nozzle geometry, the mass flow rate and the arc voltage. Recent development of a computer technology makes it possible to numerically simulate the flow field interacting with the electromagnetic field. Ao and Fujiwara carried out numerical studies of an MPD thruster using a simple cylindrical model, in which equilibrium ionization-recombination was assumed. They extended their work to a more realistic thruster model which had a pointed cathode. Chanty and Martinez-Sanchez conducted a numerical simulation of MPD flows inside and outside of the thruster. In the calculation they included Hall current effect. However, the electrical conductivity was kept constant in the entire flow field. Auweter-Kurtz et al. also conducted numerical simulations of MPD thrusters using quasi-one-dimensional and two-dimensional models. In all the above mentioned work, the inlet flow was supersonic and the temperature of the propellant gas at the inlet was of the orders of 1000 K to 10000 K.

However, the inlet flow of the actual arcjet thruster is subsonic. Moreover, in the experiment, the propellant is supplied to the thruster at room temperature and the degree of ionization of the propellant is zero. Ionization process occurs in the discharge section of the thruster. This suggests nonequilibrium ionization-recombination reaction should be included in the calculation. In the work described in Refs.5 and 6, we have developed a numerical simulation code of the arcjet thruster. In our computational code, the inlet flow could be treated as subsonic and, by adding the electron continuity equation to the flow equations, the flow was treated as an ionizational nonequilibrium flow. Consideration of the nonequilibrium ionization-recombination also makes it possible to estimate the frozen flow loss of the thruster. This was our preliminary work of the numerical study of the low power arcjet thruster.

Brophy et al. and Deininger et al. investigated the performance characteristics of a 30KW DC arcjet thruster through the use of a few nozzle profiles. The nozzles designed by an approximate analytical technique, i.e. an optimum inviscid expansion nozzle profile modified by the boundary-layer displacement thickness, were employed in the experiment. However, the flow conditions they used in the design procedure were not the flow field of the arcjet.
It is the object of the present research to develop a design technology for the numerical investigation of the arcjet performance based on our forementioned work that can simulate the unusual flow field of the arcjet. The phenomena taking place inside the thruster are complex and there still exist many problems to be studied. Therefore, it is quite difficult to quantitatively predict the arcjet performance with sufficient accuracy. However, using the reasonable assumptions, at least it seems possible to qualitatively investigate the performance. In the present study, we focused our attention on the qualitative investigation of the arcjet performance through the use of different nozzle geometries to increase the overall efficiency of the thruster by increasing nozzle efficiency.

This paper describes numerical simulation results and performance characteristics of the low power DC arcjet thruster for various mass flow rates. Moreover, the performance differences between the conical and bell-shaped nozzles, which are numerically and experimentally investigated, are also described. In this paper, the word "nozzle" and "nozzle section" mean the divergent part of the nozzle downstream of the constrictor.

2. GOVERNING EQUATIONS FOR AN ARCJET THRUSTER

The flow of the arcjet is described by the gasdynamic equations and the electromagnetic equations so that both equations need to be solved simultaneously. The following assumptions are introduced in the calculations:

1) The flow is axisymmetric.
2) The flow is inviscid. The thermal conduction and the radiation loss from the nozzle wall are neglected.
3) Argon gas is used as the propellant. Only single ionization is considered. Hence the propellant is composed of neutral argon atoms, singly ionized ions and free electrons.
4) The velocities and temperatures of all the species are in equilibrium.
5) An inflow to the thruster is subsonic and an outflow from the thruster is supersonic.
6) Ionization takes place due to inelastic collisions between neutral atoms and electrons, and recombination takes place due to three-body recombinations, i.e. the following reaction is considered:

\[ A + e^- \rightarrow A^+ + e^- + e^- \]

7) Hall current and ion slip are neglected.
8) Self-induced magnetic field is considered.

