THE STATE OF RESEARCH AND DEVELOPMENT
OF END-HALL PLASMA THRUSTERS IN THE USSR

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Abstract

End-Hall thrusters range in size from small power levels (1-20 kW) to very large power levels (200 kW or more). The physical processes and operating characteristics of these thrusters depend on power level and vary in a consistent manner with that power level. The research and development of end-Hall thrusters in the USSR are reviewed for a wide range of power levels.

Introduction

Low-power (1-20 kW) end-Hall thrusters (EHT) have been used in scientific and spacecraft-propulsion applications which did not require particularly high exhaust velocities and thruster efficiencies. The development of high power sources for spacecraft increases the probability of using plasma thrusters with a modular power of 30 kW or more. In the region from 30 to 200 kW, EHT with a solenoidal magnetic field have the highest performance.

The high-current end-Hall thruster (HCEHT) at modular power levels of 200 kW or more is distinguished from the EHT by the nature of the volumetric acceleration and the type of cathode used. These differences define the physical processes in the thruster, the interrelations between the operating parameters, and the thruster characteristics.

Features of Physical Model

and Relations Between Thruster Parameters

EHT typically use coaxial propellant introduction between the central cathode and the surrounding anode. The magnetic field is provided by an external solenoid.

In comparison, the propellant supply in an HCEHT is introduced through the cathode (made of tungsten rods and wires) in the HCEHT. The array of holes between rods provides a multihole cathode capable of current densities up to 200-300 A/cm².

The electromagnetic acceleration force in all end-Hall thrusters is of the form

\[ \mathbf{F} = j \times \mathbf{B}. \] (1)

In EHT, the magnetic field, \( \mathbf{B} \), is supplied by the external solenoid. At a modular power of 200 kW or more, in the regime of HCEHT, the magnetic field is primarily that due to the discharge current. An external magnetic field is still important, but because it can be used to control the current distribution at the anode, not because it is required for acceleration.

A change of discharge-current regime, together with the change in propellant introduction, leads to a change in flow pattern for the accelerated plasma. At the same time, the conditions at the flow boundaries (electrode surfaces) are changed.

Thus, in the EHT, the plasma flow is determined directly by the shapes of the electrodes and the operating parameters. The general solution of such a problem is not yet available, which is why it is necessary to make a quasi-one-dimensional approximation of the acceleration processes.

The above describes the electromagnetic acceleration process, but it is necessary to compare this process with the gas dynamic one. The ratio of the electromagnetic pressure to the gasdynamic one is

\[ R = \frac{p_{\text{em}}}{p_{\text{gd}}} = \frac{\mu_j}{\mu_g} \frac{j^2}{8 \pi S p_{\text{gd}}}. \] (2)

where \( \mu_j \) is the permeability of free space, \( j \) is the discharge current in A, \( S \) is the cross-sectional flow area of the thruster in m², and \( p_{\text{gd}} \) is the gas-dynamic pressure in N/m² averaged over the area \( S \). This ratio is equivalent to the similarity criterion

\[ R = k \frac{\mu_j}{\mu_g} j^2 M / 8 \pi \dot{m} a = 0.833 \times 10^7 j^2 M / \dot{m} a, \] (3)

where \( k = C_p / C_v = 1.67 \), \( M \) is the Mach number, \( \dot{m} \) is the mass flow in kg/s, and \( a \) is the sound velocity.

The electromagnetic acceleration of flow is efficient if, in each cross section, the criterion \( R > 1 \). At the exit from a cathode of a HCEHT where the plasma velocity is small, it can be assumed that \( R = 1 \): it permits the accurate definition of Mach number at the cathode exit for low plasma temperatures.
It can be seen that, at sufficiently high currents, the flow leaving the holes of a cathode is subsonic, \( M < 1 \).

In an EHT, where the current value is low, an external electromagnetic acceleration the flow is accelerated from \( M = 1 \) to \( M > 1 \). The plasma temperature does not change significantly and it is reasonable to assume the condition of \( T = \text{const} \).

