NUMERICAL CALCULATION OF A CYLINDRICAL MPD THRUSTER

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Abstract

For several years experimental investigations with magnetoplasmodynamic (MPD) thrusters have been conducted at the Institute for Space Systems (IRS). In order to assess the performance range and to achieve a better understanding of these thrusters, numerical MPD codes have been developed and applied. These calculations allow the determination of the current, electron temperature and flow distribution in these self-field MPD thrusters. The extended Ohm's law is applied to calculate the current contour lines, the electron energy equation is taken to calculate the electron temperature distribution and a two-dimensional flow code is used to obtain the velocity, pressure and heavy particles temperature distributions. By this computation method the thermal nonequilibrium in the plasma flow can be determined. The gas dynamic equations are solved by a finite volume code. The electromagnetic discharge equation and the electron energy equation are solved for the steady phase by a modified Gauss–Seidel algorithm. All equations used for the iteratively solved high enthalpy flow are coupled completely by the correlated source terms. For a better conformity with the physical phenomena of the plasma flow in the region of high specific impulses, higher ionization modes and real gas effects of argon are taken into account. The calculations with the high enthalpy flow solver HEFLOS are done for a cylindrical, self-field MPD configuration, because for this device the highest specific impulses are expected.

Nomenclature

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<thead>
<tr>
<th>Symbol</th>
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Subscripts

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1 Introduction

For planetary space missions, like the human mission to Mars, specific impulses of 2 000 s and high thrust levels of some 10 N will be required for the thruster system. MPD thrusters are considered for such missions due to their high thrust level and simplicity. But much effort is still required to increase the specific enthalpy and thrust efficiency.

Experimental investigations on magnetoplasmodynamic (MPD) thrusters are conducted at the Institute for Space Systems (IRS). In order to achieve a better understanding of the investigated thrusters, numerical procedures to calculate these MPD accelerators have been applied. Fig. 1 shows a simplified scheme of the interacting forces in an MPD accelerator.
mental results. Therefore, higher ionization modes and real gas effects were included in the calculation model to improve the numerical results at high specific impulses.

By means of the Saha equations for the different ionization levels, the conservation of mass and the neutrality of electric charge, the ionization rates up to the sixth ionization level have been iterated. Considering the higher ionization modes in the electric conductivity, the differential discharge equation up to the sixth ionization level can be calculated. It turns out that these effects influence the electromagnetical source terms of the plasma flow in the MPD thruster. By the determination of the partition function for each plasma species, the effective ratio of the specific heats is obtained and is introduced in the calculation of the heavy particles energy equation. With this real gas implementation Navier–Stokes equations for the heavy particles flow are calculated. The electron energy equation is solved to determine the thermal non-equilibrium of the plasma flow. All equations applied to the iteratively solved high enthalpy flow are coupled by corresponding source terms which correlate the different physical processes and their corresponding conservation equations to one another.

In this paper this new two-dimensional axisymmetric program system HEFLOS is presented. The results were calculated for a cylindrical MPD thruster with argon as propellant at high specific impulses.

2 Computational Model

The model for the numerical simulation of a self-field MPD accelerator was developed at the IRS. This model unites three different physical fields to one another. The three parts of this model are the extended Ohm’s law for plasma to calculate the discharge, the electron energy equation to calculate the electron temperature and the Navier–Stokes equations to obtain the properties of the flow field.