Under the above assumptions, gasdynamic equations for the flow field of the arcjet in conservation form are described as follows:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla p - j \times \mathbf{B} = 0 \]  \hspace{1cm} (2)

\[ \frac{\partial (\rho \mathbf{E})}{\partial t} + \nabla \cdot [(\rho \mathbf{v} + p) \mathbf{v}] - j \cdot \mathbf{E} = 0 \]  \hspace{1cm} (3)

\[ \frac{\partial n_a}{\partial t} + \nabla \cdot (n_a \mathbf{v}) = n_a \]  \hspace{1cm} (4)

where \( \rho \) is the density, \( t \) is the time, \( \mathbf{v} \) is the velocity, \( p \) is the pressure, \( j \) is the current density, \( \mathbf{B} \) is the magnetic flux density, \( \mathbf{E} \) is the electric field, \( n_a \) is the electron number density, and \( n_a \) is the net production rate of the electrons. Bold characters indicate vectors. \( e \) is the total energy per unit mass of the propellant and is given by

\[ e = \frac{(1 + \alpha) k_b T}{\gamma - 1} n_a + \frac{\alpha E_i}{m_a} + \frac{\mathbf{v} \cdot \mathbf{v}}{2} \]  \hspace{1cm} (5)

where \( T \) is the temperature, \( k_b \) is Boltzmann's constant, \( \gamma \) is the ratio of specific heats, \( \alpha \) is the degree of ionization, \( E_i \) is the ionization energy, and \( m_a \) is the mass of an atom. The equation of state for singly ionized gas is

\[ p = (1 + \alpha) \rho RT \]  \hspace{1cm} (6)

where \( R \) is the gas constant. The net production rate for the ionization-recombination reaction is given by

\[ n_a = K_1 n_a n_i - K_2 n_a \]  \hspace{1cm} (7)

where \( n_a \) is the atom number density, \( K_1 \) is the ionization rate constant, and \( K_2 \) is the recombination rate constant derived from the principle of detailed balancing and Saha equation. For \( K_1 \), Drawin's expression for argon is employed in the present calculation.

Electromagnetic equations for the arcjet are described as

\[ j = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \]  \hspace{1cm} (8)

\[ \nabla \times \mathbf{B} = \mu \mathbf{j} \]  \hspace{1cm} (9)

\[ \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \]  \hspace{1cm} (10)

where \( \sigma \) is the scalar electrical conductivity and \( \mu \) is the permeability. For the electrical conductivity, Spitzer's free path theory is applied. Gasdynamic equations (Eq.(1) thru (7)) are numerically solved by TVD-MacCormack scheme, and electromagnetic equations (Eq.(8) thru (10)) are solved by implicit FTCS (Forward Time Centered Space) scheme. Both equations are alternately solved until steady state solutions are obtained as asymptotic solutions. The details of the numerical calculation are described in Refs. 5 and 6.
3. RESULTS AND DISCUSSIONS

3.1 Arcjet Performance for Mass Flow Rate

The numerical simulation is conducted using the same nozzle geometry as "KU-MELCO I" laboratory prototype arcjet thruster\(^9\). Figure 1 shows the geometry of "KU-MELCO I". The thruster has the nozzle with a half cone angle of 15 degrees. The diameter of the constrictor is 2 mm and the diameter of the nozzle exit is 25 mm. The grids used in the calculation are depicted in Fig. 2. The computational region is divided into 80 x 12. The parameters used in the calculations, such as the mass flow rates and the arc current, were identical with the experimental conditions\(^9\). They are tabulated in Table 1.

Thruster characteristics for a few mass flow rates are investigated. In this investigation, the arc current is kept constant at 80 A. Figure 3 shows the thrust vs. mass flow rate. With an increase in the mass flow rate, the thrust is increased. However, the thrust is not proportional to the increase in the mass flow rate, which leads to a decrease in the specific impulse with an increase in the mass flow rate as shown in Fig. 4. The numerical results shown in Figs. 3 and 4 are qualitatively in satisfactory agreement with the experiment.

![Fig. 1 Geometry of a conical nozzle.](image1)

![Fig. 2 Grids for conical nozzle computation.](image2)

Table 1 Parameters for the calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow rate</td>
<td>0.11, 0.22, 0.33 g/s</td>
</tr>
<tr>
<td>Arc current</td>
<td>80 A</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>Inlet radial velocity</td>
<td>0.0 m/s</td>
</tr>
<tr>
<td>Inlet electron number density</td>
<td>0.0 particles/m(^3)</td>
</tr>
</tbody>
</table>

![Fig. 3 Thrust vs. mass flow rate.](image3)

![Fig. 4 Specific impulse vs. mass flow rate.](image4)
3.2 Ionization in the Nozzle

Figure 5 shows the flow fields for the mass flow rate of 0.11 g/s and the arc current of 80 A. In the figure, distribution of the temperature, electron number density and current density are shown by 256 different colors. The values of each property are divided into 256 between the maximum and minimum values in logarithmic scale. The color band used in the figure is similar to the colors of the rainbow, where red and purple represent, respectively, the maximum and minimum values. The maximum and minimum values are summarized in Table 2.

