ABSTRACT

A general methodology for the optimization of the electric propulsion orbital transfer vehicle (OTV) design was developed and programmed for computer calculations to allow evaluation of a broad set technology and mission assumptions. Based on the OTV "Existence Equation" theory, a computer code is then used to find the optimal design parameters for an OTV operations from low earth orbit (LEO) to geosynchronous orbit (GEO).

INTRODUCTION

There already exists reliable launch systems to reach low orbit (LEO), both in the Soviet Union and in the United States. At LEO the Soviet Union maintains the Space Station Complex Mir, which can provide a stepping stone to higher orbits. However, there still remains the need for an orbital transfer vehicle (OTV), that could transport payloads from the space station to higher orbits significantly less expensive and operationally more flexible than conventional stages.

Conventional wisdom employs a solid or liquid expendable upper stage that delivers the payload to its intended destination. However, mission cost using solid or liquid propulsion is extremely high. It costs 10 times as much to transfer a payload from LEO to GEO as to deliver it to LEO. By using electric propulsion we hope to sufficiently decrease the mission cost.

Several proposals for an orbital transfer vehicle have been presented in the past. One of the most realistic ones is a modular, ion-propelled OTV investigated by Hermel, etc in 1986. This modular OTV concept consists of a reusable power bus (solar arrays, power processing unit, batteries, thermal control system, tracking, telemetry, and command, and attitude control system) and a propulsion module (ion thrusters, propellant feed system, and propellant storage tanks). The power bus remains in space while the propulsion module is detached after each mission for servicing. The required electrical power could also be supplied by a nuclear reactor.

There have been previous studies to optimize the design of an EOTV operating from LEO to GEO. The present study extends the previous work by focusing on the mission cost analysis required to make the reusable OTV concept both technically and economically feasible.

FUNDAMENTAL RELATIONSHIPS AND CHARACTERISTICS

The initial (launch) mass of an electric propulsion orbital transfer vehicle is given by

\[ m_0 = m_{pl} + m_P (1 + s_{TF}) + m_{OTV} \]  

where \( m_{pl} \) is the payload mass, \( m_P \) is the propellant mass, \( s_{TF} \) is the tankage function, and \( m_{OTV} \) is the "constant mass" of the OTV. The constant OTV mass takes into account the power source \( m_{PS} \), power processor \( m_{PP} \), thrusters \( m_T \), structures \( m_S \), and other subsystems \( m_{OS} \).

\[ m_{OTV} = m_{PS} + m_{PP} + m_T + m_S + m_{OS} \]  

The constant OTV mass is proportional to the electric power supplied to the thruster subsystem

\[ m_{OTV} = \alpha_{OTV} N_{IN} \]
where $\alpha_{\text{OTV}}$ is the "specific mass of the EOTV". In other words, it is the sum of the specific masses of the power source, propulsion unit, thrusters, and other subsystems.

$$\alpha_{\text{OTV}} = \alpha_{\text{PS}} + \alpha_{\text{PP}} + \alpha_{\text{T}} + \alpha_{\text{O}} + \alpha_{\text{ES}}$$

The thrust provided by an electric propulsion thruster is

$$p = \frac{2n N_{\text{IN}}}{g I_{\text{sp}}}$$

where $I_{\text{sp}}$ is the specific impulse of thrust subsystem, $g$ is the gravitational constant, $\eta$ is the efficiency of the thruster subsystem.

$$\eta = \eta_{\text{PP}} \eta_{\text{T}}$$

where $\eta_{\text{PP}}$ is the power processor efficiency and $\eta_{\text{T}}$ is the thruster efficiency.

