Onset and Erosion in Self-Field MPD Thrusters

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Abstract

A method is outlined for the determination of cathode temperatures and hence, evaporative erosion rates using the simplest level of coupling between the electrode and flowing MPD plasma. Given the mass flow rate, total current, and geometry, a previously developed quasi one-dimensional model is used to generate profiles of all relevant quantities in the axial direction. These are then used as boundary conditions for a boundary layer model, which in turn generates the necessary boundary conditions for the electrode-adjacent sheath and electrode thermal analysis. The steady state heat conduction equation is then solved numerically to estimate cathode temperatures. The coupling between the flowing plasma and the electrode is modelled via the boundary conditions so that solutions are tractable. A simple version of this technique is then used to compare model predictions with experimental observations of cathodes in steady state MPD thrusters operating at high current levels. The model correctly predicts the observed damage to the ZT-1 and DT-2 thrusters, and shows the inter-relations between back-EMF onset, erosion, and terminal voltage oscillations.

Nomenclature

- \( \alpha \) Ionization fraction
- \( \eta \) Viscosity of heavy particles
- \( \mu \) Permeability of free space
- \( \rho \) Mass density
- \( \sigma \) Electrical Conductivity
- \( \prime \) Superscript indicating evaluation
- \( \text{at} \) the sonic point
- \( \text{i} \) Subscript denoting inlet value

1. Introduction:
The behavior of self-field MPD thrusters at high values of total current for a given propellant mass flow rate and thruster geometry, has been a subject of considerable interest. Particularly, the phenomenon encountered at high currents known as Onset has received much attention due to its significant influence on the performance of these devices. Onset is a term used to denote collectively, increased erosion of thruster components and terminal voltage oscillations that are seen to occur during operation at high current levels[1,2]. The focus of this paper is to show that the conditions leading to voltage oscillations and electrode erosion are indeed linked, and to provide...
a basis for quantitative prediction of erosion rates at and below the Onset limit.

Several theories have been successful at predicting an Onset limit[3-6]. Schrade et. al. have attempted further to explain the increased erosion observed at Onset[4]. They have analyzed the behavior and stability limits of a single current carrying channel, and have predicted the critical current at which such a channel becomes unstable subsequently forming other neighboring spots (molten, concentrated regions of high current density on an electrode). Although the erosion rate from a single spot can be predicted with such a model, generalization to an overall erosion rate from an electrode surface is an insurmountable task.

Recently, it has been shown that cathode and anode behavior are strongly controlled by a combination of variables, notably the local current and number densities[7,8]. A thermal runaway mechanism triggered by bombardment of energetic plasma electrons shown to operate at the cathode in regions where the local current density was low and the local number density was high[8]. The same mechanism was shown to be operative at the anode as well, but in regions of high current density and low number density[7]. In this paper, it is shown that these conditions are consistent with the Back-EMF theory of Onset, and that this theory is capable of explaining oscillations as well as increased erosion at Onset. Also, a methodology is outlined whereby evaporative erosion rates can be predicted purely as a function of the global parameters such as total mass flow rate, total thruster current, and thruster geometry. Such an approach is deemed a useful tool to the designer.

This paper is organized as follows. The Back-EMF Onset theory is reviewed first in the following section, followed by a detailed discussion of a simple model that couples the MPD plasma flow and electrode processes. The numerical results from this model are presented, followed by a discussion of the relationship between Back-EMF Onset and erosion, and Back-EMF Onset and oscillations, respectively. Finally, the findings of the present study are summarized and conclusions drawn.

II. Back-EMF Onset Revisited:
The idea of a high Back-EMF being responsible for Onset phenomena was first discussed within the context of a one-dimensional, steady state, frozen, fully ionized flow[6]. Although restrictive in its assumptions, this simple model revealed several important facts. First, MPD flow was parametrized by the Magnetic Force Number $S^* = B^2 / \mu_0 a^*$, where the superscript $*$ denotes quantities evaluated at the magnetogasdynamic sonic or choking point. Second, there was shown to be an upper limit on $S^*$ for sustaining supersonic flow in the thruster. Finally, $S^*$ was shown to be related to the Onset parameter $J_c^2 / m$, utilized by experimentalists to describe Onset. A limit on $S^*$ thus translated to a limit on $J_c^2 / m$, and this limit was derived to be:

$$J_c^2 / m \leq \frac{8.52 W a^*}{H \mu_0 a^*}$$  (1)

