PRE-IONIZATION PROCESSES IN A SELF-FIELD MPD ACCELERATOR

Rodney L. Burton* and Nicholas Tiliakos**
Department of Aeronautical and Astronautical Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801

ABSTRACT

Numerical models of magnetoplasma-dynamic (MPD) accelerators have often assumed an inlet ionization fraction of $\alpha_1 = 1\%$. Here, we seek to evaluate $\alpha_1$ by a numerical model of the upstream pre-ionization region of an MPD accelerator. The physical processes involved in the diffusion of electrons upstream against the neutral flow into the MPD are discussed. Models are presented based on the appropriate species conservation equations for the acceleration of partially ionized argon in a rectangular, constant area, electrically insulated channel contiguous to the accelerator. The spatial variation of charged particle ambipolar diffusion velocity $V$, electron temperature $T_e$, axial electric field $E_x$, and ionization fraction $\alpha$ is investigated for three models: constant $T_e$, constant temperature gradient, and finite thermal conductivity. The scale length for the constant $T_e$ model is found to be $kT_e/eE_x$, where $eE_x = M_i V_{veff}$, the effective ion-neutral momentum transfer rate. The latter two models, for which the temperature increases, predict a large rise in $\alpha$ for sufficiently large electric fields. A region width of a few millimeters is predicted for the finite thermal conductivity model. Boundary conditions leading to physical solutions are discussed. As a preliminary conclusion, $\alpha_1 = 1\%$ is a justifiable value at the entrance to the MPD accelerator.

NOMENCLATURE

B Magnetic field [T]
e Electron charge [coulombs]

* Associate Professor, Senior Member, AIAA
** Graduate Student, Student Member, AIAA

I. INTRODUCTION

An understanding of the physics of electric propulsion devices, such as the magnetoplasma-dynamic (MPD) thruster, is critical if these devices are to be developed for future interplanetary missions. The MPD thruster utilizes the $j \times B$ force for the production of useful thrust. Since the interaction between the electromagnetic fields and the propulsive fluid are complex, simplifying assumptions are used for numerical modeling of these devices.
A numerical model of a one-dimensional multi-fluid MPD arc was previously made by Devillers.\(^1\) This analysis is applied to a supersonic argon flow with initial conditions of \(u_n = 869\) m/s, \(T = 300\) °K and Mach = 2.3 and it is found that a continuous supersonic acceleration of the flow from inlet to exit is achieved by a constant arc electron temperature \(T_e\), of 18600±150 °K. The Devillers model is described below (Appendix), and is used to provide downstream boundary conditions for the present analysis.

The Devillers model assumes, as have others,\(^2,3\) that the ionization fraction is \(\alpha_1 = 1\%\) at the accelerator inlet, increasing by electron impact ionization in the current conduction region. In this study, we investigate the flow upstream of the MPD inlet, the so called "pre-ionization" region, through the application of multi-fluid MHD models. An understanding of the pre-ionization region should lead to an improved understanding of the complete acceleration process for the MPD thruster.

The dominant physical process involved in the transport of electrons in the pre-ionization region is the large electron pressure gradient, which creates an axial electric field \(E_x\). The resulting rates of diffusion of electrons and singly-charged ions are equal, a process called ambipolar diffusion.\(^4\) The \(E_x\) field retards the highly mobile electrons and accelerates the heavier ions, thus coupling their motions.

It is the objective of the present investigation to find the size of the pre-ionization region and the dependence of plasma properties on model boundary conditions. The spatial variation of electron diffusion velocity \(V\), \(T_e\), \(E_x\), and ionization fraction \(\alpha\) is calculated for three models: constant \(T_e\), constant gradient in \(T_e\) and finite thermal conductivity.

**II. PROBLEM DESCRIPTION**

As shown below (Fig. 1) cold argon gas flows to the right from a porous insulated wall at \(x_0\) toward the inlet of a self-field accelerator at \(x_1=0\). Electric fringe fields are neglected and no current \(j\) flows in the region \(x<0\).

