LASER FLUORESCENCE VELOCIMETRY OF AN ARCJET EXHAUST PLUME

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Abstract

We describe a two-beam, multiplexed, laser induced fluorescence (LIF) technique with which radial and axial profiles of vector-velocities, Doppler widths, and relative density profiles of excited propellant species were obtained in the exhaust plume of a 300 Watt argon arcjet. The data clearly identify a boundary layer in the radial direction, and a shock in the downstream region of the flow. The peak velocity on centerline remains roughly constant at 3 km/s throughout the expansion. From the LIF data, a specific impulse of 160 seconds is estimated.

1. Introduction

Recent studies have reemphasized that significant savings in satellite launch costs can be realized by the use of electrically powered plasma thrusters to provide orbit transfer and station keeping [1,2]. Although the concepts of many of these thrusters have been known for almost three decades [3], development has been delayed due to a lack of adequate space power and appropriate missions. As a result, several types of thrusters are only now reaching the level of maturity required for deployment. A case in point is a 1.8 kW hydrazine arcjet system [4,5], which is scheduled for a 1993 Telstar-4 launch.

Early development and optimization of electric thrusters has taken place largely by empirical methods, but new computational methods are becoming available to permit a more systematic approach to optimize thruster performance [6-8]. However, it is a considerable challenge to model accurately the properties of the plasma core and the exhaust plume of, for example, an arcjet thruster. Namely, highly non-linear plasma processes must be accounted for, including absorption of electrical power, radiative transfer, convection, conduction, and turbulence. Because the thermodynamic and transport properties of some of these processes are difficult to calculate or measure for the temperatures and pressures of interest, code validation can only be achieved with the help of detailed measurements of the plume properties at the exit nozzle. In this paper, we describe a laser induced fluorescence (LIF) technique that is capable of producing such data. Specifically, we report spatially resolved measurements of the exit vector-velocities, Doppler widths, and densities of propellant species in the exhaust plume of a low-power (300 Watt) arcjet, operated on argon.

The principle of measuring velocities, Doppler widths, and densities of flowfield species using LIF measurements has been known for some time. For example, Zimmerman and Miles developed an LIF technique for measuring velocities in a hypersonic wind-tunnel a decade ago [9]. In short, the LIF technique consists of using a narrowband laser to excite one of the plume species, and fluorescence radiation is detected as an indicator of the absorption process. Because, unlike in emission spectroscopy [10], a small effective scattering volume can be created by imaging only a small section of a focused laser beam onto a detector, spatial resolution on the order of a millimeter or less is easily achieved. For the purpose of flow diagnostics, it is of particular interest to measure the fluorescence signal as the laser is tuned across the absorption profile of the absorbing species, thereby yielding the Doppler profile of the transition. This Doppler profile is a direct measure of the velocity distribution of the absorbing species projected onto the propagation direction of the laser beam.

Quite recently, two groups have used LIF techniques to measure flow velocities in the exhaust plumes from a 30 kW class hydrazine arcjet [11], and from a 1 kW class hydrogen arcjet [12]. In both cases, excitation of the hydrogen Balmer-α line was used to extract radial profiles of the axial velocity components in the flow, just downstream from the exit nozzle. Also, in

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both cases, some qualitative features of the distribution of Doppler widths and densities in the flow were discussed, but no detailed data were presented.

Here, we describe an LIF scheme that is similar to the ones used above, except that, by two-beam multiplexing, we are able to measure radial and axial velocity components in the flow simultaneously, so that vector-velocities of the plume can be constructed. In addition, we present data on the radial and axial distributions of the Doppler widths and relative densities of the diagnostic species used. Just as in the case of the LIF measurements on the hydrogen Balmer-α line above [11,12], the diagnostic species in our work is actually an excited propellant species, so that some care must be taken in the interpretation of the data. However, several features of the flow are easily identified from these measurements of Doppler widths and densities, for example the locations of a radial boundary layer and of a shock on centerline. The choice of argon propellant, as opposed to hydrazine or hydrogen, was primarily for convenience, and the value of our data is mostly a proof-of-principle of the multiplexed LIF technique. Indeed, although some of our data may not be representative for a higher power hydrazine or hydrogen arcjet, the technique itself should be applicable to such systems without significant modification. Finally, the quality of our data is somewhat limited as a result of having available only a non-optimized arcjet system, with which true steady-state operation could not be maintained for the duration of the measurements.

The outline of the paper is as follows. In Section 2, the theory is developed that forms the basis for extracting the properties of a two-component velocity distribution in the exhaust plume from two-beam multiplexed LIF measurements. For a more detailed discussion of the principle of LIF measurements in general, the reader is referred to the literature. In Section 3, the experimental facilities are described, namely the arcjet system, the vacuum facility, the laser diagnostics system, and the data acquisition system. In Section 4, the results of the plume measurements are presented, namely in the form of vector-velocities, Doppler widths, and relative densities of the absorbing species at different radial positions in the flow immediately downstream from the nozzle exit, and at different axial positions on centerline, up to and beyond the first shock location. In Section 5, some further analysis of the data is presented, focusing on some subtle aspects of the measurements. Finally, a summary is presented in Section 5, as well as a comparison with the arcjet LIF work from Refs. [11] and [12].

2. Theory

In this section we consider the problem of determining the properties of the velocity distribution function of the propellant plume, in two dimensions, from Doppler measurements along two non-orthogonal laser beam directions in the experiment. Although, with our particular arcjet system, non-steady-state behavior of the plume could not be avoided, we assume below, for simplicity, that the data may be regarded as representing true steady-state behavior of the plume. Some possible artifacts that may have resulted from averaging over plume fluctuations are discussed in the Appendix, at the end of this section, and in Section 5.