The required propellant for any mission for constant specific impulse thrust propulsion system is given by the Tsiolkovsky equation

$$m_p = m_0 (1 - \exp(-\frac{\Delta V}{g I_{\text{sp}}}))$$

For a constant thrust propulsion system, we define the "trajectory-design function" $\Delta V^\text{min}$.

$$\Delta V^\text{min} = \Delta V(a_0, I_{\text{sp}}, \theta_0, T_0)$$

where $a_0$ is the initial thrust acceleration, $T$ is transfer time, and $\theta_0$, $T_0$ are parameters dealing with the initial and final orbits, such as inclination, eccentricity, etc. For near earth mission, the equivalent field free velocity increment, minimum delta $V$, for very low acceleration propulsion system is given by Edelbaum-Lebedev's formula (after taking into account the desired correction such as perturbation effects, drag, gravity losses, etc.).

$$\Delta V \approx \Delta V(a_0, I_{\text{sp}}, \theta_0, T_0)$$

In Eq. (9) $V_x$ and $V_y$ are the circular velocities of the beginning and final orbits and $\Delta l$ is the change in inclination.

Moreover, for near earth missions the transfer time without coasting related to the initial acceleration through equation

$$T = \frac{W}{a_0} \left(1 - \exp(-y)\right)$$

where $y = \frac{\Delta V}{W}$, $W = g I_{\text{sp}}$.

The payload mass fraction for EOTV is defined as follows

$$\mu_{\text{PL}} = \frac{m_{\text{PL}}}{m_0} = 1 - \mu_{\text{OTV}} - \mu_p (1 + y_{TF})$$

where $\mu_{\text{OTV}}$ and $\mu_p$ are the constant OTV mass fraction and propellant mass fraction, respectively. These are

$$\mu_{\text{OTV}} = \frac{m_{\text{OTV}}}{m_0} = \frac{\alpha_{\text{OTV}} a_0 W}{2}$$

$$\mu_p = \frac{m_p}{m_0} = \frac{a_0 T}{W}$$

Inserting these expressions into Eq. (11), one obtains

$$\mu_{\text{PL}} = 1 - \frac{\alpha_{\text{OTV}} a_0 W}{\eta} - \frac{a_0 T}{W} (1 + y_{TF})$$

Classical optimization theory optimizes $\mu_{\text{PL}}$ in terms of exhaust velocity and does not take into consideration the cost of mission, transfer trip time, and number of trips (OTV lifetime).

**OTV EXISTENCE EQUATION**

Instead of optimizing the payload mass fraction, let's introduce the OTV Existence Equation$^5$. Rewriting Eq. (1) as

$$m_{\text{PL}} = m_0 - m_p (1 + y_{TF}) - m_{\text{OTV}}$$

and dividing by $m_{\text{OTV}}$ one arrives at...
Equation (15) is the OTV Existence Equation and β is referred to as the OTV payload parameter. The payload parameter multiplied by the specific mass of the OTV shows how many kilograms of payload per every kilowatt of power that can be delivered for a specified mission.

Furthermore,

$$\alpha = \frac{2\pi}{0_{\text{OTP}}\alpha_{0} W} \quad \gamma = \frac{2\pi T}{\alpha_{\text{OTP}} W^2}$$  \hspace{1cm} (16)$$

When designing the OTV it is necessary to solve the Existence Equation, or to determine a payload parameter function

$$\beta = \beta (a_{0}, I_{SP}, T, 0_{0}, 0_{f}, \alpha_{\text{OTP}}, y_{TF}, \gamma)$$  \hspace{1cm} (17)$$

**ONE WAY EARTH MISSION**

In the case of a one-way mission, the mission parameters vector is

$$\alpha_{m} = (m_{PL}, 0_{0}, 0_{f}, T)$$  \hspace{1cm} (18)$$

For near earth missions, T and $a_{0}$ are related through Eq. 10 and minimum delta V is given by Edelbaum-Lebedev’s formula. Thus, the existence equation is simplified to

$$\beta = \frac{2T}{\alpha'_{\text{OTP}} W^2} x \frac{\exp(-y)(1+y_{TF}) - y_{TF}}{1-\exp(-y)} - 1,$$  \hspace{1cm} (19)$$

where $\alpha'_{\text{OTP}} = \frac{\alpha_{\text{OTP}}}{R}$

A limiting trip time exists for the case of no payload $\beta = \mu_{PL} = 0$.