In this two fluid non-equilibrium model the viscosity, electron and heavy particles conductivity are included. The basic assumptions for this model is the rotational symmetry of a quasi-neutral plasma flow, no electrodes surface processes and boundary layer effects are included, and electron and heavy particles energy are linked only by elastic transfer. An additional assumption is the input of the ohmic heating in the electron energy equation, due to the higher mobility of the electrons. The transfer of this heat to the heavy particles is contained in the compensation between the electron and the heavy particles temperature.
2.1 Discharge Code

The current distribution within a self-field MPD thruster is assumed to be two-dimensional, and no azimuthal current is to be expected. In order to calculate the current distribution of such an arc discharge, a two-dimensional computer code has been developed. The basic equation for the discharge is the extended Ohm’s law for plasmas:

\[ j = \sigma (E + \bar{v} \times \vec{B}) - \frac{\omega_e \tau_e}{B} (j \times \vec{B}) - \beta \nabla p_e \]  

(1)

Here \( \sigma \) is the electric conductivity, \( \omega_e \) the electron cyclotron frequency and \( \tau_e \) the electron collision time. Rewriting the Ohm’s law by means of the Maxwell’s equations for steady state conditions, one obtains a vector equation for the magnetic induction field \( \vec{B} \) in the form

\[
0 = \frac{1}{\mu_0} (\nabla \times (\frac{1}{\sigma} \nabla \times \vec{B}) - (\nabla \times (\bar{v} \times \vec{B}))) + \frac{1}{\mu_0} (\nabla \times (\beta(\nabla \times \vec{B}) \times \vec{B})) - \nabla \beta \nabla p_e 
\]  

(2)

with

\[
\beta = \frac{\omega_e}{B \sigma} = \frac{1}{e n_e} 
\]  

(3)

The equation (2) yields with a stream function \( \Psi = r B_\theta \) the elliptical, partial differential equation of 2nd order

\[
\frac{\partial^2 \Psi}{\partial r^2} + \frac{\partial^2 \Psi}{\partial z^2} - \frac{\partial \Psi}{\partial r} \left( \frac{1}{r} + \frac{1}{\sigma \partial \sigma} - \frac{\sigma \Psi \partial \beta}{r \partial z} + \mu \sigma \partial r \right) - \frac{\partial \Psi}{\partial z} \left( \frac{1}{\sigma \partial \sigma} + \frac{\sigma \Psi \partial \beta}{r \partial z} - \frac{2 \sigma \Psi \beta}{r^2} + \mu \sigma \partial z \right) - \frac{\sigma \mu e \Psi}{r} \left( \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} - \frac{v_r}{r} \right) + \mu_e \sigma \partial r \left( \frac{\partial \beta \partial p_e}{\partial r} - \frac{\partial \beta \partial p_e}{\partial z} \right) = 0
\]  

(4)

The function \( \Psi(r, z) = \text{const} \) now represents a current contour line, since \( B = B_\theta \) is proportional to \( I(r) \), where \( I(r) \) is the electric current carried through the cross sectional area \( \pi r^2 \).

The electric conductivity of a plasma is determined by

\[
\sigma = \frac{3}{8} \sqrt{\frac{\pi}{2}} \frac{e^2 n_e}{\sum_{\nu}(\pi \sigma^2) n_{\nu} Q_{\nu} \sqrt{m_{\nu} k T_{\nu}}}
\]  

(5)

where \( m_{\nu} \) is the reduced mass and \( T_{\nu} \) the reduced temperature. With respect to the different ionized levels the Gvosdover cross sections follow by

\[
Q_{\nu} = \frac{\pi}{4} \left( \frac{z_{\nu} e^2}{4 \pi \varepsilon_0 k T_e} \right)^2 \ln \left( \frac{144 \pi^2 \varepsilon_0^3 T_e^4 k}{n_e e^6 (z_{\nu}^2 (z_{\nu}+1))} \right)
\]

(6)

\( z_\nu \) stands for the charge number. For \( Q_{\nu} \) being the cross section between electrons and atoms in equation 5 the Ramsauer cross section is used. From these equations it is obvious that high ionization levels have a strong effect on the electric conductivity.

2.1.1 Ionization of an Argon Plasma

Due to the higher mobility of the electrons, the ionization of the argon plasma in MPD thrusters is dominated by the electron temperature. The Saha equation results in the equation for the different ionization levels

\[
n_{i} \frac{n_{i+1}}{n_{i}} = \frac{g_{i+1} (2 \pi m_e k T_e)^{3/2}}{g_{i}} e^{-\frac{\Delta E_{i}}{k T_e}}
\]  

(6)

Here \( n_i \) is the number density of the electrons, \( n_i \) the number densities of the heavy particles, \( g_i \) the weighting factor taking into account the degenerated states and \( \epsilon_i \) the ionization energy of the ionization level \( i \).

