LOCAL EROSION OF THE CATHODE-COMPENSATOR IN ELECTRIC PROPULSION

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ABSTRACT

To predict the lifetime of the electric thruster operation, the data on the local erosion of its cathode-compensator are required. Usually a cathode-compensator is a hot hollow cathode (HC). It is known that the local erosion of the active surface of HC is essentially non-uniform and this non-uniformity can not be explained only by inconstancy of the cathode temperature \( T_c \) on the cathode surface. In this paper different causes which define the local erosion of the cathode have been analyzed. A mathematical model of the local erosion has been suggested. This model takes into account the return and convectional carrying out of the evaporated atoms. This model allows to analyze the temporary evolution of the running part under long-time working. In the limit case of large Knudsen numbers, analytical expressions have been obtained. In this case the calculations of the local erosion for the plane and cylindrical geometry of the cathode have been executed. It has been shown that the increased erosion in the exit region of HC which may be observed even if \( T_c \) is constant, is explained by the decrease of the return of the evaporated atoms.

INTRODUCTION

Cylindric hollow cathodes (HCs), the inner surface of which is the operating one, are usually utilized as cathodes-compensators for electric propulsion (ETs). The lifetime of HCs mostly defines the lifetime of ETs as a whole. It is well known, that the erosion of the HC is essentially non-uniform, and this fact cannot be explained completely by non-uniformity of the temperature field on the electrode surface.

In this work the problem of HC erosion calculation is considered. The two following basic mechanism of the cathode erosion are known-evaporation and sputtering with plasma ions. The relation between these mechanisms is discussed in Ref. 1. Evaporation was considered there in more detail. However, the results obtained can be easily generalized for the sputtering of the cathode too.

Depending on pressure, the two utmost regimes of the cathode material vapors flow in the cathode cavity may be marked out - free-molecular and hydrodynamic. At the first regime, vapor atoms move from one point of the HC surface to another without collisions, and at the second regime, they are involved into the flow of the plasma-generating gas supplied into the cathode cavity. Both utmost regimes of the vapor flow are considered in this work.

It is assumed, that the distribution of the plasma flow parameters along the HC and the flow density of atoms evaporated from the operating surface \( G^+(x) \) are known. It is required to figure out the local \( G(x) \) and total \( G_t \) cathode erosion.

1. Free-molecular regime of erosion

At this regime, the cathode cavity pressure is low, so the plasma flow in this cavity almost does not influence on the erosion rate. This fact essentially streamlines figuring out the erosion. Solving the problem, thus set up and considered in Ref. 2, enables qualitative features of the HC local erosion to be found out. It is obvious that the cathode erosion \( G(x) \) at the point \( x \), is a difference between flow densities of evaporated and condensed particles, i.e.

\[
G(x) = G^+(x) - G^-(x).
\]

The expression for the condensed particles flow density \( G^-(x) \) depends on the HC geometry. The simplest expression \( G^-(x) \) for a model HC can be obtained in the form of two parallel planes provided the evaporated atoms move only in a plane, which is perpendicular to the HC planes.

If the characteristic longitudinal size of this HC is much greater than the distance between the planes, then, with accordance to Ref. 2, the following expressions are valid
In these relations \( f(z, v) \) is the function of the distribution of the evaporating particles in velocity \( v \), and it is supposed to be isotropic; \( \varphi \) is an angle between the vector velocity and normal to the HC surface, \( m_a \) - atomic mass, \( z - x/d \) is a non-dimensional longitudinal coordinate; \( d \) is the accelerating channel length the plane \( z = 0 \) coincides with the HC exit plane. If the flow density of the evaporating particles \( G^+ \) is constant along the HC, then the following simple relation for the local erosion may be obtained:

\[
G(z) = \frac{G^+}{2} \left( 1 - \sqrt{1 + \frac{z}{z'}} \right). \tag{1.4}
\]

In accordance with (1.4), the HC erosion is essentially non-uniform. The maximum erosion is observed near the exit section \( z = 0 \), and at \( z >> 1 \) the erosion decreases according to the law \( G(z) \sim G^+ / z^3 \).