Between the MPD electrodes the only current component is \(j_y\). Both \(E_y\) and \(E_x\) components exist in the pre-ionization region. The magnetic field \(B_2\) is in the +z direction and is constant for \(x<0\), with \(B_2 = 0.25\) T.

![Fig. 1. Geometry of pre-ionization region \(x_0<\!\!\!x<x_1\) in a self-field MPD accelerator.](image)

The diffusion scenario is depicted in Fig. 2, which shows ambipolar diffusion as observed in the electron-ion reference frame. The electric field force is balanced by ion-neutral momentum transfer drag.

![Fig. 2. Ambipolar diffusion in the electron/ion reference frame, with electrostatic forces balanced by ion-neutral momentum transfer drag.](image)

In this paper we assume that the flow of partially ionized argon is steady and one-dimensional, with mass-averaged axial velocity \(u\). Transverse velocity \(v\) is zero. The fluid has two temperatures: \(T_e(x)\) for the electrons and \(T\) for the heavy particles, assumed constant. Radiation and viscosity are neglected, and ionization is assumed to
occur either by charge exchange or by a two-step electron-impact excitation process. The degree of ionization is assumed sufficiently small so that the neutral velocity \( u_n \) can be regarded as constant, and electron-ion recombination is neglected in the pre-ionization region. It is also assumed that a low background level ionization \( \alpha_0 \) exists far upstream, created by photoionization or other unspecified sources.

### III. GOVERNING EQUATIONS

A strong electron pressure gradient exists for \( x < x_1 = 0 \). Throughout the pre-ionization region the ion pressure gradient and ion inertia terms are negligible compared to the other terms in the ion momentum equation. Since the conduction current \( j \) is zero in the region, it follows that \( u_e = u_i \). The initial neutral velocity \( u_n \) is assumed to be supersonic, Mach=2.3, with an initial flow velocity of 869 m/s, consistent with previously calculated conditions for the arc region.

**Ohm's Law:**

In the pre-ionization region the plasma is non-isothermal, so that an Ohm's law description of current conduction must include effects of ion slip plus temperature and pressure gradients. Neglecting ion and neutral pressure gradients, the axial electric field is given, for \( J = 0 \) by:

\[
E_x = - \frac{1}{n_{ee}} \frac{dp_e}{dx} \left( \frac{k/2eG}{1 + \Omega_e^2} \right)
\]

For typical MPD conditions \( (n_n = 4 \times 10^{22} \text{ m}^{-3}, T_e = 1 \text{ eV}) \), \( \Omega_e^2 > 1 \) in the pre-ionization region, and the second term on the right-hand side of Eq. (1) can generally be neglected. The transverse electric field is \( E_y = -u_e B_z \).

**Charge Continuity:**

Starting with the continuity equation for electrons in steady flow:

\[
\frac{d(n_e u_e)}{dx} = \Gamma_i - \Gamma_r
\]

where the recombination rate \( \Gamma_r \) can be neglected for \( \alpha << 1 \). The plasma is neutral, so \( n_i = n_e \). For steady state the electron continuity equation becomes:

\[
\frac{du_e}{dx} = -\frac{\Gamma_i}{\alpha n_0} \frac{u_e d\alpha}{\alpha dx}
\]

### Ion Momentum:

In the axial direction, with \( u_i = u_e \), the ion momentum equation is given by:

\[
\rho_i \left( u_i \frac{du_i}{dx} \right) + \frac{dp_i}{dx} = \frac{n_i M_i}{2} (V_n - V_i) v_{in} + n_i e E_x + M_i (V_n - V_i)
\]

where \( V_n = \alpha (u_n - u_i) \) and:

\[
V = V_i = V_e = -(u_n - u_i) (1 - \alpha)
\]

Neglecting the inertia and ion pressure terms, and combining Eqs. (4) and (5):

\[
n_i M_i (u_n - u_i) v_{in}/2 + n_i e E_x + M_i (u_n - u) = 0
\]

The \( \Gamma_i \) term gives the momentum change in the ion flow due to ionization of neutrals by electron impact. The ion-neutral collision frequency \( v_{in} \) includes elastic and charge exchange collisions, both of which result in a momentum rate which is balanced by the ionization rate and axial electric field.