For small thermal non-equilibrium values Veis modified the Saha equation to:

\[
n_e \frac{n_{i+1}}{n_{i}} = \frac{g_{i+1} (2 \pi m_e k T_e)^{3/2}}{g_{i}} e^{-\frac{\Delta E_{i}}{k T_e}}
\]  

(7)

where \( T_e \) is the electron and \( T_h \) is the heavy particles temperature for all heavy particles.

With the sum of the partial pressures of all plasma components as the total pressure

\[
p = \sum_{\nu} n_{\nu} k T_{\nu}
\]  

(8)

the neutrality of charge

\[
n_e = \sum_{i} z_i n_i
\]  

(9)

and either the Saha equation 6 or the modified Saha equation 7 by Veis, considering the higher ionization modes up to the sixth ionization level, a seventh order polynomial for the electron density is determined. For the transition between both calculations of the electron density a \( \sin^2 \)-function is chosen. In equation 8 \( T_{\nu} \) represents the electron temperature \( T_e \) and the heavy particles temperature \( T_h \).

The equation for the electron density is solved by a Newton iteration. With this electron density the other partial densities like the argon atoms and the different ions are determined.

For an equilibrium temperature \( T_h = T_e = T \) up to 80 \( kK \) and a constant pressure of 100 \( mbar \) in Fig. 2, a typical concentration distribution with respect to the sum of heavy particles is presented. With the model developed here it is obvious that the ionization distribution for typical MPD conditions is determined up to a temperature of nearly 80 \( kK \).
2.2 Electron Energy Equation

The electron temperature has a strong effect on the electrical and thermal conductivity and on the electron density, which again influences the discharge pattern. Therefore, a two-dimensional code for the electron temperature distribution, corresponding to the two-dimensional discharge code, was written. The electron temperature distribution is determined by the energy equation for the electron component of the plasma.

\[
\nabla (\lambda_e \nabla T_e) + \frac{5}{2} \alpha_e \frac{e}{e} \nabla \cdot \mathbf{v} = \nabla \sum \nu n_e \alpha_{ev} (T_e - T_h) - \epsilon_i \frac{\partial n_e}{\partial t} \bigg|_{rct}
\]

(10)

The subscript \( \nu \) represents the heavy particles. The first term on the left hand side represents the conductive heat flux in the electron gas, and the second term gives the convective heat flux of the electron gas due to the electron drift. The energy input due to ohmic heating is represented by the first term on the right hand side. The sum of losses due to the energy transfer from the electron gas to the heavy particles gas is calculated by the second term on the right hand side. The reaction losses are given by the last term, where \( \epsilon_i \) is given by Unsold. The heat transfer coefficient \( \alpha_{ev} \) and the thermal conductivity \( \lambda_e \) depend on the electron temperature, where \( \alpha_{ev} \) is given by Unsold. The heat transfer coefficient \( \alpha_{ev} \) and the thermal conductivity \( \lambda_e \) depend on the electron temperature, where \( \alpha_{ev} \) is given by Unsold.

\[
\alpha_{ev} = \frac{8 \sqrt{2} Q_{ev} \sqrt{\mu_{ev} k T_{ev}}}{\sqrt{\pi}} \frac{k}{m_e + m_{ev}}
\]

(11)

and the thermal conductivity:

\[
\lambda_e = 15 \sqrt{\frac{\pi}{2}} \frac{n_e k^2 T_e}{\sqrt{2} \sum \nu(Q_{ev}) n_{ev} Q_{ev} \sqrt{m_{ev}} k T_{ev}}
\]

(12)

In the sum of this conductivity equation \( \nu \) also includes the electron component. For \( \nu = e \) the collision cross section \( Q_{ee} \) can be set equal to the Gysodover cross section. If \( Q_{ev} \) is the collision cross section between electrons and ions, the Gysodover cross section is used with respect to the different ionization levels; and for \( Q_{ev} \) being the collision cross section between electrons and atoms the Ramsauer cross section is used.