For the cylindrical HC the expression for the local erosion gets complicated, and, according to Ref. 2, looks as follows:

\[
G(z) = \frac{\pi}{2} \int_0^\infty f(z, v) v^2 \cos \varphi dv d\varphi - \int_0^\infty f(z, v) v^2 \cos \varphi dv \int_0^\infty f(z', v) v^2 dv. \tag{1.5}
\]

For evaporation rate, constant in length, the expression (1.5) gets simplified:

\[
G(z) = G^+ \left( 1 - \frac{\arctan \frac{z}{\cos \varphi}}{\arctan 1} \right) \cos \varphi d\varphi. \tag{1.6}
\]

Like for the planar HC, in this case the maximum erosion is typical to be at the exit section, however, for the cylindrical HC the erosion is even more nonuniform:

\[
G(z) \sim \frac{G^+}{z^3}.
\]

Knowing the value \( G(x) \), it is possible to trace the evolution of the cathode cavity geometric characteristics in time. Thus, the alternation of the cylindrical HC radius \( R \) can be found from the following equation:

\[
\frac{dR}{dt} = \frac{G(x)}{\rho}, \tag{1.7}
\]

where \( \rho \) is a cathode material density.

Fig. 1 presents the results of calculations of the HC local erosion with the formulas (1.4) and (1.6). This figure also contains experimental data on the cylindrical HC exit erosion from Ref. 3.

The total cathode erosion value \( G_t \) is determined by integrating \( G(x) \) along HC length. For the flow density \( G^+ \), constant along the length, the required expression for \( G_t \) can be easily obtained using the analogy between the problem considered and the problem of radiation from the cavity. Since the degree of blackness of a long isothermal cavity equals to unity, similarly to the radiation, for this case it may be written down that

\[
G_t = 2AG^+,
\]

where \( A \) is the HC exit area, and the factor 2 accounts the presence of two exits.

2. Hydrodynamic regime of erosion

For this regime, the distribution of concentration of evaporated atoms over the volume is described by the following system of equation of continuity for atoms and ions of the cathode material:
\[
\text{div } J_a = \nu I n_a - \nu r n_i, \tag{2.1}
\]
\[
\text{div } J_r = \nu r n_i - \nu I n_a, \tag{2.2}
\]
where \(\nu I\) and \(\nu r\) are frequencies of ionization and recombination of cathode material particles, respectively, and
\[
J_a = -D_a v n_a + \vec{u} \cdot \vec{n}_a, \tag{2.3}
\]
\[
J_r = -D_r v n_i + (\vec{u} + \mu_1 \vec{E}) \cdot \vec{n}_i \tag{2.4}
\]
are flow densities of evaporated atoms and created in result of their ionization ions, respectively, \(\vec{u}\) is a plasma-generating gas flow velocity, \(\mu_1 \vec{E}\) is an ions drift velocity, \(D_a\) and \(D_i\) are diffusion coefficients of atoms and ions, respectively. When writing down (2.3) it was supposed that the evaporated atoms momentum relaxation length was much smaller than the cavity radius. The system of equations (2.1-2.4) allows to take into account the effect of the basic factors influencing on the erosion, videlicet convective transfer and diffusion of the evaporated particles, ionization of these particles and return of the ions to the cathode caused by the accelerating electric field.

The boundary conditions for the projections of the atom and ion flows on the normal to the HC surface are as follows:
\[
J_{an} = \sqrt{-kT_c} \frac{n_a - n_s}{2 \Delta n_a} x_a, \tag{2.5}
\]
\[
J_{in} = \chi I n_i v_n, \tag{2.6}
\]
where \(T_c\) is a cathode temperature, \(V_n\) is an ion thermal velocity, \(n_s\) is a cathode material saturated vapour density; \(x_a, 1 = 1\) are numerical coefficients. While solving this problem it was supposed that the saturated vapour concentration along the HC length is as follows (0 < \(x < L\)):
\[
n_s(x) = n_s(1 - \exp(-\lambda x)). \tag{2.7}
\]

The plasma-generating gas velocity distribution across the HC section \(u(x,r)\) was written taking into account the slipping along the wall
\[
u(x, r) = \frac{\vec{u}(x)}{1 - b/2} \left(1 - b \frac{r^2}{R^2}\right),
\]
where \(b = 1/(1 + 4x \text{ Kn})\), \(R\) is a considered cylindric HC channel radius, \(\lambda = 1\) is a numerical coefficient depending on the nature of the scattering of the gas by the cathode surface.

