NONLINEAR DEVELOPMENT OF SPACE CHARGE INSTABILITY
IN MPD-THRUSTER FLOWS

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Abstract

In the first part of this paper a review of the investigation of instabilities in multicomponent non-equilibrium MPD thruster flows is given, carried out at IRS and MAN Technologie AG. Also latest results obtained from nonlinear development of the space charge instability are presented showing that the nonlinear development of the current density actually leads to strong oscillatory behaviour depending on the time development of the linear states, particularly on the local value of the electric field strength and thus on the values of the relative gradients. Finally the connection between the specific impulse and development of the space charge instability will be discussed.

1. Introduction

New interest in the development of magnetoplasmadynamic (MPD) thrusters was awakened with the beginning of the 1980's. Despite presently low overall efficiency, their high and widely adjustable specific impulse allows better payload ratios than in the case of chemical propulsion systems.

Figure 1: Frequency spectrum of the discharge voltage oscillations (DT2-IRS)

Since the specific impulse is proportional to $I^2/m$, operating the thruster at a high ratio $I^2/m$ will improve its efficiency. However, as shown by experimental investigations, oscillations in the discharge voltage (Fig.1) occur if the thruster is running at a point beyond a critical value $(I^2/m)_{cr}$. Simultaneously, the anode fall increases which leads to a strong decrease of the efficiency.

This unstable behaviour called "onset" is the main barrier in the development of highly efficient MPD thrusters and has been object of many studies. A survey of literature to this subject can be found in Rempfer et al. or Wagner et al.

A systematic investigation of instabilities in multicomponent nonequilibrium MPD thruster flows is in progress at IRS since 1987. In continuation of this work a cooperation between IRS and MAN Technologie AG has been established since 1990. One of the main objects of study is an explanation for the onset of performance limiting instabilities in MPD thrusters. Contrary to lines of study in the USA, research at IRS and MAN Technologie AG concentrates on macroscopic and drift instabilities, as only these are considered to be able to drive such effects as charge carrier depletion and/or current chopping.

First results obtained by Rempfer et al. with a simplified three-fluid formalism showed the appearance of a macroscopic (electrostatic) electron acoustic wave instability.

In continuation of this work, a three-fluid theory including electric and magnetic fields, finite gradients for the flow variables and dissipative terms was developed. Two gradient driven instabilities were found from the solution of the linear dispersion relation of this system by Wagner et al. One of these instabilities was tentatively identified as an unstable acoustic wave mode, whereas the other instability showed properties of the so-called space charge instability. In a further work, Wagner et al. showed that a space charge instability, originally defined in a bounded plasma, can also appear in unbounded systems.

The space charge instability (also called Pierce or di-
ode instability) is of particular interest in discussing charge carrier deficit and/or arc starvation. The nonlinear development of this instability leads to the 'current chopping' instability which occurs when a critical current density is exceeded and involves rapid reductions of the current and sometimes complete disruption. The appearance of this instability may be an explanation of the onset phenomenon in MPD thruster flow.

In the quasi-linear theory, latest calculations for a two-fluid model of a (fully ionized) plasma yield approximate analytical expressions for the unstable roots of the linear dispersion relation, giving the explicit dependence of the oscillation frequencies and linear growth rates on the wave vector and the various plasma parameters. According to quasi-linear theory it should be possible to calculate the electron and ion susceptibilities as well as the anomalous transport rates, using the expressions for the isolated instabilities.

In the nonlinear theory a first calculation of the nonlinear evolution of the space charge instability was done by Maurer et al. using the quasi-analytical method QAM. It could be effectively shown that an interruption of the current density occurs in the nonlinear development up to fifth order. Thus the space charge instability leads, in its nonlinear development, to unstable current density oscillations including current interruption.

In continuation of this work the calculations of the nonlinear development of the space charge were continued extending the QAM-development to 9th order. According to previous works a reduced three-fluid model of the plasma flow in the MPD thruster was used with the components electrons, ions, and neutral particles of the working gas.

For each component the continuity equation, Navier-Stokes equations, the energy transport equation, Maxwell's equations and the equations of state have to be formulated yielding a system of 24 nonlinear partial differential equations.