As can be seen from the figure, the distribution of the temperature (Fig. 5(a)) in the discharge section is different from that of the electron number density (Fig. 5(b)). There is a gradual increase in the temperature from the inlet, and then the growth rate begins to rise, increasing rapidly in the latter half of the discharge section. On the other hand, there is a dramatical increase in the electron number density due to the atom-electron collision in the vicinity of the inlet. In the latter half of the discharge section, there is a steady but gradual increase. These results indicate that the input electric energy is consumed with the ionization of the propellant in the first half of the discharge section, and with the heating of the propellant in the latter half of the discharge section.

The maximum electron number density is found at the anode surface slightly upstream of the constrictor, where the degree of ionization is $3.6 \times 10^{-2}$. This position is corresponds to the location of the maximum temperature and the maximum current density.

In the nozzle section the propellant is cooled due to gasdynamic expansion, since Joule heating is negligibly small there. As shown in Fig. 5(c), the arc current is concentrated in the discharge section. This result is similar to the current distribution of the actual thruster which is operated at low voltage mode. From the calculation, 99.99% of the total current concentrates in the discharge section. This means that Joule heating of the propellant occurs only in the discharge section.

The electron number density in the nozzle section decreases from the entrance to the exit of the nozzle. But, there are still many electrons in the exhaust flow. The degree of ionization at the entrance of the nozzle is $2.4 \times 10^{-2}$, while at the exit of the nozzle is $1.5 \times 10^{-2}$. These data suggest that the improvement in the efficiency can be achieved by transforming ionization energy to kinetic energy in the nozzle section.

3.3 Simplified Calculation Model

The numerical code which we have developed is useful in the investigation of the thruster performance, but it is not so efficient because it takes too much time to compute the performance. Therefore, the simplified calculation model to investigate the performance through the use of different nozzle geometries is proposed here from the considerations described hereafter.

1) The velocity of the propellant is supersonic at the entrance of the nozzle section. This means that the flow in the nozzle section does not affect the flow field upstream of the constrictor. Hence we may connect a different nozzle geometry with the present constrictor at the exit of the constrictor.

2) The nondimensional value of Lorentz's force $j \times B$ is of the orders of $10^{-4}$ to $10^{-2}$ in the discharge section and below $10^{-4}$ in the nozzle section, whereas the inhomogeneous term of Eq.(2) is of the orders of $10^{-2}$ to 10 in the entire flow region. In addition, Joule heating $j \cdot E$ and the inhomogeneous term of Eq.(3) are of the orders of $10^{-1}$ to 10 and $10^{-1}$ to $10^{3}$ in the discharge section, respectively. In the nozzle section, they are of the orders of below $10^{-4}$ and 10 to $10^{4}$, respectively. These results suggest that Lorentz's force and Joule heating may be neglected in the nozzle, in other words, electromagnetic equations need not be solved in the nozzle section.

3) The dominant phenomenon in the nozzle section is recombination, so that we should still include the nonequilibrium ionization-recombination reaction in the calculation.

These considerations enable us the following simplified but reasonable nozzle section calculation for various nozzle geometries without solving full equations, Eqs.(1) thru (10):

(1) Connect any shaped nozzle with the constrictor of the thruster shown in Fig. 1.

(2) Compute nozzle section flows by using Eqs.(1) thru

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Electron number density ($\text{particles/m}^3$)</th>
<th>Current density ($\text{A/m}^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>$1.88 \times 10^4$</td>
<td>$1.40 \times 10^{22}$</td>
</tr>
<tr>
<td>Minimum</td>
<td>$3.00 \times 10^2$</td>
<td>$1.00 \times 10^{-19}$</td>
</tr>
</tbody>
</table>

Table 2 Maximum and minimum values in Fig.5
Fig. 5 Computed arcjet thruster flow field.
Fig. 5 Computed arcjet thruster flow field.
where the flow properties at the constrictor exit obtained from the computation of Eqs. (1) thru (10) are employed as boundary conditions.

To justify the simplified calculation model, the computation is performed with the same nozzle geometry as used in the full calculation model. Table 3 shows the results of the calculation at the mass flow rate of 0.11 g/s. As expected, results for both models agree quite well. This indicates the simplified model is sufficiently useful to investigate the performance of different nozzle geometries.