If to neglect the transfer coefficient dependance of the radius, then the system of equations (2.1) and (2.2) takes the following non-dimensional appearance:
\[
\frac{\partial^2 n_a}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial n_a}{\partial \tau} + \frac{\partial^2 n_b}{\partial \tau^2} = \tilde{\beta}_a \left(1 - b \tau^2\right) \frac{\partial n_a}{\partial \tau}. \tag{2.10}
\]

where \(\sigma_0\) is a cross-section area of collisions of propellant atoms with each other, and \(\sigma_a\) is a cross-section area of their collisions with evaporated atoms. At usual operating conditions of the cathode-compensators \(\beta_a = 5...20\).

The exact solution of the system of equations (2.8-2.9) is complicated enough. To obtain the concrete results, let us consider a simpler problem, where the cathode material vapour ionization may be neglected. This approach has methodical and practical interest. Firstly, it is possible to solve this model problem exactly in one practically important case. Comparing the exact solution of the problem with various approximate methods, it is possible to evaluate the accuracy of these approximations. Furthermore, as it follows from physical considerations, the ionization must reduce the erosion. Hence, the solution of the model problem enables to determine the maximum possible erosion of the HC.

The equation (2.9) is not used in this model problem, and it is assumed that the flow velocity \(u\) is constant along the HC. In this case the equation (2.8) takes the following appearance:
\[
\frac{\partial^2 n_a}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial n_a}{\partial \tau} + \frac{\partial^2 n_b}{\partial \tau^2} = \tilde{\beta}_a \left(1 - b \tau^2\right) \frac{\partial n_a}{\partial \tau}. \tag{2.10}
\]
Here $\beta_a = \bar{u}/R (1 - \frac{b}{2}) D_a$. At $b = 0$ it is possible to obtain the exact solution (2.10) with the boundary condition (2.5). This solution is as follows:

$$n_a (z, \tau) = \sum_{j=1}^{\infty} \frac{2 \mu_j J_1 (\mu_j)}{J_0 (\mu_j) + J_1 (\mu_j)} \times$$

$$\times \frac{J_0 (\mu_j / \tau)}{k_1 / - k_2 j} z_j (z),$$

where $J_0$ and $J_1$ are the Bessel’s functions of the zero and first orders, and

$$k_1, k_2 = \frac{\beta_a}{2} \pm \sqrt{\frac{\beta_a^2}{4} + \mu_j^2}, \quad (k_1 > k_2)$$

$$z_j = e^{k_2 j / z} \int_{-\infty}^{z} n_s (z') e^{-k_2 j / z'} dz' +$$

$$+ e^{k_1 j / z} \int_{z}^{\infty} n_s (z') e^{-k_1 j / z'} dz'.$$

The values $\mu_j$ proper are defined from the equation:

$$J_0 (\mu_j) = \omega a \mu_j J_1 (\mu_j)$$

and

$$\omega a = \frac{\chi a D_a \sqrt{2 \pi m a}}{k T_c}.$$

The parameter $\omega a$ is an analogue of the Bio criterium from the heat transfer theory ($\omega^{-1} = \text{Bio}$), and the following relation is valid:

$$\omega a = 3M / \beta a,$$

where $M$ is a characteristic Mach number in the HC. Under typical HC operation condition $\omega a = 0.1...0.5$. For the concentration of evaporated atoms $< n_a >$, average in terms the section and near-wall concentration $n_w$, the following equations may be obtained:

$$< n_a > = 4 \sum_{j=1}^{\infty} \frac{1}{1 + \omega \beta a \mu_j} \frac{z_j (z)}{k_1 / - k_2 j},$$

$$n_w = \sum_{j=1}^{\infty} \frac{2 \omega \beta a \mu_j z_j (z)}{(1 + \omega \beta a \mu_j) (k_1 / - k_2 j)}.$$

The difference between $n_a$ and $n_w$ determines the local erosion value and vapor flow alternation in the given section, so, having integrated $n_a - n_w$ from $-\infty$ to $z$, the flow of evaporated atoms can be obtained. We shall divide the atoms flow hereinafter by its maximum possible value:

$$G_m = \frac{\nu_t}{4} \int_{-\infty}^{+\infty} 2 \pi r n_s dx,$$

where $\nu_t$ is the average thermal velocity of evaporated atoms.