The ion momentum equation is used to derive an expression for the axial electric field. Solving for \( E_x \) from Eq. (6) gives:

\[
E_x = - \frac{M_i (u_n - u_e) v_{eff}}{n_{ee}}
\]

where \( v_{eff} = \frac{V_{in} + \Gamma_i}{2 \alpha n_0} \)

For typical conditions the ionization rate \( \Gamma_i \) is small and \( v_{eff} = v_{in}/2 \).

An equation for the variation of the degree of ionization is found from Eq. (1) for \( p_e = \alpha n_0 k T_e \):

\[
\frac{1}{\alpha} \frac{d\alpha}{dx} = \frac{e E_x}{k T_e} G \left( \frac{1}{2 (1 + \Omega_e^2)} \right)
\]

where \( G \) is the electron temperature gradient. This equation requires \( E_x < 0 \) for \( \alpha \alpha_0 \) increases monotonically with \( x \).

**Electron Energy:**
The general form of the electron energy equation for a partially ionized plasma is written:

\[
\frac{d\epsilon_e}{dx} = E_{je} - \frac{3m}{M} n_e k x \frac{\partial \epsilon_e}{\partial x} + \frac{d\epsilon_e}{dx} = E_{je} - \frac{3m}{M} n_e k x \frac{\partial \epsilon_e}{\partial x}
\]

\[
\left( T_e - T_i \right) v_c - \frac{V^2}{Y_e} = \frac{d\epsilon_e}{dx} - \frac{\partial}{\partial x} \left[ \Gamma \left( E_i + 3kT_e/2 \right) \right]
\]

The heat conduction, conduction current and total collision frequency are given by:

\[
q_e = -\kappa_e \left( \frac{dT_e}{dx} \right) + \left( 5/2 \right) n_e k T_e V
\]

\[
\frac{\partial}{\partial x} \left( n_e e V = \alpha n_0 e (u_n - u_e) \left( 1 - \alpha \right) \right)
\]

\[
v_c = v_{ei} + v_{en}
\]

The electron thermal conductivity is:

\[
\kappa_e = \frac{\sigma_0 k^2 T_e}{(1 + \Omega_e^2) e^2}
\]

where the scalar electrical conductivity \( \sigma_0 \) is:

\[
\sigma_0 = \frac{\alpha n_0 e^2}{m (v_{en} + v_{ei})}
\]

The collision frequencies \( v_{en} \) and \( v_{ei} \) are evaluated with cross sections of \( Q_{en} = 2 \times 10^{-20} \) \([m^2]\) and \( Q_{ei} = 3.9 \times 10^8 \ln \Lambda / T_e^{-2} \) \([m^2]\), where \( \ln \Lambda \) is taken as 10.

For conditions in the pre-ionization region the ionization process is represented by the rate of excitation of the first excited level above the ground state, resulting in an ionization rate:

\[
\Gamma_i = \Gamma_{exc} = \frac{S(2k)^{3/2}}{(\pi m)^{1/2}} (\alpha(1 - \alpha)n_0)^2 \times
\]

\[
T_e^{3/2}(T_{ex}/T_e + 2) e^{-T_{ex}/T_e}
\]

where \( S = 0.0044 \) \([m^2/J]\) is the slope of the excitation cross-section for energies above threshold, and \( T_{ex} = 135400 \) °K.

IV. NUMERICAL METHOD

Equations (3), (9) and (10) are highly coupled ordinary differential equations, with the electron energy equation (10) of second order. The equations are solved using a fourth order Runge - Kutta routine with adaptive step size control. For this method the electron energy equation is separated into two first order equations with the substitution:

\[
G = \frac{dT_e}{dx}
\]

making Eq. (10) first order in \( dG/dx \).