Although some progress was made in diagnostics of MPD thrusters, it is still not possible to define the distribution of fields exactly. Therefore concept of relative gradients will be used again to define a local stationary state which will give rise to the development of nonlinearities in the system.

To calculate the behaviour of the system, the quasi-analytical procedure QAM for the solution of insta- nency field equations will be used. The characteristic feature of this method is that it is based on algebraic computer codes (e.g. REDUCE). With the aid of such a code the nonlinear system will be developed in a hierarchy of linear inhomogeneous equations to be solved successively.

As already mentioned, the calculations with QAM show that in the (first order) stationary state of the system a linear dependence between the relative gradients and the local stationary electric field strength can be found. The stationary electric field gives rise to the development of instabilities in second order which have oscillatory behaviour. According to Wagner et al. these instabilities are found to be relevant for the nonlinear development. The nonlinear development of the instability is now calculated in dependence on the relative gradients. It can be shown that beyond a threshold value the current density has oscillatory behaviour, which leads to local time dependent interruptions for high relative gradients. This event is connected with oscillations of the local electric field strength. Further calculations show the different behaviour of the field variables between continuous and pulsed operation modes.

Besides the space charge instability, the electron acoustic wave instability is found to occur. A possible influence of this instability on the onset phenomenon can take place if there is a heat transfer from electrons to the neutral component. This will give rise to local gradients forcing the space charge instability and its nonlinear development.

2. Basic Equations and QAM-Development of the System

In a simplified model we consider the plasma flow occurring at a point in the anode region of the thruster. Since we are only interested in the influence of the electric field, and no external magnetic field is applied, we neglect the effects of the magnetic field and all components of the flow perpendicular to the electric field, thus obtaining a one-dimensional system consisting of three fluids:

(i) ions of the working gas (single charged),

(a) neutral particles of the working gas and

electrons.

For each component, the following equations have to be considered (a detailed discussion can be found in other reports):

- the equations of continuity

$$\frac{\partial n_k}{\partial t} + \nabla \cdot (n_k \mathbf{v}_k) - \Gamma_k = 0, \quad k = a, i, e$$
with the terms for non-equilibrium ionization:

$$\Gamma_e = \Gamma_i = - \Gamma_a = S_{se} \left[ n_a n_e - \frac{n_a^2 n_i}{K_e} \right],$$

the equations of motion

$$n_k m_k \left[ \frac{\partial \tilde{u}_k}{\partial t} + (\tilde{u}_k \cdot \nabla) \tilde{u}_k \right] + m_k \tilde{u}_k \Gamma_k + \nabla p_k - n_k Ze \left[ \tilde{E} + \tilde{u}_k \times \tilde{B} \right] - \eta_k \left[ \frac{1}{3} \nabla (\nabla \cdot \tilde{u}_k) + \Delta \tilde{u}_k \right] - 2 \left( \frac{d f(\tilde{v}_k)}{d \tilde{v}_k} - \frac{1}{3} \nabla \cdot \tilde{v}_k \right) \nabla \eta_k + n_k \sum_{l} \nu_{k, l} \frac{m_{k, l}}{m_k + m_l} (\tilde{u}_k - \tilde{v}_l) = \tilde{0},$$

with \( k, l = a, i, e \)

$$Z_i = - Z_e = 1 , \ Z_a = 0$$

$$\nu_{k, l} = \text{collision frequency between } k \text{ and } l$$

and, instead of the energy equations,

the equations of variation of the temperature

$$c_v \rho_k \left( \frac{\partial}{\partial t} T_k + \tilde{u}_k \cdot \nabla T_k \right) + \nabla \cdot \tilde{q}_k + p_k \nabla \cdot \tilde{u}_k + \chi_i \Gamma_k \delta_{k \kappa} - \Phi_k - \tilde{j}_k \tilde{E}$$

$$- \tilde{d}_{k \kappa} c_v \rho_k \sum_{l} (T_k - T_l) = 0,$$

with \( k, l = a, i, e \)