To start, we assume that the distribution of velocities of the gas, or, to be exact, of the species that is excited in the LIF scheme, is fully characterized by the distribution function \( f(v_x; v_y) \), where \( v_x \) and \( v_y \) are the velocity components along two orthogonal axes, namely, parallel and perpendicular to the arcjet centerline, respectively. With proper normalization, the density \( n_a \) of the absorbing species is given by the zeroth moment of this distribution function:

\[
n_a = \iint f(v_x; v_y) \, dv_x \, dv_y. \tag{1}
\]

(Unless noted otherwise, all integrals are from \(-\infty\) to \(+\infty\).) Using the notation \( f(d^2v) = f(v_x; v_y) \, dv_x \, dv_y \), the mean velocity components and the variances of the velocity components are now given by the first moments and the second central moments of the distribution function, according to

\[
\bar{v}_x = \frac{1}{n_a} \iint v_x f(d^2v), \tag{2a}
\]
\[
\bar{v}_y = \frac{1}{n_a} \iint v_y f(d^2v), \tag{2b}
\]
and

\[
\sigma_{xx}^2 = \frac{1}{n_a} \iint (v_x - \bar{v}_x)^2 f(d^2v), \tag{3a}
\]
\[
\sigma_{yy}^2 = \frac{1}{n_a} \iint (v_y - \bar{v}_y)^2 f(d^2v), \tag{3b}
\]
\[
\sigma_{xy}^2 = \frac{1}{n_a} \iint (v_x - \bar{v}_x)(v_y - \bar{v}_y) f(d^2v). \tag{3c}
\]

For an isotropic distribution, \( \sigma_{xx}^2 = \sigma_{yy}^2 \) as well as \( \sigma_{xy}^2 = 0 \), independent of the choice of coordinate system. For a non-isotropic distribution, it is still possible to define a coordinate system in which the cross-variance \( \sigma_{xy}^2 \) vanishes, namely a coordinate system that is rotated with respect to the original system over an angle \( \gamma \) given by

\[
\tan 2\gamma = 2\sigma_{xy}^2/(\sigma_{yy}^2 - \sigma_{xx}^2). \tag{4}
\]
If we denote the variances along the rotated axes by $\sigma_{\parallel}^2$ and $\sigma_{\perp}^2$, we can interpret such a non-isotropic distribution as an elliptical distribution, centered at the mean velocity ($\overline{v}_x, \overline{v}_y$), with major axes of radii $\sigma_{\parallel}$ and $\sigma_{\perp}$, oriented at an angle $\gamma$ relative to some preferred, fixed coordinate system, as illustrated in Fig. 1.

![Diagram](image)

Fig. 1: Schematic representation of a general, two-component velocity distribution function, with mean velocities $\overline{v}_x$ and $\overline{v}_y$, widths $\sigma_{\parallel}$ and $\sigma_{\perp}$, and orientation angle $\gamma$. The rectangular band of width $\Delta v_D$ encloses velocities with equal Doppler shifts $v_D$, for a laser beam angle $\varphi$.

Now let us consider an LIF measurement, in which a narrowband laser beam intersects the plume at an angle $\varphi$ to the centerline (see Fig. 1). Excitation of the gas occurs when the Doppler shift of the absorbing atom is equal to and opposite the detuning of the laser from linecenter. In terms of velocity, this resonance condition for the Doppler shift $v_D$ may be expressed as

$$v_D = -\lambda(\nu_L - \nu_0),$$

where $\lambda$ is the wavelength of the transition, and $\nu_L - \nu_0$ is the detuning of the laser frequency $\nu_L$ from the centerline frequency $\nu_0$. More precisely, excitation of the gas takes place if the detuning of Doppler-shifted linecenter is less than the homogeneous linewidth of the excitation process. In the experiment, this homogeneous linewidth is much less than the Doppler width of the velocity distribution of interest. In particular, the laser linewidth is three orders of magnitude smaller than typical Doppler widths of the species in the plume. Also, the laser power density in the excitation volume is sufficiently small that no significant power broadening and saturation of the transition occur. Thus, the LIF signal, for a laser beam at an angle $\varphi$ to the centerline, may be expressed as

$$S_{\varphi}(v_D) = C_{\varphi} \int_{\text{band}} f(v_x; v_y) \, dv_x \, dv_y,$$

where $C_{\varphi}$ is an experiment-dependent constant, and the “band” of integration in Eq. (6), which is shown schematically in Fig. 1, is given by the Doppler resonance condition

$$|v_x \cos \varphi + v_y \sin \varphi - v_D| < \frac{1}{2} \Delta v_D,$$

where $v_D$ is the Doppler velocity from Eq. (5), and $\Delta v_D = \lambda \Delta v_D$, where $\Delta v_D$ is the (small) homogeneous linewidth of the excitation process. It has been explicitly indicated in Eq. (6) that the LIF signal $S_{\varphi}(v_D)$ is obtained as a scan across the absorption profile of the transition, or, with Eq. (5), across the Doppler distribution projected onto the laser beam.

To proceed, it is convenient to assume a specific model function for the velocity distribution function $f(v_x; v_y)$ of the absorbing species. Let us first consider the two-dimensional, elliptical, gaussian distribution function

$$f(v_x; v_y) = \frac{n_d}{\pi \sigma_{\parallel} \sigma_{\perp}} \exp\left\{ -\frac{(v_x - \bar{v}_x)^2}{\sigma_{\parallel}^2} - \frac{(v_y - \bar{v}_y)^2}{\sigma_{\perp}^2} \right\},$$

which satisfies the normalization condition from Eq. (1), has mean velocities $\bar{v}_x$ and $\bar{v}_y$ that satisfy Eq. (2), and has variances that are given, according to Eq. (3), by $\sigma_{xx}^2 = \sigma_{\parallel}^2$, $\sigma_{yy}^2 = \sigma_{\perp}^2$, and $\sigma_{xy}^2 = 0$. Substitution of this model function into Eq. (6), and integration over the velocity band from Eq. (7), yields the corresponding Doppler profile at angle $\varphi$:

$$S_{\varphi}(v_D) = A_\varphi \exp\left\{ -\left( \frac{v_D - \bar{v}_\varphi}{\sigma_{\varphi}} \right)^2 \right\},$$

where

$$A_\varphi = \frac{\pi^{-1/2} n_d C_{\varphi} \Delta v_D}{\sigma_{\varphi}},$$

and

$$\sigma_{\varphi}^2 = \sigma_{\parallel}^2 \cos^2 \varphi + \sigma_{\perp}^2 \sin^2 \varphi.$$
That is, the variance of the Doppler profile depends on the laser angle relative to the orientation angle of the velocity distribution. In the experiment, Doppler profiles are obtained simultaneously perpendicular to the flow ($\varphi = 90^\circ$), and at an angle $\varphi = \alpha \approx 70^\circ$. By fitting, separately, the two profiles to Gaussian model functions of the general form of Eq. (9), we thus obtain, for each Doppler profile, a mean shift $v_\varphi$, a variance $\sigma_\varphi^2$, and an integrated signal $N_\varphi$, given by

$$N_\varphi = \int S_\varphi(v_D) dv_D = \pi^{1/2} A_\varphi \sigma_\varphi.$$  \hspace{1cm} (14)