The procedure solves for the first derivatives of \( \alpha, T_e, G \) and \( u_e \). Rearranging Eq. (1), Ohm's law, gives an equation for \( da/dx \), Eq. (9). Electron continuity, Eq. (3), is then integrated to give the electron species velocity \( u_e \). The electron temperature gradient, Eq. (17), is integrated to give the electron temperature.

V. RESULTS AND DISCUSSION

Solutions are sought for which the electron temperature and ionization fraction increase from a low value far upstream to arc conditions at the accelerator entrance, \( x_1 = 0 \). These conditions imply: \( G > 0, u_e > 0, E_x < 0 \) and \( da/dx > 0 \), for \( x_0 < x < x_1 \).

Constant Electron Temperature Model

It is instructive to consider a sequence of models, proceeding from the simplest toward the most realistic one. We first examine the case of constant electron temperature, for which there is no heat conduction and \( G = dG/dx = 0 \). For this case, the governing equations become:

Ohm's Law:

\[
E_x = -\frac{kT_e}{\alpha n_0} \frac{da}{dx}
\]

Momentum:

\[
E_x = \frac{M}{\alpha n_0} (u_n - u_{en}) v_{eff}
\]

Charge Continuity:

\[
\frac{du_e}{dx} = \frac{\Gamma_i}{\alpha n_0} \frac{u_e}{dx}
\]

Electron Energy:

\[
0 = E_{ke} - 3m n_e k (T_e - T_i) v_c - \frac{V^2}{Y_e} \frac{d\epsilon_e}{dx} - \frac{\partial}{\partial x} \left[ \Gamma \left( E_i + 3kT_e/2 \right) \right]
\]

where \( V \) and \( j_e \) are given by Eqs. (5) and (12).
\[ x^* = \left( kT_e/e|E_x| \right) \quad (m) \] (22)

Integrating Eq. (18) gives:
\[ \alpha(x) = \alpha_1 \exp \left[ -x/x^* \right] \] (23)

We note that the scale length can also be written in terms of the momentum transfer between ions and neutrals. Using Eqs. (5) and (7) for \( \alpha << 1 \), gives:
\[ eE_x = MV_{\text{eff}} = MV_{\text{in}}/2 \]
and \[ x^* = 2kT_e/MV_{\text{in}} \]

The collision frequency \( V_{\text{in}} \) includes both elastic and charge exchange collisions and is dominated by the latter. The charge exchange process creates ions with zero initial diffusion velocity which must then be accelerated by the \( E_x \) field, thereby creating an effective "drag" force.

Simplifying the electron energy equation and substituting the expressions for \( d\alpha/dx \) and \( dV_e/dx \) from above results in a simple quadratic equation for \( E_x \). The assumption that \( V_{\text{en}} >> V_{\text{ei}}, \) valid for \( \alpha < 10^{-3} \), leads to an expression for \( E_x \) which is independent of \( \alpha \):
\[ \left( \frac{e^2}{M_{\text{eff}}} \right) E_x^2 + (7u_{\text{en}}/2)E_x + \frac{3m_k(T_e - T_i) V_c}{M} + \frac{\Gamma_i}{\alpha n_0} (E_i + 5kT_e) = 0 \] (24)
where \( V_c = V_{\text{en}} \) and \( V_{\text{eff}} \) is given by Eq.(8).

For a given \( T_e \) only one solution of the quadratic Eq. (24) satisfies \( u_e > 0 \). For arc inlet conditions of \( T_e = 18500 \) K, \( T = 300 \) K and \( u_n = 869 \text{ m/s} \):
\[ E_x = -2490 \text{ V/m} \]
\[ u_e = 600 \text{ m/s} \]
\[ x^* = 0.64 \text{ mm} \]

For higher arc temperatures \( |E_x| \) increases therefore \( u_e \) decreases, and a theoretical limit is reached when \( u_e \) becomes zero, stalling the flow. For \( u_n = 869 \text{ m/s} \) this condition is reached for:
\[ T_e = 20,800 \text{ K} \]
\[ \text{max } E_x = -8100 \text{ V/m} \]
for which:
\[ x^* = 0.22 \text{ mm} \]