With respect to the rotational symmetry, equation 10 results in the following elliptical, partial differential equation of 2nd order:

\[
\lambda_e T_e \frac{\partial^2 T_e}{\partial \tau^2} + \lambda_e T_e \frac{\partial T_e}{\partial \tau} + 5 k_\nu \frac{T_e}{e} \frac{\partial T_e}{\partial \tau} + 5 k T_e \frac{\partial j_r}{\partial \tau} + 5 k T_e \frac{\partial j_z}{\partial \tau} + 5 k T_e \frac{\partial j_r}{\partial \tau} + 5 k T_e \frac{\partial j_z}{\partial \tau} + 5 k T_e \frac{\partial j_r}{\partial \tau} + 5 k T_e \frac{\partial j_z}{\partial \tau}
\]

(13)

The terms on the left hand side are taken in the usual fluid dynamic manner, where \( \vec{q} \) is the independent variables vector and \( \vec{F} \) and \( \vec{G} \) are the spatial derivatives vectors. The first two terms on the right hand side are the dissipative derivatives. For the impulse equations \( \zeta \) represent here the viscosity coefficient \( \mu \), which is given by:

\[
\mu = \frac{3}{8} \sqrt{\frac{\pi}{2}} \sum \nu(Q_{ev}) n_{ev} Q_{ev} \sqrt{m_{ev}} k T_{ev}
\]

(15)

Since the temperatures of the heavy particles are equal, \( T_l = T_{lv} = T_h \) and \( m_{lv} \) is the reduced mass for \( l, \nu \) as heavy particles subscripts. For the energy equation \( \zeta \)
represents in the dissipative terms the heat conductivity coefficient $\lambda$.

$$\lambda = \frac{15}{8} \sqrt{\frac{\pi}{2}} \sum_i n_i k^2 T_i$$

The transport coefficients are derived in a similar manner as in $^{34}$ and more detailed derivations for the coefficients are described in $^{28}$. The cross sections $Q_{l\nu}$ for all coefficients are taken from the references $^{29,30}$.

The source vector $\vec{H}$ is given to:

$$\vec{H} = \begin{bmatrix} 0 \\ j_{\eta} B_{\eta} \\ j_{\xi} B_{\xi} \\ \sum_{\nu} n_{\nu} n_{\nu} \alpha_{\nu}(T_h - T_e) \end{bmatrix}$$

The terms in the impulse equations represent the $j \times \vec{B}$ forces. The transfer of Joule's heat is contained in the temperature compensation between the electron and the heavy particles temperature, which is represented by the source term in the equation of energy conservation.

### 2.3.1 Real Gas Effects

Drellishak et al. $^{26}$ calculated the variable ratio of specific heats $\kappa$ derived from the partition functions for an argon plasma (see Fig. 3). By inspecting the Navier-Stokes equations it is obvious that the ratio of specific heats occurs only in the energy equation.

$$\frac{\partial \epsilon_\kappa}{\partial t} + \nabla((\epsilon_\kappa + p)\vec{v}) = \nabla(\vec{\nabla} \vec{v}) + \nabla \lambda \nabla T + H$$

The index $\kappa$ indicates that the specific energy is a function also of $\kappa$, being a variable depending on $p$ and $\rho$

$$\epsilon_\kappa = \frac{p}{\kappa - 1} + \frac{\rho q^2}{2}$$

Next, another specific energy is defined with a constant ratio of specific heats $\gamma = \text{const.}$,

$$\epsilon_\gamma = \frac{p}{\gamma - 1} + \frac{\rho q^2}{2}$$

The difference of both specific energies is then

$$Q = \epsilon_\kappa - \epsilon_\gamma = p \left( \frac{1}{\kappa - 1} - \frac{1}{\gamma - 1} \right)$$

