Analytical Study on Nonequilibrium Flows in Self-Field MPD Thrusters

T. Shoji*, K. Ogiwara† and I. Kimura‡
Department of Aeronautics and Astronautics, School of Engineering, Tokai University, Hiratsuka, Kanagawa, Japan

Abstract

A numerical simulation of one-dimensional flows in self-field magnetoplasmadynamic (MPD) thrusters is performed taking thermal and ionization nonequilibrium state into account. The model used includes the electron energy and ionization rate equations and is referred to as a two-temperature model. In this paper the calculated results based on this model are compared with those based on the one-temperature model in which the electron temperature is the same as the heavy particle temperature although ionization process is involved. It is found that: 1) the onset current for the one-temperature model is larger than that for the two-temperature model at any mass flow rate (Ar) in the range of 0.01~1g/s under a specific configuration, 2) the variation of H (distance between plane electrodes) under a given discharge current does not cause any change in the flow characteristics, when F₀ (mass flux) is adjusted to keep a fixed value, and the variation of L (channel length), under usual MPD thruster flow conditions, does not cause noticeable change in the value of onset current, for given values of H and m (mass flow rate), 3) the sonic point for the two-temperature model exists very near the inlet of thruster compared to that for the one-temperature model, 4) as for the electromagnetic operation, in both models the input power is spent chiefly on ionization of propellant in low discharge current levels and on acceleration in high discharge current levels and 5) a lowering of the thrust density due to ionization appears in the region of low current levels for the two-temperature model.

1 Introduction

The phenomena occurring in self-field MPD thrusters are rather complicated because of the coupling of thermodynamic and electromagnetic effects, so it is difficult to evaluate the performance and lifetime of the thrusters. Some investigations have been done to clarify the major physical processes in MPD thrusters. The analyses based on ideal and thermodynamic equilibrium models were made by King [1] and it was shown that self-field MPD thrusters have three operational modes under one-dimensional configurations, i.e., electrothermal, transitional and electromagnetic modes. In the electrothermal mode a plasma flow is accelerated by thermal effects and it attains a Mach number of unity at the exit of the channel. In the transitional mode there are two sonic points near the inlet and at the exit; plasma flow has smooth acceleration through the former sonic point. On the way a shock wave appears and the subsonic flow is accelerated to the exit where the Mach number is unity. In the electromagnetic mode there is a sonic point near the inlet and the flow is in supersonic state from the sonic point to the exit. In his theoretical investigation, he also derived the choking condition for smooth acceleration from subsonic to supersonic flow in the cases of ideal and equilibrium conditions.

Lawless et al. [2] proposed a back-EMF (electromotive force) theory, which can explain the destructive onset phenomenon both in frozen (fully ionized) flow and thermodynamic equilibrium flow conditions. Later, Subramaniam et al. [3] developed the back-EMF theory for the case of ionization nonequilibrium. In their work it is shown that the electric field satisfying the choking condition is affected by the ionization rate at sonic point, although it is assumed that the MPD flow is in thermal equilibrium, i.e., the electron temperature is the same as that of heavy particles (neutral and singly ionized atoms).

In actual MPD thrusters, the progress of ionization is caused by the collisions between electrons and neutral particles and the rate coefficient of the ionization reaction is controlled mainly by electron gas temperature. Consequently, the choking condi-
tion is also influenced significantly by the electron gas temperature.

In this paper the effects of thermal nonequilibrium between heavy particles and electrons on MPD flows are investigated, comparing the calculated flow fields and terminal conditions for the cases of thermal nonequilibrium and thermal equilibrium. Throughout this work, the word, two-temperature (2T) model means the one considering both ionization and thermal nonequilibrium effects in MPD flows, and the word, one-temperature (1T) model means the one considering only ionization nonequilibrium effect as in the case of the analysis by Subramaniam et al.

2 MPD flow model

2.1 Assumptions

A steady one-dimensional MPD flow, in a duct with a constant rectangular cross section, is subject to the following assumptions:

1) The propellant is argon and the species involved are neutral atoms, singly ionized atoms and electrons.