$$\rho_k = n_k m_k \text{ mass density of } k,$$

$$\tilde{q}_k = \text{heat flow vector},$$

$$\Phi_k = \text{dissipation function of } k^{12,13},$$

$$\chi_i = \text{ionization energy},$$

$$\tilde{\nu}_{k, l} = \text{energy equilibration frequency between particle } k \text{ and } l,$$

together with the equations of state:

$$p_k = n_k k_B T_k.$$ 

Since the magnetic field is neglected, only the Poisson equation is used,

$$\nabla \cdot \tilde{E} = \frac{\rho e}{\epsilon_0}.$$ 

The current density is defined by

$$\tilde{j} = e (n_i \tilde{v}_i - n_e \tilde{v}_e).$$

These expressions constitute the set of equations describing the system.

According to QAM, the dynamical variables are developed in a 'perturbation' series of the form

$$\psi(\tilde{r}, t) = \psi_0 + \sum_{n} \psi_n (\tilde{r}, t) \cdot \epsilon^n,$$

where \( \psi(\tilde{r}, t) \) stands for either of the variables and \( \psi_0 \) represents the equilibrium value which is independent of \( t \) and \( \tilde{r} \). Inserting this series in the system of nonlinear equations and ordering with respect to powers of \( \epsilon \), there results in a hierarchic system of inhomogeneous linear differential equations. This linear system now can be solved successively.

4. Initial State and
Linear Dispersion Relation

According to the QAM-method the first order part of the variables defines the initial distribution of plasma flow parameters. In our case we assume first that simultaneously the first order will be the stationary state of the system, that means the first order variables are assumed to be time independent. In a second consideration we assume that the first order variables are constant during a time interval, short in comparison to the inverse growth rate of the instability, and equal to zero for longer times. This case represents a pulsed operation mode. To describe the influence of gradients in first order in both cases the concept of "relative gradients" already mentioned in \(^4\) is used which is equivalent to the inverse scaling length.

According to the QAM-method the linear dispersion relation gives the range of instable behaviour of the system for times near the starting time of the calculation. In establishing this linear dispersion relation, the expressions for the neutral component can be decoupled due to the smallness of the interaction terms

$$n_k \sum_{l} \nu_{k, l} \frac{m_k m_l}{m_k + m_l} (\tilde{u}_k - \tilde{v}_l)$$

for the collisions of neutrals and charged particles. Hence, there follow two separate dispersion relations, one for the charged particles electrons and ions, and one for the neutrals. The latter is already discussed in Maurer et. al.\(^9\), so that only the calculations for the charged particles are considered here.

With the following parameter set of variables defining stationary state, selected as typical for thrusters operated at IRS, all examples in this paper were calculated:

$$m_i = 6.631 \cdot 10^{-26} \text{ (Argon)}$$

$$m_e = 9.109 \cdot 10^{-31}$$
\[
n_i = 2.650 \cdot 10^{21} = n_0 \\
n_e = 2.650 \cdot 10^{21} = n_0 \\
v_i = -2.823 \cdot 10^3 \bar{v}_i = v_0 + v_{1,i} \\
v_e = 7.550 \cdot 10^5 \bar{v}_e = v_0 + v_{1,e} \\
T_i = 1.267 \cdot 10^4 = T_0 \\
T_e = 1.267 \cdot 10^4 = T_0 \\
B_i = 0 \\
v_0 = -2.813 \cdot 10^3 \bar{v}_i \\
j_i = -3.217 \cdot 10^3 \bar{v}_e 
\]

This gives the following results for the transport coefficients,
\[
\sigma = 3.342 \cdot 10^3 \\
\eta_i = 0.559 \cdot 10^{-5}, \eta_e = 0.21 \cdot 10^{-7} \\
\lambda_i = 0.108 \cdot 10^{-4}, \lambda_e = 0.392
\]

Solution of first order gives a connection between relative gradients, electron velocity, and the electric field strength. Using the above parameter set, there follows
\[
\varepsilon_{n,i} = -\varepsilon_{n,e} = \varepsilon_{v,e} \\
\varepsilon_{T,i} = -\varepsilon_{T,e} = \varepsilon_{v,e} \\
\varepsilon_{v,i} = -\varepsilon_{v,e} \\
\bar{v}_e = 108.78 \varepsilon_{v,e} \\
E_1 = -1.046 \cdot 10^7 \varepsilon_{v,e}
\]

