PLASMA ACCELERATION PROCESS IN A HALL-CURRENT THRUSTER

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Abstract

Numerical analyses and experiments of a Hall-current thruster have been conducted not only to improve thruster performance but also to understand the plasma acceleration processes and its relationship with the thruster performance. Common to both models is the assumption that the ions produced are electrostatically accelerated to produce the thrust, while the electrons flow to the anode by Bohm diffusion. The thruster performance and plasma properties in the acceleration channel were calculated and compared with the experimental results.

Nomenclature

(SI units unless noted otherwise)

B = magnetic field strength
D = electron diffusion coefficient
Dg = Bohm diffusion coefficient
dC = classical diffusion coefficient
t = thermal diffusion coefficient
e = energy distribution function
 grav = gravitational acceleration
Ia = acceleration current
Ib = backstreaming electron current
Ie = ion production current
Isp = specific impulse
Lm = ion loss rate per unit surface
Lr = ion loss rate per unit volume
m = propellant mass flow rate
m = electron mass
n = ion mass
ne = plasma density
nn = neutral atom density
Qm = ion production rate per unit volume
S = cross-sectional area of the channel
t = time
T = thrust
e = electron temperature, eV
v = electron velocity
vi = ion velocity
Vn = neutral atom velocity
V = ion beam voltage
Ve = average ion beam energy
x = axial position
a = ion loss fraction
β = coefficient of ion production
γ = ratio of excitation energy to ionization energy
'F = electron flux
'F = ion flux
δ = secondary-electron emission yield
εi = coefficient of energy input by electric field
ηa = acceleration efficiency
ηI = ion beam energy efficiency
ηT = thrust efficiency
μu = propellant utilization
ν = electron collision frequency
σo = ionization cross-section
φ = space potential
φa = anode potential
φg = sheath potential drop

Introduction

A Hall-current thruster has a coaxial channel in which radial magnetic fields are applied to maintain a high voltage between anode and cathode neutralizer, and ions generated in the channel are accelerated downstream to produce thrust. Since the channel is filled with quasi-neutral plasma, there is no space charge limited current and hence this type of thruster can offer much higher thrust density than gridded ion thrusters. In addition, Hall-current thrusters are generally operated at moderate current level with high voltage, so that electrodes are hardly eroded by high arc currents nor spattered by high-energetic ions, and hence their life time may be longer than that of arcjets and gridded ion thrusters.

In our previous work on the Hall-current thruster, the thrust efficiency could be raised by changing the geometric design; shortening the channel length together with arranging the magnetic field lines to be perpendicular to the axis. However, the plasma acceleration processes have not been clarified enough due to the complicated plasma phenomena in the channel where ion production and plasma acceleration occur simultaneously.

The objective of this study is to understand physical phenomena in plasma acceleration processes and to develop a high performance thruster through numerical analyses and experiments.

Experiment

Apparatus and Experimental Procedure

Three types of thrusters, as illustrated in Fig. 1, have been designed and tested. The thrusters consist of anode, cathode neutralizer, and acceleration channel in which radial magnetic fields are applied. In Type I, the applied field whose strength is up to 0.02 T is induced by the magnetic circuit formed by three solenoidal coils and magnetic pole pieces. Two thin ceramic cylinders are inserted in the acceleration channel to prevent the inner and outer metal walls from short-circuiting plasmas. The cathode neutralizer, which is a filament-type and is located downstream of the channel, supplies electrons not only to ionize the propellant gases in the channel, but also to neutralize ion beam extracted out downstream. Type II has a shorter acceleration channel, and produces a higher magnetic field (up to 0.1 T) than Type I. In the thruster Type III, a hollow cathode is mounted on the axis of the thruster in place of the filament cathode, in order to be erodable for long operation. To measure thrust, the thrusters were mounted on a pendulum-type thrust stand. Extracted ion beam current was measured with an ion beam collector located 10 cm downstream of the thrusters.