2) The gas is in the state of ionization and thermal nonequilibrium (2T model), or in the state of only ionization nonequilibrium (1T model), although the temperature of neutral particles is equal to that of ions in both cases.

3) In the governing equations, viscous effect and thermal conduction are not taken into consideration, but the ionization collision between a neutral atom and an electron, and collisional energy transfer between an electron and heavy particles (2T model) are considered.

4) The Hall effect and ion slip are neglected.

The configuration of the one-dimensional thruster considered here is shown in Fig.1.

2.2 Governing Equations

The conservation equations for a steady one-dimensional MPD flow can be written as follows.

Overall mass conservation:
\[ \rho u = \rho^* a^* = F_0 \] (1)

where \( \rho, u, a \) and \( F_0 \) are density of flowing plasma, axial flow velocity, sonic speed and mass flux, respectively, and asterisk (*) indicates the value at sonic point.

Overall momentum conservation:
\[ F_0 u + \frac{B^2}{2 \mu_0} = F_0 a^* + P^* + \frac{B'^2}{2 \mu_0} \equiv F_M \] (2)

Overall energy conservation:
\[ F_0 \left( h + \frac{1}{2} u^2 \right) + \frac{EB}{\mu_0} = F_0 \left( h^* + \frac{1}{2} u^2 \right) + \frac{EB^*}{\mu_0} \equiv F_E \] (3)

where \( E, B, F_M, F_E \) and \( \mu_0 \) are electric field, magnetic field, total momentum flux, total energy flux and permeability in vacuum, respectively. The total pressure \( P \) is expressed in terms of the temperatures and number densities of heavy particles and electron:
\[ P = n_e k T_e + n_e k T_e = \rho R_{Ar}(T_0 + \phi T_e) \] (4)

where \( n, T \) and \( k \) are number density, temperature and Boltzmann constant, and subscripts \( g \) and \( e \) indicate heavy particles and electron, respectively. The total enthalpy \( h \) is related to the total pressure, density and degree of ionization \( \phi = n_e/n_g \):
\[ h = \frac{5}{2} \frac{P}{\rho} + \frac{\phi R_{Ar} \theta_i}{k} = \frac{5}{2} R_{Ar}(T_0 + \phi T_e) + \phi R_{Ar} \theta_i \] (5)

where \( R_{Ar} \) is the gas constant of argon and \( \theta_i = \epsilon_i/k \) is the characteristic temperature related to the first ionization potential \( \epsilon_i \) of argon.

Electrons and singly ionized atoms (ground states) are produced, while they recombine to produce neutral atoms (ground states), through the following reaction:
\[ Ar + e^- \xrightarrow{k_f} Ar^+ + e^- + e^- \] (6)

Then the rate equation for ionization is expressed as
\[ \frac{d\phi}{dz} = \frac{F_0}{m_{Ar} u^2} \left( k_f (1 - \phi) \phi - \frac{F_0}{m_{Ar} u} k_b \phi^2 \right) \] (7)

where \( k_f \) and \( k_b \) are the forward (ionization) and backward (recombination) rate coefficients, respectively, and they are controlled by electron temperature, that is:
\[ k_f = \frac{76.8}{T_e^3} \exp \left( -\frac{\theta_i}{T_e} \right), \quad k_b = 4 \times 10^{-31} \frac{T_e^{9/2}}{T_e^{9/2}} \] (8)

in MKS unit.

The electron energy equation is expressed as follows:
\[ \frac{d}{dz} \left( n_e k \left( T_e + \phi \theta_i \right) u \right) = \frac{j^2}{\sigma} - P_e \frac{du}{dz} = -\frac{3 m_e n_e k(T_e - T_0) \nu_e}{m_{Ar}} \] (9)

where \( j, \sigma \) and \( \nu_e \) are current density, electrical conductivity and electron collision frequency, respectively. The first term on the right hand side of Eq.(9) is the contribution from joule heating, the second
term is the pressure work of electron gas and the last term is the rate at which thermal energy of electrons is transferred to heavy particles. The term of energy loss due to the ionization is included in the left hand side of Eq.(9).

