AN OPTIMAL DESIGN PROCEDURE FOR A PULSE FORMING NETWORK

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ABSTRACT

Design Procedure

In this paper a procedure for the design of a PFN feeding a quasi-steady MPD thruster is presented. Given the design parameters (pulse duration, pulse rise time, pulse current amplitude and load equivalent resistance), the procedure allows one to obtain the value of the inductances and capacitances for an optimal design of an L-C ladder feeding network.

In the procedure an auxiliary network having all branches in parallel, and each branch formed by a series of a capacitor and an inductor, is used. By means of this auxiliary network, which allows a simple expression of its generated current pulse, the equivalent L-C ladder network is deduced. Finally a suitable performance index is considered in order to optimise the obtained network, taking into account the presence of parasitic elements.

NOMENCLATURE

- \( T \): total pulse duration
- \( r \): pulse rise time
- \( t_p \): pulse working time
- \( \alpha \): ratio between \( r \) and \( T \)
- \( I_d \): required pulse current amplitude
- \( i_l \): real current delivered to the load
- \( V_0 \): initial charged voltage

INTRODUCTION

A power source, capable of feeding a quasi-steady MPD thruster, must deliver current pulses of high value and a suitably long duration. These characteristics can be obtained by using appropriate "L-C ladder" networks.

The L-C ladder networks have been theoretically and experimentally studied. By means of these studies simple mathematical relations between the rise time, the duration, the amplitude of the pulse and the values of the capacitance and inductance of the branches of the network were derived.

The above relations are valid only when \( L \) and \( C \) are the same in each branch and when the parasitic parameters are negligible.

This paper deals with a design procedure that optimizes the L-C branches in order to obtain a given trapezoidal current pulse and takes into account the parasitic effects overcoming these limitations.

DESIGN PROCEDURE

a) Input data

The input data are the pulse duration, the rise time, the amplitude of the current, the number of the sections of the L-C ladder network and the equivalent resistance \( R_1 \) of the MPD thruster (see Figg. 1 and 2).

b) Auxiliary network

Starting with the input data and following Guillemin's theory, the values of the capacitances and inductances of an auxiliary network with parallel branches (see fig.3) are determined.
The auxiliary network determined above can be transformed into its equivalent ladder network by comparing the output impedances of the networks.

The impedance of the parallel branches network results:

$$z_{\text{par}} = \sum_{k=1}^{n} \left( \frac{1}{1 + s^2 L_k C_k} \right)$$

$$\sum_{k=1}^{n} \frac{n_k}{s C_k} \left( \frac{n_j}{1 + s^2 L_j C_j} \right)$$

where $n$ is the number of the branches in parallel.

The impedance of the ladder network can be written by making continued-fraction expansions of the reactance or admittance functions in the form:

$$z_{\text{lad}} = \frac{1}{s C_1} \frac{1}{s C_2} \ldots \frac{1}{s C_n}$$

An expression of the form of (6) can be derived from (5) by dividing the numerator of (5) by its denominator, which gives $1/s C_n$ inverting the remaining fraction and dividing again, which gives $C_{n-1}$ and continuing the process.

d) Parasitic parameters

The above ladder network is completed by adding the parasitic parameters. The parameter values were considered proportional to the corresponding elements. They are normally identified by a parasitic inductance $L_p$ and resistance $R_p$ in series with the capacitors and by a parasitic resistance $R_C$ in series with the inductors. Their values are given by the manufacturers, and can be controlled by measurements.

e) Optimal procedure

The complete pulse forming network (see Fig.4), is finally optimised, using a minimum square deviation performance index:

$$Q = \frac{1}{\pi} \int_{-\infty}^{\infty} \left[ I_d(t) - I(t) \right]^2 dt$$

where $I_d(t)$ and $I(t)$ are the desired and actual currents, respectively.
Fig. 4 Complete L-C ladder network.

where $W_i$ are suitable weights, $i(t)$ is the required current and $i_1(t)$ the obtained current, and $m$ is a suitably big natural number.