Using Ampere's law,

\[
\mu_0 J = 2\pi r B, \tag{7}
\]

Eq. (6) can be transformed into

\[
\frac{v^2}{2} + \int (dp/\rho) + (1/2 \dot{\rho} \dot{m}) \int v J dJ = 0. \tag{8}
\]

It follows from Eq. (8) that, for each pair of functions \( p(\rho) \) and \( J(v) \), there is a unique flow distribution. In this case it is defined \( \rho(v) \) and is, consequently, a flow cross section \( S(v) \) from Eq. (4).

The existence of two regions of acceleration: the subsonic (thermal) and the supersonic (electromagnetic), requires that their parameters be determined separately. If, in the initial subsonic region, the plasma is constrained by a magnetic field so that \( R \leq 1 \) and a current does not flow out of flow boundary \((j \parallel v)\), the work of volumetric forces is

\[
j \times B \cdot v = 0. \tag{9}
\]

In Eq. (8) the two first terms remain and, if we assume that the process is a polytropic one, \( p = \rho^p \), we can define the function \( \rho(v) \) and, consequently, \( S(v) \). The relations between corresponding parameters along a longitudinal coordinate are defined by the energy equation:

\[
\rho \dot{v} C_p (dT/dz) - (v dp/dz) = \frac{j^2}{2} \quad (j^2 >> j^2) \tag{10}
\]

In the region of electromagnetic acceleration the flow is accelerated from \( M = 1 \) to \( M > 1 \). The plasma temperature does not change significantly and it is reasonable to assume the condition of \( T = \text{const} \).

In this case from Eq. (8), reduced to non-dimensional form, it follows that

\[
\overline{\rho} = \exp\left[ -(k M_0/2)(\overline{M}^2 - 1) - 2\alpha \overline{R}_o \int \overline{M} \overline{dJ} \right], \tag{11}
\]

where \( \overline{\rho} = \rho/\rho_o \), \( \overline{M} = M/M_o \), and \( \overline{J} = J/J_o \) with \( \rho_o, M_o \) and \( J_o \) are the pressure, Mach number, and current at the start of the electromagnetic acceleration region.

The equations (8) and (11) point to the existence of a variety of possible plasma-acceleration processes. A special case is the acceleration with frozen magnetic field (the magnetic field carried with the plasma). For this condition, it can be shown theoretically that

\[
\alpha = \frac{B^2_{\text{ave}}/B^2_{\text{bound}}}{0.5}, \tag{12}
\]

where \( B_{\text{bound}} \) is the value of B at the outer boundary, and \( \overline{R}_o \) is obtained from Eq. (2) with the values for the start of the acceleration region.

The equations (8) and (11) point to the existence of a variety of possible plasma-acceleration processes. A special case is the acceleration with frozen magnetic field (the magnetic field carried with the plasma). For this condition, it can be shown theoretically that

\[
B/\rho \dot{r} = \text{const.} \tag{13}
\]

One can see that this relation corresponds to the following conditions: \( J \cdot v = \text{const} \) or \( \overline{M} \cdot \overline{J} = 1 \) at \( \overline{T} = 1 \), where \( \overline{T} = T/T_o \). For a single-stage accelerator, typically \( M_o = 1 \) and Eq. (3) becomes

\[
R = 0.833 \times 10^{-7} J^2 / \text{m} a_o. \tag{14}
\]

In two-stage accelerators at the entrance to the second stage, \( M_o > 1 \). The possible accelerating processes in the general case have been analyzed.\(^1\)

The function \( M(z) \) is defined from the generalized Ohm law:

\[
J_z = \sigma (E - v_z B_0 \hat{e}) - j \times B/e n + \nabla p/e n \tag{15}
\]

At small values of \( \omega \tau_e \) and the thermodiffusion term \( \nabla p/e n \), Eq. (15) is simplified,

\[
J_z = \sigma (E - v_z B_0 \hat{e}) = -(1/\mu)(dB_0/dz) \hat{e} \tag{16}
\]
After making it non-dimensional and integrating we obtain the dependence \( \bar{Z}(\bar{M}) \) in the form:

\[
\bar{Z} = (\Pi_0^2 / 2Rm_o) \sqrt{(kM_0^M + M^{-1} - 2\alpha A_0 M^{-2})dM/M(1 - \Pi_0^2)} \quad (17)
\]

where \( \Pi_0 \) is the ratio of flow velocity to electromagnetic drift velocity at the start of the electromagnetic acceleration region,

\[
\Pi_0 = \nu_0 B_0 / E_0,
\]

and \( Rm_o \) is the magnetic Reynolds number at the same location,

\[
Rm_o = \mu_0 \sigma_0 \nu_0 R, \quad (19)
\]

with \( R \) the conventional Reynolds number.