In principle, it is possible to obtain the density $n_a$ of the absorbing species from the integrated signal $N_\varphi$. This, however, would require an absolute calibration of the experiment constants $C_\varphi$ and $\Delta v_D$ from Eq. (10), which is not easily accomplished. Still, measurement of $N_\varphi$ at different positions in the flow gives an indication of the relative density variations of the absorbing species in the flowfield. By contrast, interpretation of the mean velocities and variances is relatively straightforward. Namely, if we denote the fitted mean velocities by $v_{90}$ and $v_\varphi$, we note that, by Eq. (11), $v_{90}$ is just the mean velocity component $v_r$ in the radial direction, and that the mean axial velocity component $v_\alpha$ follows from

$$v_\alpha = (v_\varphi - v_{90} \sin \alpha) / \cos \alpha.$$  \hspace{1cm} (15)

Similarly, the data yield two values of the fitted variances, namely $\sigma_9^2$ and $\sigma_\varphi^2$. In the experiment, there appears to exist a small systematic difference between the two values. Although measurement at a third angle $\varphi$ would be required to determine both the variances $\sigma_9^2$ and $\sigma_\varphi^2$, as well as the orientation angle $\gamma$ of the distribution, we find that the data are consistent with the assumption that the orientation angle $\gamma$ is equal to the flow direction $\beta$, which is given by

$$\tan \beta = v_\gamma / v_\alpha.$$  \hspace{1cm} (16)

To demonstrate this correlation, we use the expression

$$\left( \frac{\sigma_\varphi}{\sigma_9} \right)^2 - 1 = \mathcal{R} \cos \alpha \cos(\alpha - 2\beta),$$  \hspace{1cm} (17)

with

$$\mathcal{R} = \left( \frac{\sigma_\parallel}{\sigma_\perp} \right)^2 - 1.$$  \hspace{1cm} (18)

This result may be obtained from Eq. (13) for small flow angles $\beta$, for which $\mathcal{R} \sin^2 \beta \ll 1$. The resulting ratio of parallel to perpendicular variances, however, is rather large ($\sigma_\parallel / \sigma_\perp \approx 2.78$), and, moreover, appears to remain fairly constant throughout the expansion. In the Appendix, it is shown that the same correlation between width ratios and flow angles can also be accounted for by an alternative assumption, namely that the distribution itself is isotropic, but has a mean flow velocity whose magnitude fluctuates on a timescale that is not resolved in the experiment. In this case, the correlation between Doppler widths and flow angles can, again, be expressed by Eq. (16), except that the coefficient $\mathcal{R}$ is now given by

$$\mathcal{R} = \left( \frac{\Delta v}{\sigma} \right)^2 - 1,$$  \hspace{1cm} (19)

where $\Delta v$ is the variance of the temporal fluctuations in the magnitude of the mean velocity, and $\sigma^2$ is the variance of the isotropic velocity distribution. We will not address this issue of non-stationary flow here in detail, except to indicate that, in the case of fluctuations in the flowfield, some care must be exercised in the interpretation of the data. Particularly, a measured variance $\sigma^2$ may lead to an overestimate of the Doppler temperature $T$, given by

$$kT = \frac{1}{2} m \sigma^2,$$  \hspace{1cm} (20)

where $m$ is the atomic mass of the absorbing species and $k$ is Boltzmann’s constant. Similarly, other broadening mechanisms may lead to overestimates of the Doppler temperature. For example, in both Refs. [11] and [12], laser power broadening was shown to be significant, and, with the pulsed laser technique used in Ref. [11], linear broadening was substantial as well. In addition, Stark broadening of the absorption process due to electrons in the plume is more serious for hydrogen than it is for argon, as is broadening due to fine-structure components within the absorption line. As remarked above, we believe that neither power broadening, nor saturation broadening, nor other types of broadening contribute significantly to the observed Doppler widths of our data, with the exception of broadening due to averaging over plume fluctuations.

Finally, although, in this work, we consider only moments up to second order of the velocity distribution function, we note that higher moments may also be considered. In fact, it is merely convenient, rather than necessary, to use a Gaussian model function to analyze the experimental Doppler profiles. As discussed in Section 5, some non-Gaussian behavior has actually been observed, particularly in the radial boundary layer of the flow. In this regard, we observe that an experimental Doppler profile is a direct measure of the velocity distribution function projected onto the laser beam direction, and that mean velocities and variances can be obtained directly from the Doppler profiles, if desired.
3. Description of the Experiment

In this section, we describe the arcjet system, the vacuum facility, the laser diagnostics setup, and the data acquisition system and analysis procedure.

Arcjet and Vacuum Facility

The arcjet in the experiment was built locally, based on the same design parameters as the familiar low power arcjet that has been tested extensively at the NASA Lewis Research Center [13]. A cut-away cross-section is shown in Fig. 2. The cathode, made out of thoriated tungsten, has a diameter of 3.2 mm, a half tip angle of 30 degrees, and is centered inside a copper anode nozzle with a boron nitride insulator. The water-cooled anode section has a 0.5 mm long constrictor channel with a diameter of 0.5 mm, and conical sections upstream and downstream of the constrictor with half angles of 30 degrees and 20 degrees, respectively. The length of the expansion region is 9.5 mm, yielding a nozzle exit radius of 3.7 mm and a nozzle expansion ratio of about 200. The anode-cathode gap is adjustable, but was maintained at a constant value of 0.60 mm. Propellant is injected tangentially through helical grooves in the boron nitride insulator.

Operating conditions of the thruster were as follows. Standard purity argon (UN 1006) was used. The flowrate was set to 135 mg/s (5 sl/min), but could not be controlled actively, allowing variations in flow rate of up to 5 percent. Using an adjustable power supply, the arc current was maintained at 10 A, yielding a corresponding arc voltage of 32 V. The same power supply, originally designed to operate a flashlamp-pumped YAG laser, was used to ignite the arc. During steady-state operation, the vacuum level in the test facility was about 40 Pa (0.3 torr). However, in some cases, the arc could only be established at higher facility pressures. In any event, data were collected only after an hour of steady-state operation of the arcjet.

The vacuum chamber has a main section that is 6.0 m long and 2.7 m wide, with a smaller section on one end, in which the arcjet was mounted. The thruster could be maneuvered in three orthogonal directions with a computer-controlled xyz-stage, that has a range of travel of 0.3 m in each direction, and a positioning accuracy of better than 0.05 mm. Although the facility has cryo-pumping capability, the only pumping for these experiments was performed by a Stokes Microvac pump equipped with a blower, yielding a pumping speed of 110 l/s.