By thrust measurement, one can calculate specific impulse, Isp, and thrust efficiency, ηT, as given by

$$I_{sp} = \frac{I}{\rho g}$$

and

(1)

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CATHODE GAS INLET

BAFFLE

SOLENOIDAL COIL

ANODE

POLE PIECE

GAS INLET

IRON

CERAMIC

COPPER

Fig. 1 Hall-current thrusters designed and tested.

ANODE

POLE PIECE

SOLENOIDAL COIL

ACCELERATION CHANNEL

HOLLOW CATHODE

KEEPER

IRON

CERAMIC

COPPER

respectively. Here, \( T \) is the thrust, \( \dot{m} \) the propellant mass flow rate, \( g \) the gravitational acceleration, and \( I_a \) and \( V \) the acceleration current and voltage, respectively. Neither the excitation power for solenoidal coils nor the heating power of the cathode filament is taken into account on the calculation of thrust efficiency because, in the practical case, solenoidal coils can be replaced by permanent magnets, and cathode filaments by hollow cathodes.

To investigate thruster performance characteristics, the following internal efficiencies are introduced and defined by the equations

\[
\eta_u = \frac{\dot{m}_{\text{propellant}}}{\dot{m}_{\text{mass flow rate}}}\]

\[
\eta_a = \frac{I_a}{I}\]

\[
\eta_E = \frac{V}{\dot{m}}\]

where \( \eta_u \) is the propellant utilization, \( \eta_a \), the acceleration efficiency, \( \eta_E \), the beam energy efficiency, \( M \), ion mass, \( e \), electronic charge, \( I_a \), the ion beam current and \( V \), the average ion beam energy calculated from the ion energy distribution, and is given by

\[
V = \frac{\int f(V) dV}{\int f(V) dV^2}\]

where, \( f(V) \) is the distribution function of beam energy \( V \). Assuming that all ions produced are singly charged, the thrust can be expressed as

\[
T = \frac{\dot{m}_p}{2Mv_e}e\]

Substituting eqs. (3)-(7) into Eq. (2), the thrust efficiency becomes

\[
\eta_T = \eta_u \eta_a \eta_E \]

Experimental Results

Typical examples of thruster performance and internal efficiencies of Type I and Type II are listed in Table 1. When xenon is used as the propellant, the propellant utilization is about twice as high as that obtained with argon. This seems reasonable because xenon gas is more easily ionized than argon gas. The acceleration efficiency of Type II is about 50% higher than that of Type I. The relation between the acceleration efficiency and the thruster geometry will be discussed later. As the result, the maximum thrust efficiency of 32% was obtained with the xenon propellant. From the table, it is found that the acceleration efficiency is the predominant efficiency that determines thrust efficiency, particularly for the case of the xenon propellant.

Table 1. Thruster performance and internal efficiencies

<table>
<thead>
<tr>
<th>Type</th>
<th>gas</th>
<th>( \eta_p(%) )</th>
<th>( I_p(\text{sec}) )</th>
<th>( \eta_a(%) )</th>
<th>( \eta_u(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Ar</td>
<td>7</td>
<td>1300</td>
<td>27</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Xe</td>
<td>15</td>
<td>1350</td>
<td>28</td>
<td>80</td>
</tr>
<tr>
<td>II</td>
<td>Ar</td>
<td>15</td>
<td>1460</td>
<td>43</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>Xe</td>
<td>32</td>
<td>1370</td>
<td>40</td>
<td>93</td>
</tr>
</tbody>
</table>
In order to evaluate the acceleration efficiency, a simple plasma model is presented here. In the acceleration channel, ions are produced by collisions of Maxwellian electrons with neutral atoms. Ion production current in the channel, \( I_p \), is given by

\[
I_p = \beta I_e
\]  

(9)

where \( I_e \) is electron current backstreaming from the cathode neutralizer to the anode and the coefficient \( \beta \) is a quantity that expresses how efficiently ions are produced.