**Constant Electron Temperature Gradient Model**

We next consider the case of constant electron temperature gradient for which \( dG/dx = 0 \). For this case, Ohm's Law, Eq. (1) is rearranged using \( p_e = a_n kT_e \) to give:
\[ \frac{d\alpha}{dx} = \frac{(eE_x + kG)}{kT_e} \] (25)

where we have taken \( \Omega_e^2 >> 1 \). This expression implies a constant \( G \) scale length for \( \alpha \) of
\[ x_o = \frac{kT_e}{e|E_x| - (kG/e)} \] (26)

Eq. (26) has a singularity requiring \( |E_x| > kG/e \) for \( d\alpha/dx > 0 \), a consequence of electron continuity. Near \( x_1 \), we have \( |E_x| >> kG \), and the scale length is similar to the constant \( T_e \) model. However, \( E_x \) falls by two orders of magnitude upstream, and approaches the value \( kG/e \), thus extending the scale length considerably. This is shown in Fig 3, where the pre-ionization region extends to ~80 mm upstream of the inlet.

The ambipolar electric field \( E_x \) is shown in Fig 4 and displays an internal structure. \( |E_x| \) falls rapidly from 2500 V/m to 500 V/m in less than 1 mm, followed by a slower drop to 10 V/m over a region 80 mm wide. The degree of ionization varies by 10^3 in this range.

Fig. 3. Variation of \( T_e \) and ionization fraction \( \alpha \) throughout the pre-ionization region for \( dT_e/dx = \text{constant} = 10^5 \text{ K/m} \). Initial conditions are \( T_{e1} = 18500 \text{ K}, \alpha_1 = .002, E_{x1} = -2515 \text{ V/m} \).
The $k_e$ term in Eq. (11) is the heat conduction and the second term is the heat diffusion, proportional to $V$.

It has not proved possible to integrate the energy equation (10), as written, because of numerical oscillations in $dG/dx$. We have therefore simplified $q_e$ by relating $V$ to $E_x$ through Ohm's Law, Eq. (1), and by making the assumption that the $\alpha$ scale length $x^*$ is equal to the temperature scale length, i.e.:

$$\frac{d\alpha}{dx} = \frac{dT_e}{T_e dx}$$

With this assumption the heat conduction can be written in a form proportional to $G$:

$$q_e = -G[k_e + \kappa_D]$$

where $\kappa_D = (5\alpha_n k^2 T_e \phi)/(2 M v_{eff})$ and $1.5<\alpha<2$ is a weak function of $\Omega_e$. For proper boundary conditions the equations can now be integrated from $x_0$ to the inlet where $T_{e1} = 18500$ $\deg K$.

All solutions start with $G_0 = 0$ at $x_0$. Solutions leading from a small value of $\alpha$ ($O[10^{-5}]$) at $x_0$ to an $\alpha$ value of $O[10^{-2}]$ at $x_1 = 0$ are sensitive to the initial electric field $E_{x0}$ and to the initial temperature $T_{e0}$. Solutions are not sensitive to $a_0$ in that $a_1/a_0$ is found to be independent of $a_0$.

Solutions producing $a_1/a_0 >> 1$ result when the electric field approaches values of -7700 $V/m$, driving the charged particle velocity to a small value. A set of solutions is shown below in Figs. 6-8 for the conditions $T_{e0} = 14000$ $\deg K$, $E_{x0} = -162.5$ $V/m$, $\alpha_0 = 10^{-5}$ and $G_0 = 0$. The degree of ionization rises monotonically to 0.013 over a distance of 2.0 $\text{mm}$. The temperature profile decreases slightly over part of the region but increases overall to 18500 $\deg K$. The Hall parameter varies from 68 to 7 (Fig. 8) and $k_e + \kappa_D$ increases by $O[10^6]$. The inlet temperature gradient is $G_1 = 1.3 \times 10^7$ $\deg K/m$.