For the above operating conditions, the luminous part of the plume is about 0.35 m long. Close to the exit nozzle, the plume has several articulated features, which are sketched in Fig. 2. In particular, it appears as if there is a narrow boundary layer on opposite sides of the plume that bends inward, and crosses (shocks) on centerline about 22 mm downstream from the exit plane. Two other such crossings, much dimmer than the first one, are observed at roughly similar intervals further downstream. In the radial direction, the boundary of the luminous portion of the plume flares outward initially, but tapers off again just beyond the crossing point of the shock lines.

One undesirable feature of the arcjet plume was that significant fluctuations of the plume could not be suppressed. These fluctuations, which were observed as “jumps” in the entire flow pattern, occurred on a timescale of one to several seconds, and could not be significantly reduced by adjusting any of the operating conditions of the thruster. Fluctuations in the arc voltage were also observed, and seemed to correlate with the visual fluctuations in the plume. Despite these fluctuations, reasonable LIF data could be obtained. However, we expect that significantly better signal-to-noise ratios would be possible for a steadier plume.

Laser Diagnostics Setup

A schematic of the laser diagnostics setup is shown in Fig. 3. The main component of this setup is a tunable ring dye laser (Coherent model CR-699), with a linewidth of approximately 1 MHz, and a frequency drift on the order of 0.3 GHz per hour. The argon absorption line used in this work was the 727.3 nm line, which, although not a metastable line, originates from an excited species of the neutral argon atom (1s4→2p2.
in Paschen notation [14]). In a separate experiment, not reported here, it was verified that the signal levels, as well as the calculated mean velocities and variances of the corresponding Doppler profiles, were roughly the same for the 727.3 nm line as for the metastable line at 714.7 nm (1s4–2p4 transition), indicating that the densities in the arcjet plume must be sufficiently high that the non-metastable 1s4 level is strongly coupled by collisions to the 1s5 metastable level.

Fig. 3: Schematic of the complete LIF setup: M=mirror; BS=beam splitter; DFC=dual frequency chopper; HC=hollow cathode lamp; L1=focusing lens; L2=collection lens; PD=photo diode (with pinhole aperture); AJ=arcjet; LOCK=phase-locked amplifier. Not shown is a wavemeter that is used to tune the laser to the desired absorption line.

A small fraction of the dye laser beam is split off into a wavemeter, to aid in tuning the laser to the desired absorption line. The remainder of the beam is directed onto an optical table, mounted at the level of one of the optical access ports of the vacuum chamber. There, the beam is split into two parallel beams of equal intensity with a prism beam splitter, and chopped mechanically with a dual frequency chopper, whose two sets of slits yield chopping frequencies of 400 Hz and 333 Hz simultaneously. A further small fraction of one of the two beams is split off, and directed into a hollow cathode discharge lamp to obtain an optogalvanic reference signal. That is, as the laser is tuned through the linecenter of the argon buffer gas in the lamp, a small change in the lamp current results, which is easily detected with the help of a phase-locked amplifier [15].

The purpose of the optogalvanic reference signal is twofold. First, it provides a very convenient means of fine-tuning the dye laser to the desired absorption line. But, more importantly, it provides an accurate, independent measurement of the non-Doppler-shifted linecenter \( v_0 \) from Eq. (5). Thus, absolute calibrations of the Doppler shifts in the arcjet plume are obtained. By contrast, in Refs. [11] and [12], linecenter is assumed to be given by the center of the radial velocity distribution. We will not describe the optogalvanic reference measurement here in detail, except to note that it, also, gives rise to a Doppler profile, which is recorded simultaneously with the two LIF signals from the arcjet plume, as illustrated in Fig. 4. In a separate series of calibration measurements, which will be described elsewhere, it was determined that the true, unshifted linecenter \( v_0 \) for the operating conditions in this experiment was actually located at \( v_{\text{ref}} \approx 30 \text{ MHz} \) ± 10 MHz, where \( v_{\text{ref}} \) is the curve-fitted linecenter of the optogalvanic Doppler profile (the 30 MHz shift, we believe, is the result of a small drift velocity of neutral atoms in the discharge lamp).

Fig. 4: Typical LIF Doppler profiles at 90° and 76°, recorded 1 mm downstream from the exit plane, 1 mm off centerline (on "upstream" side of laser beam). Gaussian curve fits are shown as well. The arrow indicates linecenter, as determined from the optogalvanic reference signal (rightmost curve).

To obtain the LIF signals, the two parallel laser beams are directed into the vacuum chamber, and are focused by a 125 mm lens inside the chamber so that the two beams overlap near the arcjet nozzle exit at a relative angle of \( \alpha = 13.6 \) degrees. This angle is large enough to calculate the axial velocity component reliably using Eq. (15), but it is not so large that greatly different Doppler shifts result for the two beams. One of the two beams is parallel to the nozzle exit plane (see Fig. 2), whereas the other strikes the nozzle exit surface obliquely. This is acceptable, because the resulting scattering occurs outside of the acceptance angle of the detection system.
To collect the fluorescence (which, for excitation at 727.3 nm, consists primarily of lines at 810.8 nm, 772.4 nm, and 696.5 nm [14]), a 100 mW focal length lens with a diameter of 50 mm is mounted perpendicular to the horizontal plane in which the two laser beams cross. In this way, the fluorescence radiation is focused onto a photodiode with a good spectral response in the 700 to 900 nm range (Hamamatsu model S-1722-02). Because unit magnification of the detection system is used, the effective size of the collection volume at the overlap of the two laser beams is equal to the diameter of the pinhole on the photodiode, which was 1 mm for the experiments described here. That is, the spatial resolution of the data, along the direction of the laser beams (the y-direction in Fig. 1), is about 1 mm, whereas the resolution along the centerline (the x-direction) is roughly equal to the diameters of the focused laser beams, which are on the order of 0.1 mm.

Although the signal-to-noise ratio of the fluorescence signal might be improved by the use of suitable filters on the photodiode, or by use of a more sensitive detector, the photodiode signal was sufficiently strong that reasonable data could be obtained with phase-locked detection. Indeed, during initial testing of the arcjet and of the LIF technique in a smaller vacuum chamber, the fluorescence signal for excitation at linecenter was actually sufficiently strong that it could be observed with the unaided eye. Most of the data that are reported here were taken with a total dye laser power of about 100 mW, with an estimated 20 mW in each beam at the overlap volume in the plume. By increasing the power level momentarily, it was confirmed that, at these power levels, no significant power broadening or saturation broadening of the measured Doppler profiles occurred.