In addition to the above equations, Ampere’s and Ohm’s laws are required:

\[ j = -\frac{1}{\mu_0} \frac{dB}{dz} \]  
\[ j = \sigma (E - uB) \]

These two equations are combined into a differential equation which determines the distribution of self-induced magnetic field:

\[ \frac{dB}{dz} = -\mu_0 \sigma (E - uB) \]

The electrical conductivity is given in terms of the electron number density and electron collision frequency as

\[ \sigma = \frac{e^2 n_e}{m_e \nu_e} \]

Here, \( \nu_e \) is composed of two parts; the electron-neutral atom collision frequency and the electron-ion collision frequency given by Spitzer-Härm formula\[6\].

Six governing equations (1-3), (7), (9) and (12) can be used to evaluate six dependent variables, \( \rho, u, h, \phi, T_e \) and \( B \), being supplemented by Eqs.(4), (5), (13) and the equation of choking condition shown in next subsection (Eq.(15)).

The independent variable is transformed from the axial position \( z \) to the magnetic field \( B \) using Eq.(12), then \( z \) becomes a dependent variable.

\[ z(B) = \int_{B_0}^{B} \frac{dB}{\mu_0 \sigma (E - uB)} \]  
where \( B_0 \) is the induced magnetic field at the inlet of channel.

### 2.3 MPD Choking Condition

Two MPD choking conditions have been proposed: one is the condition of smooth acceleration at the ordinary sonic point, which is given by King\[1\] or Lawless et al.\[2\], the other is that at the magnetosonic point, which is given by Kuriki et al.\[7\]. In this investigation the former MPD choking condition is used.

The MPD choking condition in the case of ionization nonequilibrium was derived by Subramaniam et al.\[3\]:

\[ E = \frac{5}{2} a^* B^* + \frac{F_0 R_A \theta_1}{j^*} \frac{d\phi}{dz} \]  

The first term of Eq.(15) is an electric field from the frozen flow theory and the second term is one from ionization nonequilibrium theory. For the case of 2T model where the electron temperature is distinguished from the heavy particle temperature, Eq.(15) can also be used when the derivative of ionization degree at the sonic point is evaluated based on the distinguished electron temperature. In the present investigation the sonic speed involved in Eq.(15) is evaluated by the next equation.

\[ a^2 = \frac{\rho \frac{d\phi}{d\rho}}{1 - \rho \frac{d\phi}{d\rho}} = \frac{5}{3} R_A \left( T_g + \phi T_e \right) \]

### 3 Calculation Method

The one-dimensional MPD thruster considered here has a constant-area channel, and the length \( L \) is 15 cm, the distance \( H \) between the plane electrodes is 2 cm and the electrodes’ width \( W \) is 6 cm (Fig.1). Numerical calculations are made for the discharge currents \( J_0 \) in the range of 1~20kA, for mass flow rates (0.01~1g/sec).

As stated above, the independent variable \( z \) in Eqs.(7), (9) and (15) is transformed into a self-induced magnetic field \( B \) by Eq.(14). This magnetic field decreases monotonically from \( B_0 \) at the inlet to 0 at the exit of the channel if the back-EMF does not exceed the electric field determined by the choking condition of Eq.(15).

#### 3.1 Nondimensional Equations

The variables in the governing equations are nondimensionalized using characteristic quantities: the temperature is nondimensionalized by \( \theta_1 \), the axial position by the thruster length \( L \), the flow velocity by \( u_e = \sqrt{R_A \theta_1} \), the density of plasma by \( \rho_e = F_0 / u_e \), the pressure by \( P_e = \rho_e R_A \theta_1 \), the magnetic field by the inlet value \( B_0 = \mu_0 J_0 / W \), where \( J_0 \) is the total discharge current, and the current density by \( j_e = B_0 / \mu_0 L \).