For the scaling of frequency and growth rate with electron plasma-frequency, and for wave number and relative gradients with the inverse Debye length was selected
\[
\omega = 2.904 \cdot 10^{12} \bar{\omega}, \gamma = 2.904 \cdot 10^{12} \bar{\gamma} \\
k = 3.832 \cdot 10^6 \bar{k}, \varepsilon_{v,e} = 3.832 \cdot 10^6 \varepsilon_{v,e}
\]

Temperature is given in K, all other values are given in MKSA units.

The dispersion relation was solved numerically for various values of \( \varepsilon_{v,e} \). From these results, the normalized growth rates of the two relevant instabilities as functions of the normalized wave number \( \bar{k} \) for various relative gradients are presented in Figs. 2, 3 and 4 below.

The figures show a rapid increase of the maximum of the first growth rate (solid lines) with increasing relative gradient and a slower increase of the second (dashed lines). Thus only the nonlinear behaviour of the first instability is of interest in the range of relative gradients considered here (see also sec. 6). Fig. 5 shows the development of the maxima of the growth rates with increasing relative gradients. It can be seen that the first growth rate is negative until the relative gradient reaches a threshold value at about \( \varepsilon_{v,e} = 2 \cdot 10^{-5} \). Then a rapid increase of this rate occurs and beyond
$\varepsilon_{v,e} = 3 \cdot 10^{-5}$ the first growth rate dominates the instable behaviour. Fig. 5 shows the values of wave numbers correlated to the maxima of the growth rates. It is interesting to note, that the wave number of the first maximum increases to a factor 10 in the region of the threshold value. As was pointed out in previous publications$^5,9$ these instabilities found here can be interpreted as space charge instabilities.

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5. Nonlinear Development

The nonlinear calculations follows the method QAM described in detail in an earlier publication$^{10}$. The development, already presented in Maurer et. al.$^9$ was carried out up to 9th order. Excluding extreme cases, QAM-solutions already show convergent behaviour up to this order. Thus there is no need to an extension of calculation order.
In the first calculation it was assumed that the first order part in QAM defines simultaneously a stationary state of the system where the relative gradients and thus the electric field strength remain time independent. This model is equivalent to continuous operation of the thruster. The excitation of the nonlinear parts will include a time independent term. Thus the system reaches a new stationary state superposed by time dependent perturbations. Since the initial states are not exactly known, the nonlinear solutions are chosen so that for all gradients the final stationary state of particle density and velocity are equal, and the particle density is always greater than 10% of the stationary value.

The results of the calculations of the local current density for three relative gradients are presented in Figs. 7 to 9, where the current density is normalized to the Langmuir current density \(en_{\nu,\theta}/4\). For weak gradients (Fig.7), the current density shows oscillatory behaviour in the nonlinear development where for short times the influence of the higher harmonics of the QAM development can be recognized. With increasing relative gradients (Fig.8) the ground wave of the development dominates the time evolution superposed by a quasiperiodic excitation and damping of the instability. Finally, Fig.9 shows the case where the local current density goes to nearly zero at certain times. Thus in these time regions local current interruption occurs. This is connected with fluctuations of the local electric field strength (Fig.10), which varies between 30 and 90 V/cm. For stronger gradients the current density exceeds zero and becomes positive, as already shown. However, the local electric field strength then will increase to values greater than 100 V/cm which is hardly expected in MPD devices.

Considering now the frequencies in Figs.7-10, oscillations in a wide range between 0.05 MHz and 20 MHz can occur depending on the value of the relative gradient.

Figure 10: Electric Field Strength as a Function of Time for \(\varepsilon_{\nu,\theta} = 5 \cdot 10^{-4}\).

Figure 11: Normalized Current Density as a Function of Time \(\varepsilon_{\nu,\theta} = 5 \cdot 10^{-4}\) (Pulse).

Figure 12: Electric Field Strength as a Function of Time \(\varepsilon_{\nu,\theta} = 5 \cdot 10^{-4}\) (Pulse).