Once more we note that, to obtain measurements at different positions in the arcjet plume, the thruster itself could be moved by using the computer-controlled xyz-stage. This has the important advantage that alignment of all optical components has to be performed only once. Particularly, alignment of the laser beams has to be performed only once. Particularly, alignment of the detector (the x-direction) is roughly equal to the diameter of the pinhole on the photodiode, which was 1 mm for the experiments described here. That is, the spatial resolution of the data, along the direction of the laser beams (the y-direction in Fig. 1), is about 1 mm, whereas the resolution along the centerline (the x-direction) is roughly equal to the diameters of the focused laser beams, which are on the order of 0.1 mm.

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Data Acquisition and Data Analysis

Desktop computers are used both for data acquisition and data analysis. Doppler profile scans (an example of which is shown in Fig. 4) are accomplished by a single program that drives the laser scan frequency (using a digital-to-analog converter), and reads the three phase-locked amplifier signals (using an analog-to-digital converter), namely, the optogalvanic reference signal and the two LIF signals, which are derived by phase-locking the same photodiode signal to the two different frequencies from the dual frequency synthesizer. Thus, a single Doppler scan consists of repeatedly incrementing the laser frequency by a set amount, waiting for the signals of the phase-locked amplifiers to equilibrate, and then reading, in quick succession, the three Doppler signals, repeating the process until the scan is complete.

Given the rather strong, undesirable fluctuations in the arcjet plume, we chose to use rather slow scans, allowing the use of a 1 second time constant on the phase-locked amplifiers. Thus, the duration of a single, 8 GHz (0.27 cm⁻¹) scan was two minutes, yielding 52 data points per scan. Although the resulting increments are much wider than the precision of the reference signal (namely, 150 MHz versus 10 MHz), we found that, because of the plume fluctuations, the fitted mean velocities were accurate to only about 30 MHz, even for much larger numbers of samples per Doppler profile. The resulting accuracy of the data, about 30 MHz, corresponds to a velocity resolution of about 20 m/s, using Eq. (5).

Data analysis was performed separately, that is, after completion of the measurements. A simplex least-squares fitting program was used to fit each of the three signals to a gaussian model function as expressed in Eq. (9), yielding fitted values for the amplitude, width, and mean velocity of each profile. A typical example of the measured signals along with the gaussian curve fits is shown in Fig. 4.

4. Experimental Results

In this section, we report the results of two series of LIF measurements that were taken for arcjet operating conditions as described in Section 3. In the first series, the arcjet was operated for 1 mm downstream from the nozzle exit plane, in a horizontal plane through the arcjet centerline. These data were taken on two consecutive days, with excellent reproducibility. In a second series of measurements, axial scans of the arcjet plume were performed, namely on centerline.

Radial Data near Exit Plane

Results of the radial scans at z = 1 mm are shown in Figs. 5, 6, and 7. First, Fig. 5 shows the measured
mean velocities, obtained from the laser beams at angles 90° and 76° to the flow, and the axial velocity, constructed from these two sets of data using Eq. (15). The same data are also presented in Fig. 5 in the form of a vector velocity diagram. Whereas the axial velocity peaks on centerline, namely at about 3 km/s, the radial velocity reaches its largest value—about 300 m/s—at a radius of about 2.5 mm. The corresponding variation in flow angle $\beta$, as defined by Eq. (16), is from 0° on centerline to about 15° at the edges of the plume. By comparison, the half-angle of the conical diverging section of the nozzle is 20°.

![Fig. 5: Radial velocity profiles obtained 1 mm downstream from the exit plane. Data on the right are constructed from the two measured velocity components on the left. Open and closed symbols correspond to data sets taken on two consecutive days. Error bars on the open symbols were calculated from a series of three runs on the first day.](image)

![Fig. 6: Vector velocity diagram obtained from the data in Fig. 5 (using the data represented by the closed symbols only). The nozzle boundary is also indicated. Aspect ratios for velocity and length scales are chosen such that all angles are shown correctly.](image)

![Fig. 7: Radial profiles of the average widths (a) and average integrated signals (b) of the data from Fig. 5. Both sets of data clearly indicate boundary layer behavior at radii of about 2.3 mm, which is well within the 3.4 mm radius of the exit nozzle. A corresponding temperature scale is indicated in (a) as well.](image)

From Figs. 5 and 6 it is obvious that, although the measured velocity distribution is symmetric in the central region of the plume, significant deviations are observed beyond a radius of about 2.5 mm. In fact, for negative values of $y$ (that is, "upstream" of the laser beams), we actually observe a flow that is directed back toward the thruster. Although we do not have a good explanation for this result, we note that the effect is reproducible from the first to the second day of the measurements. We speculate that the observed backflow is a result of recirculation close to the nozzle exit surface, and is probably aggravated by the wide, blunt, exit surface of the arcjet nozzle. The asymmetry of the plume away from the centerline is probably the result of a non-circular constrictor channel.

Widths and integrated signals for the same radial data are presented in Fig. 7. In both cases, the values that are shown are actually averages of the corresponding quantities from the Doppler profiles at 90° and at 76°. Although some differences between the two sets
of data exist, the trends in the individual sets are the same as those apparent from the averages in Fig. 7. A discussion of the small differences is deferred to the end of Section 5.

The widths in Fig. 7(a), which are calculated as full widths at half maximum (fwhm), are obtained from the fitted variances $\sigma^2_\phi$ according to $\text{fwhm} = 2\sqrt{\ln 2}\sigma_\phi$. Just like the axial velocity profile, the width profile peaks on centerline, with a maximum velocity spread of 1.8 km/s. However, unlike the velocity distribution, the widths display a sharp minimum at a radius of about 2.3 mm. For larger radii, the width increases, particularly for negative values of $y$, where backlight toward the thruster is found. Note also the relatively large variations in the widths at the edges of the plume as evidenced by the error bars in Fig. 7, which indicate the spread in the three series of radial scans from the first day. Again, we must speculate about the origin of the unexpected behavior at the edge of the plume. Our guess is that the same recirculation phenomenon that is responsible for the net backlight toward the thruster above, is also responsible for the anomalous widths at the edges of the plume.

Given such effects as recirculation near the edges of the plume, and plume fluctuations that are, to a large extent, averaged out over the duration of a Doppler scan, it is, of course, difficult to assign a temperature to the flow, simply on the basis of the observed variance. Still, using Eq. (20), we have indicated such a temperature scale in Fig. 7(a). We believe that, while, due to broadening effects other than Doppler broadening, these temperatures may underestimate the actual temperature of the absorbing species—and almost certainly do so significantly near the edges of the plume—reasonable temperature values result near the center of the plume. Using the temperatures indicated in Fig. 7(a), and using the value 1.67 for the specific heat ratio $\gamma$, we conclude that the Mach number on the arcjet centerline is about $M = 3.1 \pm 0.1$.