In this case the transfer time is

$$T_{\text{lim}} = \frac{\alpha_{\text{OTP}} W^2}{2} x \frac{1-\exp(-y)}{\exp(-y)(1+y_{TF}) - y_{TF}}$$  \hspace{1cm} (20)$$

The payload mass fraction is defined as follows

$$\mu_{PL} = 1 - (1+y_{TF})(1-\exp(-y))$$  \hspace{1cm} (19)$$

Another limiting solution exists for the case in which the trip time is infinite. In this case (Fig. 1)

$$\mu_{PL} = 1 - (1+y_{TF})(1-\exp(-y))$$  \hspace{1cm} (21)$$

From Eq. (19) it is possible to find that OTV payload mass is a linear function of a trip time (fig. 2)

**ROUND TRIP MISSION**

In the case of a round trip mission, the mission parameters vector becomes

$$\alpha_{m} = (m_{PL}, \bar{m}, 0_{0}, 0_{f}, T)$$  \hspace{1cm} (22)$$

where $\bar{m}$ is the ratio of the payload mass in the return leg of the trip to the payload mass in the up leg $m_{PL}$, $T = T_{up} + T_{ret}$. Assuming that delta $V_{up} = \delta V_{ret}$ , the OTV Existence Equation becomes

$$\beta = \frac{2TD}{\alpha'_{\text{OTP}} W^2 (1-\exp(-2y))} - 1$$  \hspace{1cm} (23)$$

$$x \frac{1+\exp(-y)}{\bar{m}+\exp(-y)}$$  \hspace{1cm} (24)$$

* The solution of the Existence Equation in form (24) is not straightforward and can be found in Ref. 5.
where

\[ D = 1 - (1 + y TF)(1 - \exp(-2y)) \]

In the case of no payload, the limiting trip time is

\[ T_{lim} = \frac{\alpha_{OTV} W^2}{2D} x (1 - \exp(-2y)) \]  \hspace{1cm} (25)

In addition we have

\[ T_{up} = \frac{\alpha'_{OTV} W^2}{2} \times [\beta + \gamma(1 + y TF) + 1] x (1 - \exp(-y)) \]

\[ \mu_{PL} = \frac{\beta}{\beta + \gamma(1 + y TF) + 1} \]  \hspace{1cm} (26)

\[ \mu_p = \frac{\gamma}{\beta + \gamma(1 + y TF) + 1} \]

\[ \gamma = \frac{2T}{\alpha'_{OTV} W^2} \]

The above analytic solution of the existence equation are very useful for computer calculation to allow evaluation of a broad set of technology and mission assumptions without any iterations.

For example, when designing an OTV project for specified mission parameters vector \( \mathbf{x} \), it is easy to find the constant OTV mass

\[ m_{OTV} = \frac{m_{PL}}{\beta(I_{SP}, T, 0, 0, \alpha_{OTV}, y TF)} \]  \hspace{1cm} (27)

propellant mass

\[ m_p = \gamma m_{OTV} \]  \hspace{1cm} (28)

and the electric power supplied to the thruster subsystem

\[ N_{IN} = \frac{m_{OTV}}{\alpha_{OTV}} \]  \hspace{1cm} (29)

For specified initial mass of OTV \( m_0 \) instead of (27) we have

\[ m_{OTV} = \frac{m_0}{[\beta + \gamma(1 + y TF) + 1]} \]  \hspace{1cm} (30)

REUSABLE FEATURE

When designing the k-trip OTV with specified lifetime \( \tau \) instead of Eq. 13 total payload mass fraction is defined as follows ( \( y_{TF} = 0 \))

\[ \mu_{PL} = \frac{\sum m_{PL}}{\sum m_0} \]

\[ = \frac{\sum \beta}{1 + \sum \beta + \sum \gamma} = \frac{\beta}{1 + \beta + \gamma} \]

and trip time

\[ T = \frac{\tau}{K} \]  \hspace{1cm} (32)

Performance for k-trip OTV is indicated in Fig. 3,4. A greater number of round trips per vehicle with specified lifetime \( \tau \) shows substantial total payload disadvantages.