Equation (1) showed excellent agreement with the data of Malliaris et. al. on quasi-steady MPD thrusters[1]. The measured dependence of $J_c^2 / m$ on the propellant atomic mass as well as on the thruster geometry are correctly predicted by (1)[6]. Subsequently, the assumption of frozen, fully ionized flow was removed[9]. This meant that closed form analytical solutions could not be obtained for MPD channel flow. However, numerical solutions of the governing equations showed an upper limit on $S^*$ analogous to that given by equation (1). This upper limit was interpreted to be the Onset limit, and showed agreement with the experimentally measured Onset limits in quasi-steady, straight coaxial thrusters[10]. Physically, Back-EMF Onset is caused by a conflict between the electric field necessary to draw all the applied current (as per Ohm's law)
and that amount of the electric field necessary to sustain supersonic flow in the thruster (as dictated by the magnetogasdynamic sonic or choking condition). Both constraints on E could be met at low currents, but beyond a critical value of the current (or $S^*$), both requirements could not be simultaneously satisfied at steady state. For a more recent discussion of this theory, see ref.[11].

Although this Back-EMF theory can quite satisfactorily predict the limits of steady operation of self-field thrusters, electrode phenomena must be included to study erosion processes in select regions of the thruster. This is because electrode processes together with the plasma-electrode interactions influence erosion. It is to these that we now focus our attention.

III. A Simple Model:
The structure of MPD flow has been examined previously within the context of a one-temperature core flow, with a two-temperature boundary layer, including finite-rate ionization and recombination[12]. These models have revealed that the MPD flow is highly viscous with entry lengths on the order of a few centimeters[12]. The primary reason for this high viscosity is due to the effects of decreasing ionization fraction on the viscosity[12]:

As one proceeds from the flow centerline toward the electrodes (walls), the ionization fraction and charged particle number density drop due to wall-driven recombination. The consequences of this can be seen from equation (2) when $\alpha$ varies from a number near 1 towards a value near 0. The dominant cross section for momentum transfer changes from that due to Coulomb interactions in the core flow, to that due to ion-neutral collisions in the boundary layer. Since these two cross sections typically differ by orders of magnitude, the viscosity gains increasing importance in the near-wall regions of the flow[12]. It must be pointed out that several authors have earlier[13,14] and more recently[15] included two-temperature effects, but primarily in the axial direction. This has an effect on the boundary layer but is relatively weak when compared to the effect of changing $\alpha$, since $\eta_{H}$ varies as the square root of $T_{H}$ and thus the boundary layer thickness then only varies as $T_{H}^{1/4}$.

With electrode phenomena in mind, let us now consider the following model. Under the assumption of axisymmetry, the steady state temperature distribution in the cathode satisfies the heat conduction equation, which in polar coordinates is:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r k_c \frac{\partial}{\partial r} (k_c T) \right) + \frac{1}{\sigma} \frac{\partial}{\partial \theta} \left( \frac{\partial}{\partial \theta} \sigma \right) = 0$$

(3)

where $\sigma$ is the current density through the cathode, $\sigma$ is the electrical conductivity, and $k_c$ is the thermal conductivity. The boundary conditions are:

$$\frac{\partial T}{\partial r} \bigg|_{r=0} = -q_{Surface}$$

(4)

$$-k_c \frac{\partial T}{\partial r} \bigg|_{r=rc} = q_{Surface}$$

(5)

$$-k_c \frac{\partial T}{\partial \theta} \bigg|_{\theta=0} = -q_{Base} \quad \text{or} \quad T(x=0,r) = T_{Base}$$

(6)

$$-k_c \frac{\partial T}{\partial \theta} \bigg|_{\theta=L} = q_{End}$$

(7)

where $q_{Surface}$ is taken to be positive when flowing into the surface at $r=rc$, $q_{Base}$ is positive when flowing out of the surface at $x=0$, and $q_{End}$ is positive when flowing out of the surface at $x=L$. The condition (4) is a symmetry condition valid only for a solid cathode. Since the centerline, $r=0$, is a line of symmetry, it is an adiabat. With these considerations, the maximum temperature should be expected to occur somewhere along the centerline, i.e along $r=0$. Because of the Neumann boundary conditions (4) through (7), there is a constraint that must be met in order for a steady state to exist:

$$\int_0^L q_{Surface} C_d dx = \int_0^{rc} \int_0^{r_c} \frac{r^2}{\sigma} dr dx$$

$$= \int_0^{rc} q_{Base} r dr + \int_0^{r_c} q_{End} r dr$$

(8)