Average integrated signal levels, given by Eq. (14), are shown in Fig. 7(c). Unlike results that were reported for a hydrogen arcjet [11,12], we find that the integrated signal, and, therefore, presumably, the relative density of the absorbing species, is not actually highest on centerline, but at the edges of the plume. We must recall that the absorbing species is not actually the ground state of the argon atom, but an excited species, whose relative population depends, possibly, on the flow conditions, and, therefore, on the position in the flow. Another possible mechanism that could lead to an integrated signal whose magnitude depends on the flow conditions, even for constant relative densities of the absorbing species, would be that non-radiative quenching of the upper state of the transition is more serious on centerline, due to the larger temperatures of the flow.

Without speculating on the ramifications of such processes for the interpretation of the data, we note that, in any event, the integrated signal data, combined with the data on the widths of the distribution, clearly indicate that there is a boundary layer in the flow at a radius of about 2.5 mm, which is well inside the $r_0 = 3.4$ mm exit nozzle radius. Actually, we note that the peaks of the integrated signal data are located at a slightly larger radius than the sharp minima in the Doppler width profile. We also want to recall that the spatial resolution of the LIF data is only about 1 mm along the laser beams, which is roughly the width of the sharp features in Figs. 7(a) and 7(b). Thus, the widths of the actual features, that is, the widths as they would be observed in an experiment with better spatial resolution, are possibly much narrower than 1 mm, which is not unexpected for a boundary layer in the flow. In this regard, we also note that, indeed, a thin, luminous layer is observed at the edge of the plume, originating at a radius not much different from the exit nozzle radius.

Finally, we note that, from the above data, we are able to estimate the specific impulse $I_\text{sp}$ of the thruster. Namely, if we assume that the axial velocities and the integrated signals for the absorbing species are characteristic of, respectively, the bulk axial velocity of the flow, and the bulk density of the flow, we arrive at the mass-flux-averaged value of $1.6 \pm 0.1$ km/s for the axial velocity, so that the corresponding specific impulse is $160 \pm 10$ seconds. In our facility, we have no way of measuring the thrust directly. However, the value of 160 seconds appears to be within reason (for example, a value of 185 seconds is reported in [16] for operation of a larger thruster on argon, namely with mass flows of 0.1 mg/s, at a power level of 1 kW).

Axial Data on Centerline

A second series of data was taken on the centerline of the arcjet, over the range 0.5 mm to 30 mm from the exit nozzle. The results of these data are shown in Fig. 8. Before discussing the data in detail, we refer, again, to Fig. 2, which shows a sketch of the plume pattern as it appears to the eye. In particular, we want to recall that the two converging, bright boundaries that originate at the edge of the nozzle exit radius cross on the centerline at about 22 mm downstream from the exit nozzle, presumably causing shock behavior at this location.
Figure 8(a) shows the measured values of the radial velocity component, and the deduced value of the axial component. The latter is remarkably constant throughout the expansion, dropping only slightly from the value 3 km/s close to the exit nozzle. The radial component is negligible, as expected, until the position of the shock is reached at \( x = 22 \) mm.

Unlike for the radial profiles from Figs. 5 and 7, the (average) widths of the distributions, shown in Fig. 8(b), do not actually follow the same behavior as that of the axial velocity component. Indeed, the Doppler width drops from an initial 1.8 km/s at the exit plane to about 1.25 km/s at the shock location \( (x = 22 \) mm), but rises rapidly to the original value of 1.8 km/s immediately behind the shock. If we interpret, just like for the data from Fig. 7(a), the width of the Doppler profile as a direct measure of the kinetic temperature of the gas, we find that, on centerline, the corresponding drop in temperature from the exit plane (2800 K) to the first shock location (1400 K) is roughly a factor two. The corresponding rise in axial Mach number is from \( M = 3.0 \) to \( M = 4.1 \).

Finally, we present, in Fig. 8(c), the average integrated signals for the axial data. Somewhat surprisingly, the signal rises initially, then drops, and, at the location of the shock at \( x = 22 \) mm, continues to rise once more. Although a density increase following a shock is expected, the initial rise is more puzzling, and we must suffice by stating that more experiments and/or modeling are required to explain this behavior. However, it is still gratifying that, at the exact location in the plume where, by mere visual inspection, shock behavior appears to exist, all experimental quantities clearly reflect changes in flowfield properties as well.

5. Further Data Analysis

We now consider a few more aspects of the data, which serve, to a large extent, as a check on the principles of the two-beam, multiplexed, LIF technique. First, we consider, again, the determination of Doppler widths. Although we have presented, in Section 4, only the average values of the Doppler widths from the two LIF measurements, there is, in fact, a small, systematic difference between the widths for the Doppler profiles perpendicular to the beam, and the widths at the angle \( \alpha \approx 70^\circ \). This is illustrated in Fig. 9, in which, according to Eq. (17), the quantity \( (\sigma_\alpha / \sigma_{90})^2 - 1 \) is plotted as a function of the quantity \( \cos(\alpha) \cos(\alpha - 2\beta) \), where \( \beta \) is the mean flow angle as determined by Eq. (16). In other words, Fig. 9 shows the anisotropy

![Fig. 8: Axial profiles of radial and axial velocity (a), average velocity spread (b), and average integrated signal (c) on centerline. All data reflect clearly the shock behavior that is observed visually as the crossing of two shock lines, 22 mm downstream from the exit plane (see Fig. 2).](image)

![Fig. 9: Dependence of the Doppler width ratio \( \sigma_\alpha / \sigma_{90} \) on the flow angle \( \beta \) for the data from Figs. 5, 6, and 7. Specifically, the data are plotted and curve fitted in accordance with Eq. (17), yielding a slope \( R = 1.87 \pm 0.28 \). Choice of closed and open symbols is the same as in Figs. 5, 6, and 7. The diamond with error bar indicates the average and spread of the corresponding quantities from the axial data from Fig. 8.](image)
of the Doppler widths as a function of flow angle (note, for example, by inspection of Fig. 6, that the flow angle is essentially proportional to the radial position in the flow). Although there is a considerable scatter of the data, a correlation of anisotropy and flow angle is evident, and a one-parameter linear fit, according to Eq. (17), which is indicated by the solid line in Fig. 9, yields a slope $R = 1.87 \pm 0.28$.