Replacing $\epsilon_\kappa$ by $\epsilon_\kappa = \epsilon_\gamma + Q$, the calculated energy equation reads then

$$\frac{\partial \epsilon_\gamma}{\partial t} + \nabla((\epsilon_\gamma + p)\vec{v}) + \nabla(Q\vec{v}) = \nabla(\vec{\nabla} \vec{v}) + \nabla \lambda \nabla T + H$$

Since only the steady state is of interest the partial time derivation $\partial Q/\partial t$ would be neglected. Due to this implementation of the real gas effects, the Riemann solver for the flow field need not be changed since $\gamma$ is constant.
The transformation are chosen such that the grid spacing in the curvilinear space is uniform and of unit length (see Fig. 4). The original cylindrical space will be referred to as the physical domain. There is a one to one correspondence between a physical point in space and a computational point. By means of this construction a computational code can be produced for a wide variety of physical geometries.

2.5 Boundary Conditions

The proper boundary conditions for the discharge equation follow from the geometry of the thruster walls and electrodes. At the insulator inside the thruster and at the inflow boundary $\Psi$ is set to $-\frac{\rho L}{I}$. For the electrodes the electric field is assumed to be normal to the surfaces $\ddot{E}(\Psi) \cdot \vec{n} = 0$. In accordance with the curvilinear grid, where the $\eta$ coordinates are perpendicular to the electrodes, it is equal to $E_\xi = 0$, and it follows:

$$\frac{\partial \Psi}{\partial \eta} \left[ \frac{\partial \Psi}{R} (\xi \eta_r - \xi_z \eta_z) + \frac{1}{\sigma} (\xi_z \eta_r - \xi_r \eta_z) \right] +$$

$$\frac{\partial \Psi}{\partial \xi} \frac{\partial \Psi}{R} (\xi_r^2 - \xi_z^2) + \mu_0 \Psi (\xi_r u - \xi_z v) = 0$$

(23)

At the other boundary sections and at the symmetry axis $\Psi$ is set to 0.

At the outflow boundary $T_e$ is set to a value in accordance with measurements. The solid bodies of the thruster are treated as thermal insulators. Therefore the boundary condition inside the thruster is given by $\nabla T_e \cdot \vec{n} = 0$, where $\vec{n}$ is the normal vector of the surfaces. Due to the axial symmetry, $\frac{\partial T_e}{\partial \eta} = 0$ on the axis. At the inflow boundary $T_e$ is set to a constant value of 7000 K in accordance with the measurements.

For the flow field equations all values at the inflow boundary are determined from cold gas measurements and are fixed during the complete calculation. The outflow boundary is considered as a free stream boundary and either the independent flow variables are complete extrapolated for the case of supersonic conditions or the inner energy $e$ is calculated with the other extrapolated independent variables and a given pressure for the case of subsonic conditions. For all other boundaries, the solid body and symmetry axis, a no flux condition through the boundary is taken. This is done by an extrapolation of the density and inner energy, $\nabla \rho \cdot \vec{n} = 0$ and $\nabla e \cdot \vec{n} = 0$, a reflection of the tangential velocity and putting the normal velocity to be zero, $\vec{v} \cdot \vec{n} = 0$.

3 Outline of the Computational Method

For the calculation of the self-field MPD flow, a modular program system was developed which connects three different physical fields to one another. The three program system parts described in the previous sections are the discharge, the electron energy equation and the heavy particles flow field code. The high enthalpy flow solver HEFLOS is used in this paper for a cylindrical configuration of an MPD accelerator. The MPD flow is calculated in general curvilinear coordinates, so, a good adjustment to different geometries is possible.