The similarity criteria \( M, \Pi, R, \) and \( Rm \) are interconnected and their values are restricted to narrow ranges. Their values at the beginning of the electromagnetic acceleration region are: \( M_0 = 1, \Pi_0 \leq 1, R_o > 1, Rm_o > 1 \).

In different operating regimes of current and a propellant mass flow, when the condition \( R_o = \text{const} \) is obtained, there is self-similarity of flow distribution, which is supported by the solution of the same problem in the two-dimensional approximation.

The results of measurements of magnetic fields between the electrodes of an EHT by a three-component Hall-type magnetic probe clarified the physical picture of current distributions. As shown in Fig. 1, three characteristic zones are found: 1) the cathode current tube, where the current leaves the cathode in a primarily axial direction; 2) the intermediate region where the current flows in a primarily radial direction between the outer cathode diameter and the internal diameter of the exit anode; and 3) and the anode current layer, where electrons are constrained to move along the magnetic force lines.

This division into three characteristic zones facilitates the mathematical calculation of electromagnetic forces and, as a result, the performance and operating characteristics of EHT. From experimental probe measurements of electric and magnetic fields, the most intense plasma acceleration occurs in the intermediate region, where the currents are nearly radial.

**Instabilities in Thrusters**

The incompatibility of electrode geometry with the flow distribution in some thruster operating regimes can cause disturbances in the acceleration process. Two types of instabilities were found in experimental studies: magnetogasdynamic and ion-sonic instabilities.

The magnetogasdynamic one is a connected to the disruption of continuous flow acceleration from subsonic to supersonic one with a transition through sound velocity, Alfven velocity, fast sound velocity. This type of instability disturbs the acceleration process due to possible flow drift, and leads to appearance of local thermal vorticity and to the plasma deceleration. The magnetogasdynamic stability boundaries are the lower boundaries shown in Fig. 2 for a coaxial thruster with constant cross section.
The amplitude increase of ion-sound waves is the best indicator for the onset of the critical regimes, rather than the shape of the volt-ampere characteristic.

The Overall Characteristics of Thrusters

Experimental thrust, specific impulse, electrode voltage, efficiency as functions of current, mass flow and magnetic field strength in the HCEHT and the EHT show substantial differences, as well as certain similarities.

The variations of discharge voltage and efficiency with discharge current for the EHT and the HCEHT are generally different. One similarity in the volt-ampere characteristics of both types is the onset of critical current (ion-sound instability) with an increase in magnetic field, or a decrease in mass flow. This phenomenon also defines the maximum efficiency that can be reached by increasing discharge current.

Comparisons of the EHT thrusters characteristics with known calculated functions, such as those for thrust, show that the thrust coefficient, \( K_F = \frac{F}{J^2} \), is a variable value and depends on the regime of thruster operation. This is the consequence of boundary motion for the accelerated flow. (Equation (22) is also known as Mekker's equation.)

Using the momentum equation and taking into account the features of the physical model and several parameters of flow and electromagnetic field irregularities it is possible to obtain the following formula for the EHT thrust

\[ F = 10^7 \left[ 0.75 + \ln\left(\frac{r_a}{\rho_c}\right) J \right]^2 \]  

(22)

The purely electromagnetic thrust component with no account for pressure force on the cathode, which is equal \( 0.5 \times 10^{-7} J^2 \), will be

\[ F_{em} = 10^7 \left[ 0.833 + \left( 1/2 \frac{R_o}{J} \right) \right]^2 \]  

(24)

Equations (23) and (24) are similar in structure to Mekker's formula, but instead of \( \ln\left(\frac{r_a}{\rho_c}\right) \) they contain a term which depends on the operating regime and decreases with an increasing current or decreasing propellant flow.