As the volume recombination can be neglected as small, the ions produced in the channel are either lost to the inner and outer walls or exhausted downstream of the channel. Ion beam current is expressed as

\[
I_b = (1-a)I_p
\]  

(10)

where \( a \) denotes a fraction of ions produced that are lost to the wall.

The sum of the ion beam current and the backstreaming electron current is equal to the acceleration current, as

\[
I_a = I_b + I_e
\]  

(11)

Substituting Eqs. (9)-(11) into Eq. (4), the acceleration efficiency can be expressed as in the form

\[
\eta_a = \frac{\beta(1-a)}{\beta(1-a)+1}
\]  

(12)

From this equation, it is found that, to obtain a high acceleration efficiency, ion loss fraction, \( a \), should be as low as possible, together with an effort of efficient ion production. Besides, \( \beta \) is considered to be influenced by the operating parameters such as propellant mass flow rate and acceleration voltage, while \( a \) is determined from the geometric design parameters as the aspect ratio of the acceleration channel and the magnetic field configuration.

The plasma properties, such as space potential, plasma density and electron temperature were measured by scanning Langmuir probes in the radial and axial directions. From the distributions of these properties, one can calculate the ion production rate and the ion flux toward the channel exit and walls, assuming that the ions produced in the channel are accelerated only by electric fields.

Figure 2 shows the magnetic field configuration, the plasma property distributions and the ion extraction domain in the case of Type I. As seen in the figure, the shape of equipotential lines is similar to that of the magnetic field lines. This is because electrons move easily along the magnetic field line and eliminate the potential difference along it. Although the electric fields have only axial components in most of the region, they are distorted in the radial direction in the upstream region from the anode to the middle of the channel. Because of this distortion, most of the ions are lost to the wall. Figure 2 shows the magnetic field configuration, the plasma property distributions and the ion extraction domain in the case of Type I. As seen in the figure, the shape of equipotential lines is similar to that of the magnetic field lines. This is because electrons move easily along the magnetic field line and eliminate the potential difference along it. Although the electric fields have only axial components in most of the region, they are distorted in the radial direction in the upstream region from the anode to the middle of the channel. Because of this distortion, most of the ions are lost to the wall. Figure 2 shows the magnetic field configuration, the plasma property distributions and the ion extraction domain in the case of Type I. As seen in the figure, the shape of equipotential lines is similar to that of the magnetic field lines. This is because electrons move easily along the magnetic field line and eliminate the potential difference along it. Although the electric fields have only axial components in most of the region, they are distorted in the radial direction in the upstream region from the anode to the middle of the channel.
produced there are lost to the wall, resulting in a high ion loss fraction of 0.7.

Figure 3 shows the result of the thruster Type II. The ion loss fraction could be reduced to 0.4. This is mainly due to the better arrangement of magnetic field lines with the shorter channel length. Both the electron temperature and the plasma density are at a maximum in the center of the channel, and decrease toward the channel walls. As the result, the ion production rate peaks in the middle of the channel.

Figure 4 shows a comparison of acceleration efficiency of the thruster Type III between with the hollow cathode and with the filament cathode. As seen in the figure, there seems no significant difference of the efficiency of the thruster operated between with the hollow cathode and with the filament cathode. From the result, it may be concluded that hollow cathodes can be used without any degradation in performance and be appropriate for long operation tests.

One-Dimensional Plasma Analysis

Governing Equations

In this section, both the plasma properties and thruster performance are numerically analyzed using a one-dimensional plasma model which comprises equations of mass conservation, energy conservation, and electron diffusion.

The following assumptions are made in this analysis: 1) all the plasma properties vary only in the axial direction, 2) electron axial flow is described by the Bohm diffusion including the effects of potential and electron temperature gradients, and 3) the ion loss rate per unit volume L is proportional to the ion production rate per unit volume Q and is given by

\[ L = \alpha Q \]  

where the ion loss fraction \( \alpha \) is assumed constant and is given as a known parameter.