Equation (8) is an overall energy balance on the cathode and states that the heat removed from the cathode base, $q_{Base}$, must satisfy it in order to maintain a steady state.
The surface boundary at \( r = r_c \) couples the solid cathode to the adjacent plasma. The surface energy flux consists of ion and electron bombardment, surface electron emission, and radiative energy exchange with the anode. This surface and sheath model has been discussed in a previous paper\[8\]. \( q_{\text{surface}} \) is given by:

\[
q_{\text{surface}}(r = r_c, x, T) = \left( k_f V_c - \phi \right) + \frac{1}{2} f \left( \phi + \frac{2kT}{e} \right) \]

\[
- \frac{\sigma_{SB} T^4}{2} \left( T^4 - T_{\text{anode}}^4 \right)
\]

where the cathode sheath voltage drop \( V_c \) is determined from overall current conservation\[8\]. The plasma electron current density \( j_e \) is also dependent on \( V_c \) and can be a dominant source of heating under MPD conditions\[7,8\]. As \( V_c \) decreases, \( j_e \) increases exponentially leading to increased surface heating. The cathode tip is assumed to radiate to cold space, so that:

\[
q_{\text{end}}(r) = \varepsilon \sigma_{SB} T^4(r, x = L)
\]

A method can now be outlined to determine cathode surface temperatures. A similar approach can be used to determine anode temperatures as well. As in any design or experimental situation, the total current, propellant mass flow rate, and geometry are assumed to be specified. Given these quantities, a suitable flow model may be used to provide axial profiles of the plasma temperature, ionization fraction, current density, and velocity along the length of the channel\[9,16\]. Next, a two-temperature boundary layer model can be used to predict charged particle number densities, current densities, electron and heavy particle temperatures outside the sheath edge\[12\]. An appropriate sheath model can then be utilized to calculate the ion number densities, electron number densities, sheath voltage drop, electron temperatures, and heavy particle temperatures along the electrode surface\[8\]. These provide the necessary information to determine the right hand side of equation (9). The system (3) through (10) is then solved numerically to calculate the temperature profiles within the electrodes as well as on the surface. Knowing the electrode surface temperature, evaporative erosion rates can be determined from vapor pressure data. In this manner, plasma discharge and electrode processes are coupled. It must be pointed out at the outset that the method just outlined is only and approximate solution procedure for considering the fully coupled plasma flow and electrode processes. The fully coupled problem is extremely tedious to solve, even numerically.

In the following section, the approximate method just outlined is applied to study the ZT-1 steady state thruster at the University of Stuttgart\[17\].

**IV. Numerical Results**

Recently, there have been reports of tests conducted at the University of Stuttgart on steady-state, straight, coaxial, self-field MPD thrusters\[17\]. The results of these tests showed severe cathode damage in the middle as opposed to the tip (see Figures 1 and 2). It appears that the cathodes melted and exploded from within suggesting that the internal temperatures far exceeded the surface temperatures at this location. The authors of ref. [17] report that they believe the damage to be caused by ohmic heating in selected regions of the thoriated tungsten cathode. However, this phenomenon can and will occur at Onset according to our theory, and is intimately linked to the recently reported thermal runaway mechanism\[7,8\].

The method discussed in the previous section can be applied to the ZT-1 and DT-2 thrusters used in the Stuttgart experiments. Since specific details were not reported for the DT-2 thruster, we focus on the ZT-1 thruster.

This thruster had a cathode approximately 18 cm. long and 1.8 cm diameter. The thruster consisted of a straight constant area channel 15 cm. long, with three segmented anode sections accounting for the latter 9 cm. of the channel length. The argon propellant was introduced at an axial location of 15 cm. before the exit, at a
rate of 2g/s. This thruster according to ref.[17], was operated at steady state up to about 8000 A. This information regarding the geometry, propellant mass flow rate, and total current, provides enough information to obtain electrode temperature distributions using the method of section III. An additional simplification is made here. The boundary layer analysis is omitted so that the core flow quasi one-dimensional axial profiles are directly applied as boundary conditions to the electrode thermal analysis.

