A NUMERICAL CODE FOR CUSPED ION THRUSTERS

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Abstract

A computer code for cusped ion thrusters has been developed in order to not only improve discharge performance but also to provide chamber design data. This code can be used to calculate both ion production cost and propellant utilization for a cylindrical discharge chamber with any axisymmetric magnetic field configuration. It contains four calculation sections; namely ones that are used 1) to compute the direction and strength of magnetic fields, 2) to compute grid transparencies to ions and neutral atoms, 3) to compute primary electron confinement length, and 4) to compute the fraction of ions produced that are extracted as an ion beam. In the first section, a two-dimensional magnetic analysis code using Finite Element Method was employed to compute the magnetic field configuration. In the second, a two-dimensional ion optics code is used to compute ion beam trajectories and to optimize the grid system design. In the third calculation section, a plasma flow model in which ion flow in directions across and along magnetic field lines are assumed to be governed by Bohm or classical diffusion and ambipolar diffusion, respectively, is used to compute the extracted-ion fraction. Combining these with Brophy's performance model based on the conservation equations of mass, charge, and energy, one can compute plasma properties and then calculate the discharge performance.

Nomenclature

(SI units unless noted otherwise)

A θ : component of magnetic vector potential
A S : grid surface area
B θ : magnetic flux density
D θ : diffusion coefficient
D N : diffusion coefficient normal to magnetic field lines
D Π : diffusion coefficient parallel to magnetic field lines
E θ : i,j-th component of matrix defined in Eq. (9)
E θ : primary electron energy (eV)
E θ : electronic charge
F θ : extracted-ion fraction
J θ : fraction of ions produced that go to cathode potential
L θ : ion beam current
L θ : discharge current
M θ : Lagrangian appeared on Eq. (1)
F θ : angular momentum
M θ : electron mass
M θ : propellant flow rate (A-eq)
M θ : ion density
M θ : Maxwellian electron density
M θ : neutral atom density
m θ : primary electron density
m θ : ion density at the sheath edge
p θ : probability
Q : ion production rate per unit volume
r : radial position
T : electron temperature (eV)
T θ : ion temperature (eV)
T θ : neutral temperature (eV)
V D : discharge voltage
V θ : Bohm velocity
v n : neutral atom velocity
v θ : radial component of velocity
v θ : axial component of velocity
v θ : tangential component of velocity
z : axial position
Γ : diffusion flux
Δ t : time step
ε θ : ion production cost
κ : baseline ion energy cost
μ : total inelastic collision cross section
θ : tangential position or angle
λ : mean-free-path
σ : total inelastic collision cross section
φ : potential
φ θ : transparency of grids to ions
φ θ : transparency of grids to neutral atoms
φ θ : rate factor for ionization by Maxwellian electrons
φ θ : rate factor for ionization by primary electrons
( ) θ : component normal to the surface
( ) : relative value
[ ] : vector
{ } : tensor

Introduction

Ion thrusters based on magnetic cusp plasma confinement are being considered for on-orbit propulsion functions and orbit transfer propulsion for large space systems. Since thruster performance, typically expressed by specific impulse, thrust efficiency, and thrust-to-power ratio, strongly impacts the trip time, payload, and power requirements of such missions, it is necessary to obtain discharge performance as high as possible by minimizing the ion production cost while maintaining a high propellant utilization.

In order to improve the discharge performance and to provide chamber design data, it is desirable to have a computer code that can be used to calculate the ion production cost and the propellant utilization for a given chamber design with a given magnetic field configuration.

This paper presents a numerical computer code for cusped ion thrusters having a cylindrical discharge chamber with a magnetic ring-cusp configuration. This code contains a magnetic field analysis, an ion optics simulation, a primary electron trajectory simulation using the Monte Carlo method, a plasma flow analysis by the Finite Element Method, and a discharge performance analysis using Brophy's performance model that is basically comprised of the conservation equations of mass, charge, and energy.

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Computer Modeling

This code contains four calculation sections. One is a two-dimensional ion optics code that simulates ion beam trajectories in an axisymmetric beamlet of two or three grid systems. The others are calculation codes that analyze discharge plasmas produced in an axisymmetric discharge chamber with a magnetic ring-cusp.