The mass conservation equations for electrons, ions, and neutral atoms are given by

\[ \frac{dN}{dx} = Q - L \]
\[ \frac{dR}{dx} = Q - S (15) \]

and

\[ \Gamma_1 = n_1 n - \frac{v}{\eta} \text{MS} \]

respectively. \( \Gamma_1 \) and \( \Gamma_e \) are the ion and electron flux, both of which are taken positive in the x-direction. \( S \) is the cross-sectional area of the channel and \( v \) is the neutral atom velocity assumed constant in the channel. When ions are produced by electron-neutral ionization collisions, the ion production rate \( Q \) is given by

\[ Q = n_1 n <\phi> \]

where \( n_1 \) and \( n_\text{en} \) are the plasma and neutral atom densities, and the ionization rate factor \( <\phi> \) is obtained by averaging the product of ionization collision cross-section \( \sigma \) and the electron velocity \( v \) over velocity space. From the conservation equations for ions and electrons, one obtains the equation

\[ -\frac{e}{\eta} + \frac{e}{\eta} \frac{T_\text{e}}{S} = \frac{1}{8} \]

where \( T_\text{e} \) is the acceleration current. The energy conservation equation for electrons assumed in a Maxwellian distribution is

\[ \frac{5}{2} \frac{dT_\text{e}}{dx} = \frac{T_\text{e}}{\eta} \frac{d\phi}{dx} - (1+r/\eta)Q_1 \]

\[ = 2 \frac{L(\frac{1}{8} T_\text{e} + \frac{1}{6} \phi)}{T_\text{e}} \]

where \( T_\text{e} \) and \( \phi \) are electron temperature and space potential, respectively. The right side terms represent the energy input by the electric fields, the energy expended on inelastic collisions, and the energy associated with the electron loss to the wall surface. When the wall surface is at a floating potential the potential drop at the sheath \( \phi_w \) is given by

\[ \phi_w = T_\text{e} \ln \left( \frac{(1-\epsilon)}{\epsilon} \right) \exp \left( \frac{1/2}{\eta} \sqrt{BM} \right) \]

where \( m \) is the electron mass and \( \epsilon \) is the secondary-electron emission yield. The energy conservation equation for ions accelerated without elastic collisions is

\[ \frac{1}{2} M v_1^2 = e \phi \]

where \( v_1 \) is the ion velocity and \( \phi \) is the potential difference from its birthplace. Using Eqs. (15) and (21), the plasma density at a position \( x \) can be expressed as

\[ n_\text{en} = \int_{-\infty}^{x} \frac{Q(x') - L(x')}{2M} \frac{d\phi}{\phi - \phi_2} \frac{d\phi_2}{\phi_2 - \phi} + \frac{n_1(0) v_1(0)}{\sqrt{2M (\phi(0) - \phi_1(0))}} \]

The first term comes from the ionization in the channel, and the second term is due to the ion flux from the anode surface with an ion velocity of \( v_1(0) \). The electron diffusion equation is

\[ \frac{d\phi}{dx} = -D \frac{d\phi}{dx} + \mu \frac{d\phi}{dx} - D \frac{\phi(0) - \phi_1(0)}{v_1(0)^2} \]

The diffusion coefficient \( D \), mobility \( \mu \), and thermal diffusion coefficient \( D_T \) are given by

\[ D = T_\text{e} \]

\[ \mu = 16eB \]

\[ D_T = T_\text{e} \]

respectively. When the propellant mass flow rate \( \dot{m} \), acceleration current \( I_a \), and magnetic field strength \( B \) are given as input parameters, the Finite Difference Method was used to compute the plasma properties such as electron and ion flux, plasma density, electron temperature, space potential, and neutral atom density. Specific impulse \( I \), and internal efficiencies are then calculated from Eqs. (1), (3), (4), and (5). Thrust efficiency \( \eta \) is obtained as the product of these internal efficiencies.