The loop currents in the network (see Fig. 4) can be expressed by the recursive expression:

$$i_j = \frac{(s^2C_j^2+1LC_j+1+SC_j^2+1RC_j+1+C_j)}{(s^2C_j^2+1(L_j+LC_j+1+C_j)+C_j+1)\text{den}(i_{j-1})^-1}$$

$$V_0C_n\text{den}(i_{n-1}) = \frac{sC_jC_{j+1}(R_{L_j}+R_{C_j}+C_j+1)\text{den}(i_{j-1})^-1}{[s^2C_n^2+1(L_n+LC_n)+sC_n(R_{L_n}+R_{C_n}+R_{\text{load}})+1]}$$

where $\text{num}(i)$ and $\text{den}(i)$ are respectively the numerator and the denominator of the expression of the current, with $\text{num}(i_0)=0$ and $\text{den}(i_0)=1$, and $V_0$ is the initial charged voltage.

From the knowledge of the loop currents the currents in the branches of the network can be easily derived, since the current in the inductor of the horizontal branches is equal to the corresponding loop current, and the current in the capacitor of the j-th vertical branch is equal to the difference between $i_1$ and $i_{j-1}$.

$$i_1 = \frac{1}{\pi} \int_0^\pi [i_d(t) - i_1(t)]^2 dt$$

$$A = \frac{1}{\pi} \int_0^\pi [i_d(t) - i_1(t)]^2 dt = \frac{1}{\pi} \int_0^\pi \left[1 + \frac{A}{RE_1^2 + IM_1^2}\right] dt$$

$$+ \frac{n-1}{\pi} \int_0^\pi \left[C \frac{E}{D} + F\right] dt$$

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where $A = RE_1 \cos(2IM_1 t + 2PH_1) + IM_1 \sin(2IM_1 t + 2PH_1)$.

$$B = RE_1 + RE_1$$

$$C = (RE_1 + RE_1) \cos((IM_1 - IM_1) t + PH_1 - PH_1) + (IM_1 - IM_1) \sin((IM_1 - IM_1) t + PH_1 - PH_1)$$

$$D = (RE_1 + RE_1)^2 + (IM_1 - IM_1)^2$$

$$E = (RE_1 + RE_1) \cos((IM_1 + IM_1) t + PH_1 + PH_1) + (IM_1 + IM_1) \sin((IM_1 + IM_1) t + PH_1 + PH_1)$$

$$F = (RE_1 + RE_1)^2 + (IM_1 + IM_1)^2$$

$$G = e^{RE_1 t} \cos(RE_1 t - 1)$$

$$H = \cos(IM_1 t + PH_1)$$

$$J = \sin(IM_1 t + PH_1)$$
\[ K = R_E \cos(I_M t + \Phi_H) + \Im I_M \sin(I_M t + \Phi_H) \]
\[ L = R_E \sin(I_M t + \Phi_H) - \Im I_M \cos(I_M t + \Phi_H) \]
\[ M = (R_E)^2 + (\Im I_M)^2 \]
\[ \tau_d = \tau_r + \tau \]

The obtained lossless network was completed with the parasitic parameters derived by indications of manufacturers:

\[ L_{C1} = 1.3 \mu F \quad R_{C1} = 20.9 \text{ m}\Omega \quad R_L = 9.4 \text{ m}\Omega \]
\[ L_{C2} = 0.6 \mu F \quad R_{C2} = 10.5 \text{ m}\Omega \quad R_L = 6.4 \text{ m}\Omega \]
\[ L_{C3} = 0.5 \mu F \quad R_{C3} = 8.8 \text{ m}\Omega \quad R_L = 6.2 \text{ m}\Omega \]
\[ L_{C4} = 0.5 \mu F \quad R_{C4} = 8.7 \text{ m}\Omega \quad R_L = 5.6 \text{ m}\Omega \]
\[ L_{C5} = 0.4 \mu F \quad R_{C5} = 7 \text{ m}\Omega \quad R_L = 5 \text{ m}\Omega \]
\[ L_{C6} = 0.5 \mu F \quad R_{C6} = 8.6 \text{ m}\Omega \quad R_L = 8.2 \text{ m}\Omega \]

Starting from these values the optimisation procedure was performed, either with the parasitic parameters fixed and proportional to the corresponding inductance and capacitance values, or with the parasitic values free to vary.