Two-dimensional Ion Optics Code

A computer simulation code for axisymmetric ion beams has been developed to design two or three grid systems, and, in particular, to compute the values of grid transparencies \( \phi \) and \( \phi' \). In the simulation model, the governing equations are Poisson's equation, the continuum equation, and Newton's second law. The ion extraction surface that divides the discharge plasma and the ion sheath is determined self-consistently in such a way that the ion saturation current density of the plasma at the surface is equal to that of the space-charge-limited current density. The problem is solved iteratively. First, the solution of Laplace equation gives a set of potentials between the grids when known potentials are applied to two or more grids of a known shape. In order to determine the position of the extraction surface, one chooses two different potential values \( \phi_c \) and \( \phi' \), both of which are close to the screen grid potential. The surface of the higher potential \( \phi_c \) is chosen as the extraction surface in the first iteration where ions start their trajectories following the Newton's second law. The trajectories and velocities are computed by the Runge-Kutta method to determine the charge density. In the second iteration, Poisson's equation is used to compute the potentials and to find the surface of the lower potential \( \phi' \), lying within the charge flow. Using the distance between the potentials \( \phi_c \) and \( \phi' \), one determines the new surface of the potential \( \phi_c \) and choose it as the extraction surface. This iteration is continued until a convergent solution is obtained. The transparencies \( \phi \) and \( \phi' \) are determined by measuring the areas of the extraction surface and the aperture of the acceleration grid, respectively.

Assumptions in Plasma Analysis

The discharge plasma is produced in ionization collisions of primary and Maxwellian electrons with neutral atoms. The primary electrons have an energy equal to the discharge voltage. Temperatures of electrons, ions and neutral atoms are constant throughout the discharge chamber. Both ions and neutral atoms have the same temperature as the chamber surface (around 0.05 eV). This temperature is much less than that of the electrons. The neutral atom density is also constant throughout the discharge chamber. The primary electron energy \( E_0 \) is constant even when the electron changes its direction by collisions. As for collisions, only elastic collisions with neutral atoms are considered since the mean-free-path is much shorter than the mean-free-path of Coulomb collisions. The migration of both ions and Maxwellian electrons across magnetic field lines is described by either Bohm or classical diffusion and their migration along the lines is described by ambipolar diffusion. Ions are lost to the chamber surface at the Bohm velocity. The Lagrangian for a primary electron of mass \( m \) and charge \( e \) can be written as

\[
L = \frac{1}{2} m \left| \mathbf{v} \right|^2 - e \mathbf{A} \cdot \mathbf{v}
\]

where \( \mathbf{v} \) is the velocity and \( \mathbf{A} \) is the magnetic vector potential which is calculated by the magnetic analysis program. Substituting Eq. (1) into the Euler-Lagrange equations, one obtains

\[
\frac{d\mathbf{v}}{dt} = \mathbf{A} \times \mathbf{v} - e \mathbf{A} \left( \mathbf{A} \cdot \mathbf{v} \right)
\]

Plasma Flow Analysis

The governing equation for the particle balance within the discharge chamber is

\[
-Q \left( \mathbf{V} \cdot \left[ \mathcal{D} \left( \mathbf{V} \right) \right] \right) = \mathbf{Q}
\]

where the matrix \( \mathcal{D} \) describes the spatial variation of diffusion coefficients within the discharge chamber. The ion production rate per unit volume \( \mathbf{Q} \) is given by

Monte Carlo Simulation of Primary Electron Motions

The objective of this simulation is to compute the confinement length and density distribution of primary electrons within the discharge chamber by using the Monte Carlo method. The primary electron, emitted by a cathode and accelerated in the cathode sheath up to an energy corresponding to the discharge voltage, is supplied and travels in the electromagnetic fields with the direction altered by the Lorentz force. The Monte Carlo method, in which the position and velocity of primary electrons are surveyed as a function of time, is used in this simulation. The trajectory of a primary electron is computed using Eqs. (2) and (4) in order to preserve the angular momentum of the electron. The time step \( \Delta t \) is chosen so that the travelling distance of the electron during \( \Delta t \) is much smaller than the Larmor radius whose value varies with the field strength of the electron position. The Runge-Kutta method was used to calculate the trajectory. The probability of a collision during \( \Delta t \) is given by the equation