Calculation Results

Figures 5 and 6 show respectively the calculated results and the experimental results of the thrust efficiency as a function of specific impulse for two different propellants, xenon and argon, when the magnetic field is 0.1 T, and the channel length is 8 mm. For xenon as shown in Fig. 5(a), the calculated \( \eta - I \) curves are almost linear until the propellant utilization reaches approximately 100%. In the linear segment, the thrust efficiency increases with specific impulse, which can be explained by an increase in propellant utilization as the acceleration current increases, but at the same time both the acceleration efficiency and the beam energy efficiency are relatively insensitive to acceleration current. For example, in case the ion loss fraction, \( \alpha = 0.4 \), the propellant utilization increases from 38% to 95% as the specific impulse rises from 500 sec to 1200 sec. After this, there is almost no increase in thrust efficiency, until it finally decreases around \( I = 1700 \text{ sec} \). This indicates complete ionization and corresponds to the peaks and decreases of the thrust efficiency in the experimental results as shown in Fig. 6(a).

For argon propellant as shown in Figs. 5(b) and 6(b), the thrust efficiency increases with the specific impulse but it is much less than that of xenon propellant, because of low propellant utilization. Both Xe and Ar calculation results qualitatively agree with the experimental results and it is indicated that this one-dimensional model is useful for the estimation of the thruster performance. Figure 5 also shows that a reduction of the ion loss fraction improves the thrust efficiency. This is caused by increases in all of the internal efficiencies. As a changes from 0.7 to 0.1 at \( I = 1000 \text{ sec} \), \( \eta \) increases from 41 to 56%, \( \eta_1 \) from 59 to 59%, and \( \eta_2 \) from 56 to 93%. This indicates that the ion loss fraction has a great influence on the thrust performance and that it should be reduced as much as possible in order to develop a high performance Hall-current thruster.
Two-Dimensional Ion Flow Analysis

Governing Equations

The experimental results shown in this study suggest that the ion loss fraction changes with the magnetic field configuration and with the channel geometry. Therefore, a two-dimensional, numerical model for the ion flow in the acceleration channel is necessary to be developed, to compute the effect of the thruster configuration on the ion loss fraction.

The assumptions used in this analysis are almost the same as those of the one-dimensional analysis except that the distribution of ion production rate is given as an input parameter, thereby no energy conservation equation is employed.

Ions produced in the channel are accelerated in both axial and radial directions owing to the two-dimensional electric field configuration. The kinetic equation for ions is given by

\[
\frac{d\mathbf{v}}{dt} = -e\mathbf{E}
\]  

where \(\mathbf{v}\) denotes the ion velocity. The mass conservation equation for ions is given by

\[
\frac{d\mathbf{v}}{dt} = -e\mathbf{E}
\]  

where \(Q\) is the ion production rate per unit volume and is given as an input parameter. As the volume recombination is negligibly small in the channel, all of the ions produced there are extracted as an ion beam or are lost on the wall surface. Using Eq. (27), one can compute the ion beam trajectories, and also obtain both the plasma density and the ion loss rate by combining Eq. (27) with Eq. (28).

\[
\mathbf{v} \cdot (\mathbf{n} \times \mathbf{v}) = Q
\]  

Electron diffusion equation is given by

\[
\Gamma_e = [\mu]n_e \mathbf{v}_e - [D]n_e
\]  

where, the vector \(\Gamma_e\) is the electron flux, and \(\mu\) and \(D\) are the electron mobility and the diffusion coefficient, respectively. The mobility and the diffusion coefficient are expressed in tensors because these coefficients are anisotropic in the presence of magnetic field. The diffusion coefficient parallel to magnetic field lines is assumed to be expressed by the classical diffusion coefficient.
and the diffusion coefficient perpendicular to magnetic field lines is also expressed by the Bohm diffusion coefficient