Table 3 Results obtained from both models

<table>
<thead>
<tr>
<th>Model</th>
<th>Thrust (N)</th>
<th>Specific impulse (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>0.416</td>
<td>386</td>
</tr>
<tr>
<td>Simplified</td>
<td>0.414</td>
<td>384</td>
</tr>
</tbody>
</table>

3.4 Bell-Shaped Nozzle Performance

Figure 6 shows a flow velocity field of the thruster with the conical nozzle at the mass flow rate of 0.11 g/s and the arc current of 80 A. The flow velocity has a radial component at the nozzle exit. The radial velocity component does not contribute to the axial thrust. To increase the overall efficiency of the thruster, it is necessary to reduce the radial component of the velocity, i.e., the flow should be parallel to the axis at the exit of the nozzle. From this point of view, bell-shaped nozzle is supposed to be superior to the conical nozzle.

Figure 7 shows a geometry of the bell-shaped nozzle used in the present study. The nozzle length, constrictor diameter, and exit diameter are the same as the conical nozzle. The performance of the bell-shaped nozzle is calculated using the simplified calculation model. The computational grids with $65 \times 12$ are illustrated in Fig. 8.

The calculated velocity field for the bell-shaped nozzle is shown in Fig. 9. The mass flow rate is 0.11 g/s and other parameters are the same as in Table 1. As it is expected, the flow becomes almost parallel to the axis at the nozzle exit. The performance of the thrust and specific impulse for the mass flow rates are shown in Fig. 10. Circles correspond to the conical nozzle and squares correspond to the bell-shaped nozzle. The thrust is increased by 4% with the bell-shaped nozzle compared with the conical nozzle at the mass flow rate of 0.11 g/s.

The performance of the conical and bell-shaped nozzle were also experimentally investigated. The experimental procedure and test facility are the same as used in Ref. 10. The thrust measurements were conducted for
Fig. 10 Calculated performance.

Fig. 11 Measured performance.
the mass flow rates of 0.11 g/s to 0.55 g/s, using argon as the propellant. Arc current was kept constant at 80 A. Table 4 shows the arc voltage vs. mass flow rate characteristics for both nozzles. From these data, it is presumed that the arc voltage is relatively independent of the nozzle geometry. It may be mentioned that the behavior of the discharge is almost the same for both nozzles. This is expected because the geometries in the discharge section and constrictor are the same for both nozzles. As a consequence, the difference in performance only depends on the nozzle geometry.

Table 4 Arc voltage characteristics

<table>
<thead>
<tr>
<th>Mass flow rate (g/s)</th>
<th>Arc voltage (V)</th>
<th>Conical</th>
<th>Bell-shaped</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>19.6</td>
<td>18.8</td>
<td></td>
</tr>
<tr>
<td>0.22</td>
<td>20.0</td>
<td>20.1</td>
<td></td>
</tr>
<tr>
<td>0.33</td>
<td>21.8</td>
<td>22.3</td>
<td></td>
</tr>
<tr>
<td>0.44</td>
<td>23.3</td>
<td>24.1</td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>24.0</td>
<td>25.7</td>
<td></td>
</tr>
</tbody>
</table>

The measured performance of the thrust and specific impulse for the mass flow rates of 0.11 g/s to 0.55 g/s are given in Fig. 11 for both nozzle geometries. Solid marks correspond to the result for discharge-off and open marks correspond to the result for discharge-on. These figures indicate that both thrust and specific impulse for the bell-shaped nozzle are improved at both discharge-off and discharge-on. Approximately 6% increases in the thrust and the specific impulse are found from the data. The performance for both nozzles obtained from the numerical simulation (Fig. 10) and the experiment (Fig. 11) are qualitatively in good agreement.

4. CONCLUSIONS

The investigation on the low power DC arcjet thruster was made using the numerical simulation code, in which subsonic inlet conditions and nonequilibrium ionization-recombination reaction could be treated. The thrust vs. mass flow rate characteristics qualitatively agrees well with the experiment.

To examine the performance improvement by the use of different nozzle geometries, the simplified calculation model was utilized in the computation. We compared the performance of the thruster having a conical nozzle with that having a bell-shaped nozzle in the computational simulation and experiment. As a result, it is found that the bell-shaped nozzle is superior to the conical nozzle. The bell-shaped nozzle used in the present work is not an optimum one. Nevertheless, the possibility of the performance improvement by the use of the bell-shaped nozzle is revealed, and it may be mentioned that, as the design technology, the calculation model proposed here is useful.

REFERENCES