\[
P = \exp \left( - \frac{\Delta \mathbf{R}}{\lambda} \right)
\]
Here, \( n_n \) is neutral atom density, \( n_p \) and \( n_e \) are densities of primary and Maxwellian electrons, respectively, and \( \langle r \rangle \) is the ionization rate factor. The atom density in the chamber \( n_n \) is determined by a neutral atom flow balance and is given by

\[
\dot{\theta} = \frac{(1 - \eta)}{e V_n A_r} \tag{8}
\]

where \( \dot{\theta} \) is the propellant flow rate expressed in equivalent amperes, \( \eta \) is the propellant utilization and \( \phi \) is the effective grid transparency to neutral atoms. When \( \theta \) is the angle between the magnetic line of force and the chamber axis, the diffusion matrix can be written as

\[
[D] = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}
\]

Substituting Eq. (9) into Eq. (6), the following differential equation is obtained.

\[
\frac{\Delta n_D}{\Delta z} = \frac{\Delta n_{D_2}}{\Delta z} + \frac{\Delta n_{D_1}}{\Delta z} + \frac{\Delta n_{D_1}}{\Delta z} + \frac{\Delta n_{D_1}}{\Delta z} + Q = 0
\tag{10}
\]

Since plasma potential is typically higher than that of chamber wall surface and screen grid, the ions arriving at the sheath edge should be accelerated to the Bohm velocity \( V_B \) in order to assure that the sheath will be stable. Consequently, the boundary condition at the chamber and grid surfaces becomes

\[
\langle \vec{\Gamma} \rangle = n_p \vec{V}_B
\tag{11}
\]

where \( \langle \vec{\Gamma} \rangle \) represents the vector component of the flux normal to the surface and \( n_p \) is the plasma density at the sheath edge.

**Calculation of Discharge Performance**

When both the extracted-ion fraction and the primary electron confinement length are given, the discharge performance (ion production cost vs propellant utilization) can be described by using Brophy's performance model, which is basically comprised of the conservation equations of mass, charge, and energy. In the model, the ion production cost \( \epsilon_B \) and the propellant utilization \( \eta_u \) are defined by the equations

\[
\epsilon_B = \frac{J_B}{B_2 V_D}, \quad \eta_u = \frac{J_B}{V_D}
\tag{12}
\]

respectively. Here \( V_D \) is the discharge voltage, \( J_B \) the beam current, and \( J_B \) the discharge current. In the model the following equation describing the performance curve is given as

\[
\epsilon_B = \frac{e}{e_B} \left( 1 - \exp\left( -\frac{1 - \eta}{\eta_u} \right) \right) + \frac{1}{\epsilon_B} V_D
\tag{13}
\]

where

\[
\epsilon_B = \frac{\eta_u}{n_p A_r n_v}
\tag{14}
\]

The parameter \( \epsilon_B \), called the baseline ion energy cost, is the average energy cost of producing an ion in the chamber where the neutral density is high enough so that all primary electrons have inelastic collisions before they reach an anode. Note that \( \epsilon_B \) is independent of the propellant utilization and flow rate and depends on only the electron temperature and the discharge voltage (see Ref. 3).

From Eq. (14) the parameter \( C_0 \) is proportional to the primary electron confinement length \( l_p \) and inversely proportional to the grid transparency for neutral atoms \( \phi_n \).

**Calculation Procedure**

The outline of calculation procedure is shown in Fig. 1. There are two kinds of input data. The first kind is related to the geometric design of discharge chamber and grid system. Given the chamber design, the magnetic field analysis program is used to compute magnetic vector potentials inside the chamber and then to produce the magnetic field configuration. The second kind of data consists of operational parameters, such as propellant flow rate \( \dot{\theta} \) and propellant utilization \( \eta_u \), and discharge voltage \( V_D \). With the input data of the magnetic vector potential \( A_r \) and the primary electron energy \( E_e (=eV_D) \), the Monte Carlo simulation program computes the electron confinement length \( l_e \) and the primary electron density distribution \( n_e (r,z) \). In addition, the ion optics simulation code is used to compute ion beam trajectories and the effective grid transparencies \( \phi_i \) and \( \phi_n \). Using \( A_r (r,z) \), \( \phi_i \), and \( \phi_n \),