The three codes are connected iteratively in the following manner: for a given flow field and an electron temperature distribution, the current, and hence the magnetic field distribution, was determined. With the ohmic heating, as a result of the current distribution, an electron energy calculation follows and determines a new electron temperature distribution. With these results the flow field equations were integrated. In the next time step, this new flow field and the new electron temperature distribution were taken to calculate the new electromagnetic field distribution, and so on.

The nonlinear, elliptical, partial differential equations for the discharge (4) and the electron energy (13) are solved with a finite difference method. The discretization yields a nonlinear equation which is iteratively solved with a modified Gauss-Seidel algorithm. The nonlinear, hyperbolic, partial differential equations for the flow field (14) are solved with the finite volume solver NSFLEX. Here this code was modified for a non-equilibrium state and for the electromagnetic source terms interaction. The subsystem NSFLEX uses a Godunov upwinding scheme for the flux discretization. With the Newton method an implicit formulation is achieved, and the relaxation is done by a point Gauss-Seidel procedure.

All equations used for the iteratively solved high enthalpy flow are coupled completely by corresponding source terms which correlate the different physical processes and their corresponding conservation equations to one another. The fact that the conservation equations are solved by three individual solution procedures does not reduce the coupling intensity among the different equations, it represents only a solution and modular program developing strategy. These individual solution procedures enable an easy implementation of different physical processes and a faster and more exact calculation for several processes. However, a faster calculation is possible, since not all processes need extensive calculation procedures, or the calculation step size need not be reduced for the required value of the slowest conservation equation. With an individual procedure the calculation and the boundary conditions can be adjusted more exactly for various problems.

4 Discussion of Results

The next figures show results for the cylindrical MPD thruster geometry ZT3-IRS (see Fig. 5). The grid used for the calculation is shown in Fig. 4 as the physical
domain. In this figure only every second point in both directions is shown. The calculation grid is also extended in the outflow direction up to \( z = 360 \text{mm} \) at the symmetry axis. So, the total number contains \( 153 \times 27 \) grid points. The grid was created numerically, and the distortion of the grid is almost less than 1.04. The flow inlet boundary conditions were iterated to coincide with an experimentally obtained cold gas thrust of 1 N at a mass flow rate of 2 g/s.

![Fig. 5: Cylindrical MPD thruster ZT3–IRS.](image)

For a given current of 10 kA the computation yields the current density distribution as illustrated in Fig. 6. The calculated current contour distribution corresponds with the current contour distribution at a continuing mode if the cathode is hot glowing and is emitting electrons thermionically along its complete length.

![Fig. 6: Calculated current contour lines for 10 kA.](image)

The heavy particle (top) and the electron temperature distribution (bottom) within and outside the ZT3 thruster is shown in Fig. 7 for a given current of 10 kA. The maximum temperature value of the heavy particles occurs at the tip of the cathode on the symmetry axis. The maximum electron temperature value occurs also at the tip of the cathode, but there is also a week increase of the electron temperature at the beginning of the anode.

![Fig. 7: Heavy particles (top) and electron (bottom) temperature contours for 10 kA.](image)

The pressure map (Fig. 8) demonstrates the expansion flow with the flow described in section 2.3, in which the relatively low pinch effect was taken into account.

![Fig. 8: Pressure distribution.](image)

The velocity distribution corresponding to the temperatures and density distribution is shown in Fig. 9 as a vector graph. It shows a high increase from the inflow boundary downstream. The radial velocity components inside the channel are quite small, what is also an indication for a relatively low pinch effect there.

![Fig. 9: Velocity distribution.](image)

The calculated thrust is shown in Fig. 10. The marked points are the calculated values, and they are linked together with the given cold gas thrust of 1 N for 0 kA. The dash line represents in this figure the magnetic thrust \( \frac{m v^2}{4 \pi} \ln \left( \frac{r_e}{r_c} + \frac{3}{4} \right) \). The effective anode radius
Acknowledgement

This work was supported by the Air Force Office of Scientific Research through the European Office of Aerospace Research and Development under Grant AFOSR 91-0118.

References