The characteristic collision frequency is defined as \( \nu_e = u_e / d_e \), using the Debye length \( d_e = (m_e e^2 \epsilon_0 k \theta_1 / 2 \rho_e) \), where \( \epsilon_0 \) is permittivity), and the characteristic electrical conductivity as \( \sigma_e = e^2 \rho_e / m_e m_n \nu_e \).

After the substitution of Eq.(5) into Eq.(3), Eqs.(1-3) can be expressed in the following forms (superscript \(^* \) indicates nondimensional variable):

\[ \bar{P} \bar{u} = 1 \]  
\[ \bar{u} + \bar{P} + R_e \bar{B}^2 = \bar{a}^* + \bar{P}^* + R_e \bar{B}^{*2} = \bar{F}_M \]

\[ \frac{5 \bar{P}}{2} + \phi + \frac{1}{2} \bar{u}^2 + 2R_e \bar{E} \bar{B} \]
The calculation for solving Eqs.(23) and (24), for a specified mass flow rate and total current, is proceeded in the following manner: a value of magnetic field at the sonic point is selected to be $0.9 \leq B^* < 1$, and three quantities at the sonic point ($T_e^*, \phi^*$ and $T_i^*$) are assumed to determine the total momentum flux $\overline{F}_M$ and total energy flux $\overline{F}_E$ in Eq.(20), and to give the boundary conditions for Eqs.(23) and (24). Using these conditions the electric field $\overline{E}$ is determined from Eq.(26) and it is also used to evaluate $\overline{F}_E$. Next, Eqs.(23) and (24) are solved from the sonic point both upward to inlet of the channel ($\overline{B}_e = 0$) and downward to the exit ($\overline{B}_e = 0$) by Euler-Romberg method. When all variables in the flow fields are determined over the thruster (we should note that the independent variable is the magnetic field), the necessary length of channel $L'$ is calculated from the following equation using Eq.(14):

$$L' = \frac{L}{R_m} \int_0^1 \frac{dB}{\sigma(E - \vec{u}B)}$$

If the error of $L'$ is larger than $3\%$ (thruster length $L=15\text{cm}$), a new set of $\overline{B}^*$, $\overline{T}_e^*$, $\overline{T}_i^*$ and $\phi^*$ are assumed. Furthermore, the solutions which have a proper value of $\overline{T}_e$ at the inlet are selected.

4 Calculated Results and Discussion

4.1 MPD Flow Field Characteristics

The calculated results of MPD flow presented in Figs.2~5 are of the case that the discharge current is $5\text{kA}$ and the mass flow rate of argon is $0.25\text{g/s}$. The axial velocity and Mach number profiles are shown in Fig.2. The velocity is seen to increase monotonically up to the exit, where it decreases. This is because ohmic heating is dominant near the exit and the addition of heat to a supersonic flow causes it to slow down. It is noticed that the location of sonic point in the case of $2T$ model is much closer to the channel inlet than in the case of $1T$ model.

The temperature profiles are shown in Fig.3. It is seen that the electron temperature for $2T$ model increases sharply from the inlet to the sonic point, in the most parts of thruster it is kept nearly constant ($\sim 17,000\text{K}$) and at the exit region it increases again. This temperature rise is due to the increase of current density at the exit region, caused by the decrease of the back-EMF ($B_{(e=L)} = 0$). On the other hand, the heavy particle temperature for $2T$ model increases slowly toward the exit from the inlet region.
4.2 Mode of Utilization of Input Power

In this investigation the calculations were made only for the electromagnetic mode which has one sonic point near the channel inlet, because of the significance of this operational mode on MPD thrusters. By comparison of the flow properties at the exit with those at the inlet, it is possible to evaluate the mode of utilization of the input power in thrusters. Using Eq.(3) multiplied by the cross section of the thruster, \( A = W \cdot H \), and Eq.(5), the following equation is obtained,

\[
\dot{m} \left( \frac{1}{2} u_e^2 - \frac{1}{2} u_0^2 \right) + \dot{m} \left( \frac{5 P_e}{2 \rho_e} - \frac{5 P_0}{2 \rho_0} \right) + \dot{m} (\phi_e - \phi_0) R_A \theta, \]

\[
= J_0 V_0 \tag{28}
\]

where \( \dot{m} \) is the mass flow rate, and subscripts \( e \) and \( 0 \) represent the exit and inlet conditions. The first term of the left hand side of Eq.(28) is the power utilized for acceleration of plasma flow, the second term is the power used for thermalization of it and the third term is the power absorbed in ionization process.