The solution is sensitive to the value of $E_{x0}$. Reducing $E_{x0}$ slightly from -162.5 to -160.0 $V/m$ removes the region of $G<0$, as shown in

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**Finite Thermal Conductivity**

In the constant temperature gradient model we legislated $V \cdot a = 0$, and ignored the thermal conductivity $k_e(T_e)$. The variation of $k_e$, Eq. (14), is shown in Fig. 5 for the conditions relevant to Figs. 3-4, together with the Hall parameter $\Omega_e$. The thermal conductivity is seen to increase by 4 orders of magnitude through the region, with $\Omega_e >> 1$ everywhere.

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**Fig. 4.** Variation of $T_e$ and axial electric field $E_x$ throughout the pre-ionization region for $dT_e/dx = \text{constant} = 10^5 \deg K/m$. Initial conditions are $T_{e1}=18500\,\deg K$, $\alpha_1=.002$, $E_{x1}=-2515\,V/m$.

**Fig. 5** Thermal conductivity and electron Hall parameter for constant electron temperature gradient, $G = 10^5 \deg K/m$.

For the finite thermal conductivity model we include the thermal conductivity in the energy equation (10), with $q_e$ given by Eq. (11).
Fig. 9. This small change in $E_{X_0}$ also reduces the degree of ionization by about a factor of 12.

The observed width of the pre-ionization region (2.0 mm) is considerably smaller than that for the constant G case. This result is consistent with the higher values of $V$ and $G$ predicted in the model.

Fig. 9 Sensitivity of solution to $E_{X_0}$, displaying $G < 0$ region for $-162.5$ V/m and $G > 0$ everywhere for $-160.0$ V/m.

VI. SUMMARY AND CONCLUSIONS

The constant electron temperature model, permits an estimate of the size of the pre-ionization region and the magnitude of $E_X$ for a given set of arc conditions $T_e$ and $u_n$. The region scale length $x^*$ is inversely proportional to $E_X$ and therefore inversely proportional to the ion-neutral momentum transfer rate. This result is expected, since increasing $E_X$ reduces $u_e$ and increases $\alpha$, as can be seen from electron continuity, Eq. (3).

Models for which the temperature increases predict a large increase in $\alpha$ for sufficiently large electric fields. The mechanism for increasing $\alpha$ seems not to be the ionization rate, which is a small factor, but the deceleration near $x = 0$ of the ions and electrons by $E_X$ to a low value of $u_e$. Since $d(\alpha u_e)/dx > 0$, decreasing $u_e$ implies increasing $\alpha$.

A region width of a few millimeters is predicted for the finite thermal conductivity model. This model also allows $G_0 = 0$, so that the heat flux is zero at the upstream boundary. Assuming that an initial ionization fraction $\alpha_0 = 10^{-5}$ exists from photoionization or other sources, $\alpha_1 = 1\%$ is a justifiable value at the entrance to the MPD accelerator.
VII. ACKNOWLEDGEMENTS:

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VIII. REFERENCES


APPENDIX

The Devillers multifluid model describes the flow of argon plasma into a region between two electrodes with a current transverse to the flow and a magnetic field in the +z direction. The geometry is shown in Fig. A.1:

![Fig. A.1. Geometry of the 1-D Devillers model.](image)

The model has the following assumptions:

1. Steady, 1-D flow with constant area.
3. The flow is accelerated by $j_y B_z$.
4. $E=(E_x, E_y, 0)$, $B=(0,0,B_z)$.
5. The plasma is singly, partially ionized.
6. "Newly-born" particles are quickly brought to equilibrium with respect to the main swarm through collisions and represent a negligible contribution to the conservation equations.
7. $T_e$ is constant in the arc discharge.
8. No ion slip; $u_n=u$.