As discussed in Section 2, there are at least two possible explanations for such a correlation. Firstly, assuming steady-state plume behavior, this correlation can be interpreted as being due to different parallel and perpendicular variances of the velocity distribution function, for example, as illustrated in Fig. 1. Secondly, as calculated in the Appendix, the correlation may be explained by assuming that, in the experiment, averaging over plume fluctuations takes place, specifically, over fluctuating values of the magnitude of the mean flow velocity.

From the linear-fit value of the slope $R$, the first interpretation, using Eq. (18), would lead to a variance ratio $(\sigma_\parallel/\sigma_\perp)^2 = 2.87 \pm 0.28$ (or a width ratio $\sigma_\parallel/\sigma_\perp = 1.69 \pm 0.30$). Interpreting, in turn, the variance ratio as a kinetic temperature ratio, according to Eq. (20), we find that the parallel temperature is thus almost three times higher than the perpendicular one. Such a large difference is not very plausible for the high densities at the exit nozzle, where, as a result of collisions in the flow, parallel and perpendicular temperatures are expected to be roughly equal. Furthermore, almost no systematic variation of the variance ratio of the Doppler profiles at 90° and at 76° is observed for the axial data from Fig. 8. An average value for these data is indicated in Fig. 9 by the diamond-shaped symbol at $\beta = 0°$, along with an error bar which indicates the spread of the axial data around the average value. Note that the axially averaged value is consistent with the linear fit to the radial data. However, if there were any mechanism at work that yields unequal parallel and perpendicular temperatures at all, one would expect the ratio to vary as conditions in the flow change.

The second interpretation, namely that different Doppler widths at 90° and 76° are essentially an experimental artifact that is caused by averaging over fluctuations in the magnitude of the mean velocity, yields, with Eq. (19), a value $2.87 \pm 0.28$ for the ratio of the variance of the fluctuations to the variance of the steady-state, isotropic velocity distribution function. In light of the observed fluctuations of the arcjet plume, and the intentionally long measurement times to produce data with a good signal-to-noise ratio, this explanation for the different variances at laser beam angles of 90° and 76° is probably more plausible than the one above, yielding the large, axially constant, parallel-to-perpendicular temperature ratio. Additional measurements will be required to resolve this issue (for example, non-averaged, high sample rate data at a few select positions within the Doppler profile). However, from the calculated value of the magnitude of the fluctuations, we can estimate how much the Doppler temperatures that are indicated in Fig. 7(a) overestimate the actual kinetic temperatures in the flow. Namely, assuming that the fluctuations are adequately described by the model in the Appendix, the relative contribution of the fluctuations to the measured variance of the Doppler profile is equal to $R \cos^2(\varphi - \beta)$, where $\varphi$ is the laser beam angle. From the fitted value of $R$, it thus follows that the temperatures in Fig. 7(a) overestimate the pure kinetic temperature (assuming no other broadening mechanisms) by an average of 10 percent, with some dependence on the flow angle. Thus, although the variance of the magnitude of the fluctuations appears to be almost three times as large as the variance associated with the kinetic temperature of the distribution, the resulting overestimate of the kinetic temperature is not very serious. The reason for this is that both LIF measurements are performed roughly perpendicular to the flow, and, therefore, roughly perpendicular to the direction of the fluctuations. It is not difficult to see, however, that for other types of fluctuations of the flow, for example, for fluctuations lateral to the flow direction, the broadening of the Doppler profiles can be more severe, and the estimate of Doppler temperature more badly flawed. In general, one must recognize that in any LIF measurement, averaging takes place over some time interval, so that it will be difficult to argue that the measured Doppler width is strictly a reflection of the kinetic temperature of the absorber, and is unaffected by averaging over fluctuations in the flow, including, for example, turbulence.

Next, let us consider briefly some systematic differences between the integrated signals of the Doppler profiles at the two different laser beam angles. The ratios of these integrated signals are plotted in Fig. 10, both for the radial data from Figs. 5, 6, and 7, and for the axial data from Fig. 8. Just as for the widths of the profiles, only average values of these integrated signals are presented in Figs. 7(b) and 8(c). In fact, from Eqs. (14) and (9), it follows that the ratio of these signals should depend only on the experiment constants $C_\varphi$ at the two angles $\varphi = 90°$ and $\varphi = 76°$. Indeed, for the axial data on centerline from Fig. 10(b), a constant value of the ratios of the experiment constants $C_\varphi$ is found, namely $C_{90°}/C_{76°} = 0.87 \pm 0.03$ (note that this value does not necessarily have to be unity,
since it involves ratios of a number of quantities which are not necessarily equal for the two LIF signals, for example, laser intensities, effective sizes of the detection volumes, and sensitivities of the phase-locked amplifiers). Considering the substantial variations in widths and magnitudes of the Doppler profiles for this range of axial positions, as evident in Figs. 8(b) and 8(c), this constant signal ratio is very satisfactory. The radial data from Fig. 10(a), also, yield roughly the same signal ratio, at least, within the radius of the boundary layer. However, outside of the boundary layer, namely for radii larger than about 2 mm, substantial deviations from this constant signal ratio are encountered. In particular, the signal at the laser angle \(76^\circ\) increases by almost a factor of two with respect to the signal perpendicular to the centerline. Although we do not have a good explanation for this result yet, we believe that it is a consequence of having a spatial resolution (1 mm along the directions of the laser beams) that is, probably, significantly wider than the characteristic width of the boundary layer. We would like to point out that the variation in signal ratio is not, likely, caused by averaging over plume fluctuations. Namely, for the model that is used in the Appendix, the increase in width of the Doppler profile is compensated by an equivalent decrease in the amplitude, so that the integrated area under the Doppler profile remains constant. Again, further experiments are required to resolve the issue.

![Graph](image)

**Fig. 10:** Ratios of the integrated signals \(N_{90}/N_{\alpha}\) for (a): the radial data from Figs. 5, 6, 7; and (b): the axial data from Fig. 8.

Finally, we present one more result that indicates that some complicated physical phenomena are taking place in the radial boundary layer of the flow. That is, the Doppler profiles close to the exit nozzle, at radii of about 2.5 mm, are manifestly non-gaussian. This is illustrated in Fig. 11, which shows one Doppler profile that was taken close to the boundary layer. Comparison with the more typical data from Fig. 4 shows that the gaussian fit to the data is actually too wide, and does not have a sufficient amplitude to give a good fit to the data. This behavior was found to be quite reproducible, and be the same on both sides of the centerline. Let us recall that we have introduced gaussian fit functions merely as a convenient tool to analyze the data, and that the Doppler profiles are a direct measurement of the projection of the velocity distribution function of the absorbing species onto the propagation direction of the laser beam. Although we not have an explanation for this non-gaussian behavior of the velocity distribution function in the boundary layer, we present it here as another demonstration of the ability of the LIF technique to provide detailed information about the nature of the flow.