As mentioned above a second case is considered where the first order variables are constant during a time interval, short in comparision to the inverse growth rate of the instability, and equal zero for longer times. This model is equivalent to pulsed operation of the thruster. The results for the current density and the electric field strength are shown in Figs. 11 and 12. Using the same initial states as defined in the continuous mode there are only fluctuations in the electric field strength but no relevant fluctuations in the current density. Thus the current interruptions disappear even for high relative gradients.

With these calculations the two extreme cases of the behaviour of the first order states are treated. The real
behaviour of these states should be a mixture of the two cases. However, to calculate this, more information on the behaviour of the field variables in the quasi-linear approximation is necessary which actually is not available. One possible solution of this problem is to calculate the field distribution in the quasi-stationary case by appropriate numerical methods like FCT-procedures which are under development at IRS\textsuperscript{14}.

6. Calculations of the Second Instability

In completion the nonlinear behaviour of the second instability (dashed lines in Figs.2,3,4) was calculated. As expected, the contribution of this instability is at least two magnitudes weaker for all gradients compared to the results of the first instability. Thus this contribution can be neglected and is therefore not presented here.

The reason for this behaviour can be given as follows: The growth rate of the second instability is connected to a negative frequency whereas that of the first instability has a positive frequency. Without relative gradients the plasma is a homogeneous medium with no preferred direction. By introducing a relative gradient (equivalent to the electric field), the plasma becomes inhomogenous and thus polarizable. In the quasi-linear theory the field variables which are plane waves in our model are decomposed in two circular polarized waves. In the region of interest, the dispersion relation shows that the right-hand polarized part of the variables have stronger growth rates than the left-hand polarized part. This means that in the nonlinear development e.g. for the electric field the right-hand polarized part dominates the behaviour of the system. The left-hand polarized part is excited only weakly, and is negligible in the calculations done here.

7. Neutral Component

Some rough calculations of the influence of neutral component on the nonlinear development of the space charge instability do not show a relevant change in the results obtained here. However, the influence of the electron acoustic wave instability is not implemented yet. Including this instability it is expected that the nonlinear development should be enhanced leading to current interruption already for lower gradients. This work is under way and will be presented in the future.

8. Connection of the Space Charge Instability to Specific Impulse

The dependence of the development of the space charge instability on the specific impulse can be seen from Sec.4. Since there is an interdependence between relative gradients and the stationary current density, which again is connected to the total current in the system, every increase of total current results in an increase of the stationary current density and the relative gradients which can drive the instability. On the other side the specific impulse is proportional to $I^2$. Thus a clear dependence between the specific impulse and the local gradients is given.

To get an exact connection the calculation model has to be expanded to a three dimensional model where also the influence of the magnetic field can be taken into consideration. This expansion of the model is planned for the near future.

9. Conclusions

It was shown that the time development of local instabilities can be calculated with the QAM-method. These calculations are possible for every available set of parameters which are defining an initial state. Some data sets can be found for which the local current density will disappear. Thus it could be shown that the space charge instability leads, in its nonlinear development, to unstable current density oscillations including current interruption. This behaviour, known since some time as diode or Pierce instability, is believed to explain what is generally termed 'onset' in MPD thrusters.

To determine the parameters governing onset is, however, rather difficult at present. Since for the nonlinear calculation the knowledge of the k-spectrum of the system is important, the concept of relative gradients is not exact enough for the modelling of the local variation of the initial state in a thruster. Thus the coupling of nonlinear QAM-calculations to appropriate numerical codes working in the quasi-linear regime seems necessary. This will be carried out in the near future. Some further work will also have to be done in connection with the instability appearing for the neutral component.

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References


3 D. Rempfer, M. Auweter-Kurtz, H.J. Kaeppeler, M. Maurer: Investigations of Instabilities in
MPD Thruster Flows Using a linear Dispersion Relation. IEPC-88-071, 20th IEPC, Garmisch-Partenkirchen, 1988


9 M. Maurer, H.J. Kaeppeler: Development of Nonlinear Phenomena Due to Instabilities in MPD-Thruster Flows. AIAA 92-3294, 28th JPCE, Nashville, TN, USA, 1992