$$D_e = \frac{T_e}{\mu_e}$$  \hspace{1cm} (30)$$

where $B$ is the magnetic field strength, which is calculated prior to this analysis as a solution to the magnetostatic equations and boundary conditions. The mobilities parallel and perpendicular to the field lines are derived from the Einstein relation and are given by

$$\mu_e = \frac{1}{T_e} D_e, \quad \mu_\perp = \frac{1}{T_e} D_\perp$$  \hspace{1cm} (31)$$

respectively. The mass conservation equation for electrons is given by

$$\nabla \cdot \mathbf{J_e} = 0$$  \hspace{1cm} (33)$$

When the channel wall surface is non-conducting, the electron flux to the wall surface should be equal to the ion loss flux. Therefore, the electron flux at the wall surface becomes

$$\left(\mathbf{J_e}\right)_n = L'$$  \hspace{1cm} (34)$$

where $(\mathbf{J_e})_n$ represents the electron flux normal to the wall surface, and $L'$ is the ion loss rate per unit surface, which is determined by the ion trajectory calculations. Other boundary conditions are that the space potential is equal to the anode potential $\phi$ at the entrance of the channel, and to zero at the exit, as given by

$$\phi|_{\text{entrance}} = \phi_A, \quad \phi|_{\text{exit}} = 0$$  \hspace{1cm} (35)$$

By solving Eqs. (29)-(33) under the boundary conditions (34) and (35), the space potential distribution can be obtained using the Finite Element Method. The sequence of the calculation is illustrated in Fig. 7.

**Calculation Results**

Figure 8 shows the calculated distributions in the acceleration channel for the thruster Type I. The magnetic field and ion production distribution are also indicated in this figure. In accordance with the magnetic field lines, equipotential lines are curved toward the anode in the middle of the channel, and hence there is a large distortion in the electric field. Owing to this distortion, most of ions are accelerated toward the wall, and the ion loss fraction is computed to be 0.55. Figure 9 shows the calculated results for Type II. The ion loss fraction could be reduced to 0.25 because of a better arrangement of magnetic field lines, which are almost perpendicular to the thruster axis. Although the axial electric field is dominating in the channel, a radial component of the electric field is induced near the walls. This component is set up of such a polarity as to retard the electron flux toward the wall, and is balanced with the density gradient which is made as to drive

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**Fig. 7** Calculation sequence of the two-dimensional ion flow analysis.

**Fig. 8** Calculated distributions in the acceleration channel of thruster Type I.
it because the electron flux is limited to be small at the wall due to the boundary condition (see Eqs. (29) and (34)). These profiles show good agreements with the experimental ones.

Figure 10 shows the case where ion production rate varies only in the axial direction. In this case, the plasma density is distributed uniformly in the radial direction, and hence the radial component of the electric field is not induced. The equipotential lines are curved only slightly, and the computed ion loss fraction was improved to 0.06. This suggests that the ion loss flux may be reduced still more with the uniform ion production rate in the radial direction.

**Conclusion**

Experimental and analytical studies on Hall-current thrusters have been performed to improve the thruster performance and to investigate the plasma acceleration processes. From the plasma diagnostics, it was found that the ion loss fraction can be reduced from 0.7 to 0.4 by the better arrangement of magnetic field lines with the shorter acceleration channel. The measurement of the thruster performance with a hollow cathode suggests that hollow cathodes can be used without any degradation in performance and be appropriate for long operation tests.

One-dimensional plasma analysis model was made to calculate specific impulse and thrust efficiency. A qualitative agreement was found between the calculated and the measured thruster performance. The results also showed that the performance can be improved by the reduction of ion loss fraction.

Using a two-dimensional ion flow model, plasma properties such as space potential, plasma density, and ion loss rate were calculated and compared with the experiments. As the result, it was found that this model can be used to estimate ion loss fraction and that the electric field distortion, which is the main cause of ion loss to the walls, is induced not only due to the curved magnetic field lines, but also due to the gradient of the ion production rate.

**References**