![Graph](image)

**Fig. 11:** Example of a non-gaussian Doppler profile in the radial boundary layer. Note that the best gaussian fit is too wide and has insufficient height to achieve the same quality of fit that is evident in the more typical data shown in Fig. 4.

6. Discussion and Summary

We have demonstrated that an LIF technique, that has recently been employed to measure axial velocities in hydrazine and hydrogen arcjets, can be extended to measure radial and axial velocities in the arcjet plume simultaneously, and that, in addition to measuring vector velocity distributions in the plume, both qualitative and quantitative information can be extracted from the widths and integrated signals of
the measured Doppler profiles. Particularly, the signatures of a radial boundary layer and of an axial shock downstream from the exit nozzle are clearly evident in the data. Also, we have demonstrated that simultaneous recording of the optogalvanic signal in a small discharge lamp is a very convenient method for establishing an absolute calibration of the Doppler shifts in the arcjet plume.

Although, in Refs. [11] and [12], only a qualitative discussion of the widths and strengths of the LIF signals is given, it appears that our data are qualitatively different from the work in Refs. [11] and [12] in that the LIF signals in our experiment are not largest on centerline, but rather in the boundary layer at the edge of the plume. Because absorption of excited argon, rather than of excited hydrogen, is used in our work, it is difficult to explain this qualitative difference without further research. Qualitative agreement with the work in Refs. [11] and [12] was found in that the axial velocity is largest on centerline, and drops off, uniformly (at least, until the boundary layer is reached) with increasing radius.

One undesirable feature of our experiments is that fluctuations of the plume could not be completely suppressed. We have discussed some possible artifacts in the data, that may have resulted from averaging over such fluctuations, particularly broadening of the measured Doppler profiles. With a steadier plume, it should be possible to obtain better signal-to-noise ratios for individual data points, allowing scanning times per Doppler profile to be reduced significantly, perhaps to as little as a few seconds per scan. Also, the blunt arcjet nozzle used in this work appears to give rise to some recirculation close to the nozzle exit radius, as evidenced by a net backflow on one side of the nozzle.

Extension of our two-beam multiplexing technique to the hydrogen Balmer-α line should be straightforward, although, as stated in Refs. [11] and [12], a somewhat more careful analysis would be required to take into account Stark broadening and broadening due to fine structure components of the transition.

We conclude that we have successfully demonstrated that two-beam multiplexing can be used as a powerful diagnostic tool for characterizing two-component velocity distributions in an arcjet exhaust plume.

Acknowledgments

This work was supported jointly by NASA under grant NAGW-1195, by the Air Force Office of Scientific Research under grant AFOSR-91-0200, and by Boeing, Defense and Space Group, Seattle, Washington, with grant monitor J. Meserole. For technical assistance, we are much indebted to J. Hornkohl, C. Parigger, F. Schwartz, and, in particular, N. Wright. WMR gratefully acknowledges discussions with A. Eraslan and F. Collins, and J. Lewis. We also acknowledge assistance from T. Gogel and A. Sedghinasab in an early stage of this work.

Appendix

Because visual inspection of the arcjet plume in the experiment indicates that significant plume fluctuations occur, we describe here a simple model to calculate the effect of such fluctuations on the measured Doppler profiles. Specifically, we assume that the magnitude of the mean flow velocity fluctuates with a characteristic frequency $\omega$ according to

$$v(t) = \bar{v}_0 + 2^{1/2} \Delta v \cos \omega t,$$

where the mean velocity $\bar{v}_0$ is given by the mean average velocity components $\bar{v}_x$ and $\bar{v}_y$ from Eq. (11) according to $\bar{v}_0^2 = \bar{v}_x^2 + \bar{v}_y^2$, and $\Delta v$ is the variance of the fluctuations with respect to temporal averaging. Repeating the calculation of the experimental Doppler profile which leads, in Section 2, to Eqs. (9)–(12), and assuming, for simplicity, an isotropic velocity distribution with variance $\sigma^2 = \sigma^2_x = \sigma^2_y$, we find that the Doppler profile becomes time-dependent, namely

$$S_\varphi(v_D;t) = A_\varphi \exp\left\{ -\left( \frac{v_D - v_\omega(t)}{\sigma} \right)^2 \right\},$$

where

$$v_\omega(t) = (\bar{v}_0 + 2^{1/2} \Delta v \cos \omega t) \cos(\varphi - \beta),$$

and where $A_\varphi$ is given by Eq. (10), upon replacing $\sigma_\varphi$ by $\sigma$. Note that, with the definition of the flow angle $\beta$ from Eq. (16), Eq. (A3) reduces to Eq. (11) in the limit $\Delta v = 0$. To carry out a temporal average of the time-dependent Doppler profile from Eq. (A2), we use the fact that, for small $\epsilon$,

$$\langle \exp(-a + \epsilon \cos \omega t)^2 \rangle \simeq \exp(-\frac{1}{2} \epsilon^2) \exp\left( -\frac{a^2}{1 + \epsilon^2} \right),$$

where the brackets on the left indicate temporal averaging. From this result, the temporally averaged Doppler profile from Eq. (A2) becomes

$$\langle S_\varphi(v_D;t) \rangle = A'_\varphi \exp\left\{ -\left( \frac{v_D - \bar{v}_0}{\sigma'} \right)^2 \right\},$$

where

$$A'_\varphi = A_\varphi \exp\left[ -\frac{\Delta v^2}{2\sigma^2} \cos^2(\varphi - \beta) \right],$$

(45)
and
\[ \sigma'^2 = \sigma^2 + \Delta v^2 \cos^2(\phi - \beta). \tag{A6} \]

Thus, as a result of temporal averaging, the Doppler profile is broadened, and has a smaller amplitude. However, the integrated area under the Doppler profile is independent of the fluctuations, which follows from the fact that \( A^2 \sigma^2 = A_2 \sigma^2 \) (to second order in \( \Delta v \)), or, alternatively, by the observation that the order of integration and time-averaging may be interchanged. Note that, in this calculation, the choice of the fluctuation frequency \( \omega \) is irrelevant for the final result, which, therefore, holds also for fluctuations with a more complex temporal behavior, but with the same variance \( \Delta v^2 \).

References