OTV Missin Analysis

Comparison of the various OTV design was made on the basis of "delivery cost". It is defined as the total life cycle cost in a particular vehicle alternative in the delivery of the payload from the earth to its final orbit (excluding the cost of the payload itself), divided by the payload mass per trip. The life cycle cost was calculated by taking the sum of the development, procurement, launch, operations costs and payload cost of money associated with the mission time. For reusable vehicles, the hardware and launch costs were amortized over the total number of reuses.
The analysis of EOTV characteristics identified delivery cost as functions of specific impulse $I_{sp}$, trip time $T$, and number of trips $K$ (OTV lifetime). The reuse number was estimated by dividing the power source lifetime by the round trip time of each flight. The additional feature here is that technologies will progress in such a way as to decrease the per unit cost and specific mass of hardware for large OTV and increase them for large lifetime.

The computer program "OTV" was developed to automatically optimize the specific impulse and trip time for fixed OTV lifetime (the reuse number) (Fig.5). The algorithm is designed to consider an expendable and reusable OTV scenarios.

Software provides the evaluation of a broad set of technology and mission assumptions. Comparison of the various EOTV design was made on the basis of delivery cost.

Software allows:
- to analyze the EOTV characteristics as functions of specific impulse, trip time, and reuse number (OTV lifetime)
- to consider an expendable and reusable OTV scenarios from low earth orbit and high earth orbit and moon orbit
- to take into account mass of the payload per trip (or initial mass of OTV) and the total number of trips OTV fleet as well as specific masses and costs, thruster efficiency, initial and final orbits parameters, etc.

The typical dependance of transfer time and specific impulse on delivery cost (in units of launch cost) are shown in figure 6,7.

Figure 8a,b shows the typical dependences of the objective function versus the number of trips of a single OTV $k$ (the reuse number) for a different specific cost and mass of power source and "cost-lifetime" dependance ($K=1$ corresponds to expendable OTV).

**SUMMARY**

A general methodology based on the EOTV Existence Equation theory and mission studies, cost modeling, and technology assessments was developed and computer code is then used to analyze the EOTV characteristics as functions of specific impulse, trip time, and reuse number.

**REFERENCES**

Figure 1  Payload Fraction vs Trip Time

Figure 2  OTV Payload vs Trip Time

Figure 3  $k$-Trip OTV Total Payload Fraction vs Number of Trips (OTV Lifetime is Fixed)

Figure 4  $k$-Trip OTV Optimal Specific Impulse vs Number of Trips (OTV Lifetime is Fixed)
Mission selection $x_m$ (n-trip mission, $\kappa$-trip OTV)

$\Delta V$ requirements ($O_0$, $O_2$)

OTV systems performance initial estimation $\alpha_{OTV}$, $\eta$, $\delta_T$

Limit trip time $T_{lim}$

$T_{lim} < T$ No

Yes

Payload parameter $\beta$
Propellant parameter $\gamma$

Total power $N_{EN}$
Propellant mass $m_p$

OTV systems performance estimation

Mass models redundancy degradation models

No

Specific mass consistent

Yes

Specific cost estimation (cost per payload mass)

Cost models

Optimization $I_{sp}$, $T$, $\kappa$

Output (OTV design data)

Figure 5 $\kappa$-trip EOTV Analysis Program
Figure 6  OTV Specific cost vs Trip Time

Figure 7  OTV Specific Cost vs Specific Impulse

Figure 8  OTV Specific Cost vs Number of Trips of a Single OTV