GOVERNING EQUATIONS:

**Charge Continuity:**

$$\frac{d(n_0 u_n)}{dx} = \frac{d(Q_0)}{dx} = 0 \quad (A1)$$

**Electron Momentum:**

$$n_0 M_e \frac{d u}{dx} = -n_0 k \frac{d(\alpha T_e + T)}{dx} + \sigma (E_0 - u B) \quad (A2)$$

**Neutral Particle Energy:**

$$n_0 \left[ \alpha u + (1 - \alpha) u_n \right] \frac{d T}{dx} = -k \left[ \alpha n_0 \frac{d u}{dx} + (1 - \alpha) n_0 \frac{d u_n}{dx} \right] +$$

$$4c_e n_0 k (T_e - T) \left[ \alpha n_0 Q_{ei} + (1 - \alpha) n_0 Q_{en} \right] \quad (A3)$$

where $c_e$ is the electron thermal speed and $\gamma$ for argon is 5/3.

**Electron Energy:**

The electron energy equation is essentially discarded for $T_e = \text{constant}$ within the current sheet by requiring that the terms balance approximately. This is accomplished by non-dimensionalizing the energy equation, adding to it an electron energy error term $f$, and selecting $T_e$ so that $f \ll O(1)$.

Equations (A1-A3) are nondimensionalized as follows: velocities by the Alfvén critical velocity: $u_c = (2eV_i/M)^{1/2}$; the magnetic field by the inlet value $B_1$; electric field by $u_c B_1$; temperature by the ionization potential of argon, 15.76 eV; and distance $x$ by:

$$x = x^* \left[ \frac{Q_0 S / (\pi m)^{1/2}(2kV_{ion})^{3/2}}{u_c^2} \right]$$

where $S=0.044$ m$^2$/J is the slope of the first excitation cross-section. Collision cross sections are nondimensionalized by the quantity:

$$Q_0^* = \frac{M}{m_e} (eV_{ion}) S$$

where $V_{ion}$ is the first ionization potential.

The system of equations contains the parameters $Q_0$, $B_1$ and $E_0^*$. The value of $B_1$ is chosen to be typical of MPD accelerators, whereas $E_0^*$ and $Q_0$ are dependent on the operating conditions of the arc. If the model is valid then solutions will be found to exist only in a narrow range of $E_0^*$ and $Q_0$. In order to test this possibility, $E_0^*$ is retained as a parameter and a second non-dimensional parameter is introduced, the flow coefficient:

$$q = \left( Q_0 M u_c / (B_1^2/2u_0) \right)$$

**Method:**

The equations are written with $B$ as the independent variable and $u$ and $x$ as dependent variables. Eqs (A1)-(A3) are nondimensionalized and solved numerically. The initial conditions are fixed by $u_1^*$, $B_1$ and $T_1^*$. The non-dimensional parameters $T_e^*$, $E_0^*$ and the flow coefficient $q$ are varied, generating solutions for each integration of the equations. Depending on the parameters two
accelerating regimes can be defined: a subsonic electrothermal acceleration and a supersonic acceleration regime. There can also be subsonic/supersonic deceleration but these solutions are not desirable for MPD accelerators. In some cases the flow may pass through the sonic point.

The routine solves for the spatial variation of the heavy particle temperature $T$, the non-dimensional electron velocity $u^*$, the degree of ionization $\alpha$ and the size of the current region $x$. Constraints on the solution are:

1. $T^*>0$.
2. $u^*B^* \leq E_0^*$ (no current reversal).
3. Sheet of width between 1 and 50 cm.
4. Flow is accelerating supersonically and monotonically in the arc.
5. The energy balance correction term satisfies $f \leq O(1)$.

Results:

A narrow range of acceptable solutions is found in the region of the parameter space $E_0^*-q-T_e^*$, as shown in Fig. A.2, in which $T_e^*$ contours are projected on the $E_0^*-q$ plane. A typical result meeting the above constraints is shown in Fig. A.3, for $B_1=0.25$ T:

Fig. A.3. The variation of non-dimensional magnetic field, ionization fraction and dimensional heavy particle temperature as a function of current sheet length, for initial conditions of: $\alpha_1=0.1$, $u_1=869$ m/s, $u_e=8690$ m/s, $E_0=1300$ V/m, $T_1=300$ oK, $T_e=18500$ oK, and $q=1.0$.

Fig. A.2. The region of the $E_0^*-q$ plane corresponding to solutions satisfying $f \leq O(1)$. 

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