It should be noted that Eqs. (23) and (24) constitute a special case, corresponding to the condition \( B/\rho r = \text{const} \), or to a relation \( JM = 1 \) at \( \bar{T} = 1 \). The equations
are applied at $R_o \geq 1$ and show a decreasing value of thrust coefficient, $K_F$, as $R_o \sim J \sqrt{J/m}$ increases. In addition to the experimental data obtained by the authors of this paper, similar conclusions can be found in other publications.5-7 There is also a more general expression for $F(J)$ that does not exclude different patterns of current flows.1

The knowledge of the thrust coefficient as a function of the thruster working regime permits the calculation of thrust and other overall thruster characteristics. The volt-ampere characteristics are convenient to define using equivalent voltage drops for the main processes in a thruster. The minimal voltage is determined by the ionization of the propellant, the acceleration of the plasma flow, and the electrode losses. The anode voltage drop increases sharply after the critical current is reached. The variations in anode potential drop are not completely understood and will require additional study.

Fig. 4. The EHT design.

The comparison of calculated and experimental data has carried out for end-Hall thrusters with a design close to that shown in Fig.4. The difference between the EHT with external magnetic field and the HCEHT is in the source of external magnetic field and size of the main components. The volt-ampere characteristics for HCEHT with a power up to 500 kW are presented in Fig. 5, which includes calculated (theoretical) values with currents up to 9.5 kA at $T_e = 1$ and 2 ev.

In Fig. 6 the variation of thruster efficiency, $\eta_T$, is shown as a function of exhaust velocity for two thrusters of different geometry, in addition to the calculated values for two electron temperatures. The lower curve is for a thruster with a power up to 250 kW. With this thruster, voltage oscillations were observed at a current $J \geq 6.5$ kA. These voltage oscillations were related to an increase in anode voltage drop and the efficiency decrease shown at high exhaust velocities in Fig. 6.

Fig. 5. Volt-Ampere characteristics for HCEHT.

A similar voltage oscillation and drop in efficiency developed in a thruster with a power up to 500 kW at a current greater than 9 kA. Just below this current, the thruster efficiency was 0.65 with the voltage up to 60 v.

Fig. 6. Thruster efficiency for HCEHT.

In Figs. 7 through 9, the characteristics of EHT (with external magnetic field) are shown. The typical variation in efficiency at a magnetic field of 0.112 T and a mass flow of 20 mg/s is from about 0.12 at a power of 20 kW to 0.4 at a power of 40 kW. The efficiency increases at higher magnetic fields, with the increase due both to changes in the current distribution at the anode and other design changes.

The computational methods that have been developed predict EHT characteristics with an accuracy of ±15-20 percent. Additional corrections are required for wider ranges of operating parameters.8 Plasma sources of small power (1-6 kW) have been developed based on the EHT configuration9 and used for the study plasma-flow interaction with the surrounding medium (both its own atmosphere and with the space background) and with spacecraft surfaces.
deposition of alkali metal on spacecraft surfaces, but confirmation requires longer operating times for the plasma sources.

Conclusions

Theoretical and experimental studies of EHT and HCEIHT support a physical model with moveable boundaries between flow regions that has been developed for the plasma acceleration. This model has been used to calculate operating parameters and characteristics.

Experimental results show a very high efficiency at a power of more than 40 kW for an EHT (with external magnetic field) and at 500 kW for the HCEHT. Preliminary estimations for the HCEIHT at power levels of 1 MW and higher shows that using gaseous working materials (H₂, NH₃, and N₂) at high specific impulses (5-6 x 10⁴ m/s), the necessary voltage exceeds the capability of energy sources using thermionic diodes and having efficiencies of 0.25-0.35.

References


Such plasma sources were tested in the "Major" experiment on the spaceship "Progress 4-M" on September 20, 1990. The preliminary analysis of experimental results show that there is very little