Given the total current, channel geometry, and propellant mass flow rate, the governing equations of quasi one-dimensional MPD flow including ionization and recombination, are solved using a method previously reported[9]. This analysis yields profiles of all the relevant quantities. Figures 3 - 8 show the profiles of B, u, T, α, the back-EMF u_B, and j as a function of distance for the supersonic section (approximately latter 12 cm.) of the channel. These have been computed for a total current of 8030 A, and mass flow of 2 g/s. These profiles then provide the necessary boundary conditions for solving the system of equations (3) through (10), for the cathode temperature distributions. Computations using the method outlined in section III were carried out on a 10x10 uniform grid. Finer grids (50x50 and 100x100) resulted in quantitatively different temperatures, but trends were identical. Furthermore, the significant results obtained (see Fig. 9) were independent of the grid size. As can be seen from Fig. 9, the maximum temperature occurs in the middle of the thruster and not at the cathode tip. Also, the internal temperatures are higher than the surface temperatures, due to significant Joule heating. The surface, once molten due to the thermal runaway caused by excessive electron bombardment[7,8], evaporates. The interior of the cathode, on the other hand once molten continues to change phase and build up pressure within the material. This explains the observed damage to the ZT-1 and DT-2 Stuttgart thrusters.

V. Back-EMF Onset and Erosion

Let us review the phenomena occuring as the total current is increased for a fixed mass flow rate, and a given geometry:

1. The back-EMF climbs from a small value at the inlet, reaches a maximum somewhere in the middle of the channel, and then decreases toward the exit (see Fig. 7).

2. This causes the net current density to reach a minimum in the middle of the channel, while being large near exit region (see Fig. 8).

3. Earlier cathode and anode sheath theories [7,8] have shown that the diffuse mode behavior becomes unstable (possibly transitioning into the spot mode) at the cathode at low current densities, and at the anode at high current densities.

Thus, the center of the cathode and the exit regions of the anode are prime candidates for regions of significant erosion.

VI. Back-EMF Onset and Voltage Oscillations

It can be shown from current conservation, that the sheath voltage drop follows the same trend as the net current density[8]. The regions of low current density at the cathode then have small sheath voltage drops, thereby making the thermal runaway more likely. However, the exit region of the cathode has higher sheath voltage drops. It is therefore clear, that as the total current is increased, the sheath voltage drop in the middle of the cathode continuously decreases, while the sheath voltage drops near the exit region increases. When the sheath voltage drop reaches a value comparable with the energy of the first excited state of the propellant atoms, the electron energy distribution function is modified near the electrode. Two principal groups of electrons then result. These are the "low" energy plasma electrons, and the energetic
electrons emitted by the electrode and which are accelerated through the cathode sheath. Interaction between these two groups of electrons can then lead to electric field or voltage oscillations arising from the well known beam instability[18].

VII. Summary

A method has been outlined that allows determination of the MPD flow structure and electrode temperature distributions, given the global parameters, mass flow rate, Total current, and geometry. This theory extends previous work on back-EMF Onset, to include erosion. The general predictions of this theory appear to be in agreement with experimental observations at Stuttgart[17].

Acknowledgements

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References


Fig. 1: Schematic of blown cathode from DT-2 thruster. For photograph, see ref.[17].

Fig. 2: Schematic of blown cathode from ZT-1 thruster. For photograph, see ref.[17].

Fig. 3: The calculated values of magnetic induction are shown here versus distance along the cathode, for the ZT-1 thruster.

Fig. 4: The calculated values of velocity are shown here versus distance along the cathode, for the ZT-1 thruster.

Fig. 5: The calculated values of Temperature are shown here versus distance along the cathode, for the ZT-1 thruster.

Fig. 6: The calculated values of ionization fraction are shown here versus distance along the cathode, for the ZT-1 thruster.
Fig. 7: The calculated values of the back-EMF (μB) are shown here versus distance along the cathode, for the ZT-1 thruster.

Fig. 8: The calculated values of net current density are shown here versus distance along the cathode, for the ZT-1 thruster.

Fig. 9: The calculated centerline and surface temperatures are shown here for the ZT-1 thruster. For reference, the melting temperature of tungsten (3680 K) has been plotted as a horizontal line. Note that the temperatures in the middle region are in excess of the melting temperature, and that centerline temperatures are always larger than the surface temperature.