Concrete calculations of vapor parameters and HC erosion distribution were conducted for a cylindric cathode of 0.8 mm inner diameter and 0.4 mg/s plasma-generating gas (xenon) consumption. The cathode was considered to be endless, but the evaporation from its surface took place only within a finite distance – from the sections $x_1 = 0$ and $x_2 = 12$ mm. In the remainder of the HC channel atoms condensed on its walls. The gas flow was directed from $x_1$ to $x_2$. Fig. 2 shows typical longitudinal distributions of cathode material vapor concentrations.

![Fig. 2. Distribution of vapor equilibrium concentration $n_s$, near-wall concentration $n_w$, and average in terms cathode section concentration $\bar{n}_a$ ratios along hollow cathode at $\alpha = 1/2$ at (2.7) hereinafter](image)

The calculations conducted showed that most prone to erosion are the cathode zones placed at its inlet ($x_1 = 0$) and outlet ($x_2 = 12$ mm) sections. Therewith, the most strong and non-uniform erosion takes place at the outlet section. The erosion can vary by a factor of $10^4$ within this zone.

It is convenient to characterize the total erosion $G_t$ by the effective evaporation coefficient determined as

$$q_t = G_t / G_m.$$

In Fig. 3 $q_t$ is plotted against the defining parameters $\beta a$ and $\omega a$. The results obtained witness that for the typical operation conditions of the considered HC $q_t \leq 0.1$.

The exact solution of the equation (2.10) was obtained with the help of the method of the separation of variables. The radius dependance of parameters was
expressed through the Bessel functions of the zero order $J_0(\mu / r)$. It is very difficult to use these functions for the solution of the equations (2.8) and (2.9). It is advisable to use simpler approximate expansions of the following appearance:

$$n_a(z,r) = n_{a0}(z) + n_a(z) \frac{r^2}{R^2}.$$  \hfill (2.17)

The use of this expansions for the solution of the equation (2.10) results in the following values of the vapor characteristics:

$$n_a(z,r) = \frac{r^2 - 1/2}{2 \omega_a + 1/2} n_s(z) +$$

$$+ \frac{2 \omega_a + 1 - r^2}{2 \omega_a + 1/2} < n_a(z) >,$$

$$n_w(z) = \frac{1}{2 \omega_a + 1/2} \left( \frac{n_s(z)}{2} + 2 \omega_a < n_a(z) > \right),$$

$$< n_a(z) > = \frac{1}{k_1 - k_2} \left( k_2 (k_2 - \beta_a) e^{k_2z} \int n_a(z') b^{-k_2'} dz' + \right.$$  

$$+ k_1 (k_1 - \beta_a) e^{k_1z} \int n_s(z') e^{-k_1'} dz'),$$  \hfill (2.20)

where $k_{1,2} = \beta_a / 2 (1 - b + 6 \omega_a) \pm$

$$\pm \sqrt{\beta_a^2 / 4 \left( 1 - \frac{b}{3} + \frac{6 \omega_a}{1 + 4 \omega_a} \right)}.$$  \hfill (2.21)

The comparison of the approximate solutions (2.18...2.20) with the exact ones (2.11...2.14) at $b = 0$ demonstrates high accuracy of the approximation (2.17). Thus, the approximate profiles of parameters presented in Fig. 2, within a graphical accuracy coincides with the exact values everywhere except the exit section, where the error of (2.17) achiever 10%. The same relates to the effective evaporation coefficient (Fig. 3): the data for $q_t$, obtained from the exact and approximate solutions of the equation (2.10), coincide at $\omega_a \geq 0.3$ within graphical accuracy.

Fig. 4 is a plot of the evaporation coefficient $q_t$ vs parameters $b$ and $\omega_a$.

**CONCLUSIONS**

The local erosion of hollow cathodes used as cathodes-compensators of electric propulsion has been considered. In the approximation of the free-molecular regime of the cathode erosion products flow, analytical expressions have been obtained pointing to essential
non-uniformity of the local erosion on the cathode surface. A hydrodynamic model for calculation of vapor parameters and hollow cathode local erosion has been stated. This model accounts the basic determining factors. The features of the hollow cathode erosion have been calculated without taking into consideration erosion product ionization processes. The influence of the determining factors on the HC erosion rate has been analysed. The data for spatial distributions of vapor parameters in the cathode cavity have been obtained. A possibility of utilization of simple model distributions has been analysed.

REFERENCES