In Table I and II the results for both cases are reported.

The deviation from the assigned pulse waveform, evaluated by (7) resulted in pulses with fixed or free parasitic parameters.

In Fig. 5 the pulses delivered with the optimised network are reported. In the same figure the pulse obtained with a ladder network whose parameters were determined using the Simplex Method 4, imposing that the sum of the capacitances must remain constant and equal to the starting value, obtained by the process described above.

### RESULTS

The presented optimal design procedure was used to obtain a trapezoidal pulse having the following characteristics:

- Pulse duration \( \tau [\text{ms}] \) = 1
- Pulse rise time \( \tau_r [\text{ms}] \) = 0.1
- Pulse fall time \( \tau_f [\text{ms}] \) = 0.1
- Pulse current amplitude \( I_d [\text{KA}] \) = 10

To obtain this pulse a ladder PFN was chosen, having six sections and initial charged voltage \( V_0 \) equal to 2 kV. These values were deduced by the simplified \( I_\text{r} = \_ \) analysis reported in bibliography 4.

The parameters of the obtained auxiliary network were:

- \( C_1 = 3987 \ \mu\text{F} \)
- \( L_1 = 25.4 \ \mu\text{H} \)
- \( C_2 = 386.6 \ \mu\text{F} \)
- \( L_2 = 29.1 \ \mu\text{H} \)
- \( C_3 = 103.2 \ \mu\text{F} \)
- \( L_3 = 39.3 \ \mu\text{H} \)
- \( C_4 = 30.4 \ \mu\text{F} \)
- \( L_4 = 67.9 \ \mu\text{H} \)
- \( C_5 = 5.5 \ \mu\text{F} \)
- \( L_5 = 228.7 \ \mu\text{H} \)
- \( C_6 = 3 \ \mu\text{F} \)
- \( L_6 = 279.6 \ \mu\text{H} \)

The corresponding values of the equivalent ladder network were:

- \( C_1 = 1464 \ \mu\text{F} \)
- \( L_1 = 9.4 \ \mu\text{H} \)
- \( C_2 = 737 \ \mu\text{F} \)
- \( L_2 = 6.4 \ \mu\text{H} \)
- \( C_3 = 613 \ \mu\text{F} \)
- \( L_3 = 6.2 \ \mu\text{H} \)
- \( C_4 = 407 \ \mu\text{F} \)
- \( L_4 = 5.6 \ \mu\text{H} \)
- \( C_5 = 490 \ \mu\text{F} \)
- \( L_5 = 5 \ \mu\text{H} \)
- \( C_6 = 603 \ \mu\text{F} \)
- \( L_6 = 8.2 \ \mu\text{H} \)

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mined with the procedure reported in 1 and having six equal sections is shown. In this last case the performance index \( Q \) resulted 7431 A·s.

Fig. 5 allows the evaluation of the influence of the parasitic parameters on the delivered pulse waveform and underlines the importance of a proper evaluation of the actual parasitic parameters utilised in the design procedure.

CONCLUSIONS

The design procedure for a pulse forming network presented in this paper allows the determination of the values of the parameters of a L-C ladder network, in order to obtain a current pulse waveform very close to the required one. The analysis of the results has shown the influence of the parasitic parameters on the waveform of the delivered pulse. The parasitic parameter values of each circuital element must be, compatibly with their technological feasibility, very close to the values derived by the design procedure, in order to obtain a good current pulse waveform.

REFERENCES