![Fig. 1 Block diagram of computer code](image-url)
transparencies $\phi_1$ and $\phi_2$. Using $n(r,z)$, $\phi_1$, and $\phi_2$ with an initially estimated value of electron temperature $T_e$, one can calculate the extracted-ion fraction $f_B$ and the ion density distribution $n_i(r,z)$ in the plasma flow analysis program. The results are then used to recompute the electron temperature from the energy conservation equation in the subsequent discharge performance analysis. The recomputed value of $T_e$ is compared to the estimated value of $T_e$ and the above calculations are repeated until both the values agree.

**Numerical Examples**

As the first example, consider the discharge plasma produced in a 7 cm diam. ion thruster having 3 ring magnets. The chamber and computed magnetic field configurations are shown in Fig. 2. With this configuration, the Monte Carlo simulation was performed to compute both the confinement length and the primary electron density distribution for the case where the xenon propellant flow rate is $150 \text{ mA-eq}$, the discharge voltage $V_d = 40 \text{ V}$, and the grid transparency to neutral atoms is $0.26$. The computed confinement length was $0.73 \text{ m}$ at the diam. ion thruster developed at Colorado State University.

![Fig. 2 Magnetic field configuration in the 7 cm diam. ion thruster developed at Colorado State University.](image1)

As seen in the figure, the distribution has a large peak that appears at a position corresponding to the location of the cathode. This is due to the fact that the primary electrons cannot move easily around the central, field-free region near the cathode but they cannot have access to the outer region near the wall surface by traveling across the magnetic field lines. This result suggests that the ion production associated with primary electrons occurs mainly around the cathode. As the primary electron density distribution $n_e(r,z)$ is shown in Fig. 3. As seen in the figure, the density distributions of primary electrons and plasma ions are shown in Figs. 7 and 8, respectively. Both the shapes are similar to those obtained in the 7 cm diam. discharge chamber. The computed and measured discharge performance are shown in Fig. 9. In comparison with the previous case, both the computed and measured curves shift down and to the right, that is, show better performance and have more sharply defined knees than those obtained in the previous case.
The final example demonstrates that the ion optics simulation can be used to investigate not only the ion beam divergence but also the effect of grid system design on the discharge performance. As shown in Fig. 10, the ion beam simulation was conducted for a two-grid system; one with an acceleration grid with large holes and the other having smaller holes. The larger hole appears to give better convergent ion beams than the smaller one. Such a trend remains unchanged when the beam current density is changed. As seen in the figure, the ion extraction surface is concave toward the plasma region. Such a shape enhances the grid system transparency to ions $\phi_i$. In this case, $\phi_i$ is around 1.15 times the open area fraction of the screen grid and is almost insensitive to the hole size of the acceleration grid. The effect of the hole size on the discharge performance is shown in Fig. 11. The better performance is obtained for the smaller hole. This is due to the fact that, as described by Brophy and Wilbur, the decrease in the hole size decreases the grid transparency to the neutral atoms $\phi_n$ and correspondingly increases $C_0$. 

Fig. 5 Discharge chamber performance of the 7 cm diam. ion thruster.

Fig. 6 Magnetic field configuration in the 30 cm diam. ion thruster developed at Hughes Research Laboratory.

Fig. 7 Primary electron distribution in the 30 cm diam. ion thruster.

Fig. 8 Plasma density distribution in the 30 cm diam. ion thruster.

Fig. 9 Discharge chamber performance of the 30 cm diam. ion thruster.


Fig. 10 Computed ion beam trajectories (the beam divergence angle is 9.5 and 15.0 degrees for the upper and lower cases, respectively).

Fig. 11 Effect of the hole size of acceleration grid on discharge performance.

Conclusion

A numerical code in cusped ion thrusters has been developed which is used to estimate discharge performance for cylindrical discharge chambers having ring-cusp configurations. Several numerical examples have been employed to demonstrate that this code can be used to compute ion production cost and propellant utilization. From the results, it is concluded that this code is a useful tool of optimizing the chamber geometry and the magnetic field configuration for good discharge performance.

References