In Figs.6 and 7 the ratio of each term in the left hand side of Eq.(28) to the input power \( J_0 V_0 \) is shown as a function of the discharge current. For the case of 2T model (Fig.6), in the region of lower current levels (1.5~3kA) of 65~45% of input power is spent on the ionization process, which is larger than on acceleration or thermalization process, although in the case of 1T model (Fig.7), the ionization process spends less percentage (50~35%) in the region. For both models, in the current levels higher than 4kA, the fractions of power spent on the ionization process become smaller than those used in acceleration or thermalization, and the fractions used in them take a nearly same value, although they increase with discharge currents. The increase of the fraction for thermalization is due to the assumption of constant area channel; if a divergent nozzle is introduced into the thruster, the thermal energy of plasma can be transformed into the kinetic energy and the exhaust speed can be increased further.

Based on the above, the electromagnetic mode of operation can be divided further into two regimes, which were pointed out in the experiment of Kuriki et al.[8], one is a regime at lower current levels in which the ionization process is predominant and the other is a regime at higher current levels in which the acceleration of plasma flow is principal.

4.3 Onset, Discharge Voltage and Thrust Density Characteristics

Fig.8 shows the curves of onset discharge current versus mass flow rate for both temperature models. It is seen that the onset discharge currents for 1T model have larger values than those for 2T model for all mass flow rates, and one of the causes of this phenomenon will be the high discharge voltage for 1T model at the region of high discharge current (Fig.9), since the onset in this simulation is caused by the back-EMF approaching to the applied electric field at somewhere in the thruster[2][3]. Each curve in Fig.8 is a nearly straight line, and these curves show that
$J_{0}^{2}/\dot{m} \simeq \text{const.}$ for 1T model and $J_{0}^{2.7}/\dot{m} \simeq \text{const.}$ for 2T model, which are similar to the relation observed experimentally, $J_{0}^{3}/\dot{m} \simeq \text{const.}$, under the fixed thruster configuration\textsuperscript{[9]}. The forms of Eqs.(23) and (24), supplemented by Eqs.(20) and (26), show that the variation of $H$ (distance between plane electrodes) does not cause any change in flow characteristics when mass flux $\dot{F}_0$ is kept constant. This fact suggests that the value $J_{0}^{2}/\dot{m}$ increases proportionally with $1/H$. It was confirmed by calculation that the variation of channel length $L$ (5cm~45cm) does not cause noticeable change in the value of onset current, for given values of $H$ and $\dot{m}$. This fact suggests the change of $\phi$ and hence $\sigma$ caused by the variation of stay time of flowing plasma in the channel does not play an important part in the occurrence of back-EMF onset phenomenon.

The voltage–current characteristics are shown in Fig.9. The upper limits of discharge current for the given mass flow rate (0.25g/s) exist at 7.3kA for 2T model and at 8.0kA for 1T model. In Fig.9, in the region of lower discharge currents, the discharge voltage is proportional to the discharge current; in this region the plasma in the thruster behaves as an ohmic medium\textsuperscript{[10]}. On the other hand, in higher current region, the discharge voltage is proportional to about 2.5 power of current for 1T model and to about 2.2 power of it for 2T model. On the ideal electromagnetic thrust model, the discharge voltage is related to the discharge current through the following equation\textsuperscript{[10]}

$$V_0 = \frac{T^2}{2m_0^2} \propto \frac{J_0^3}{\dot{m}} \tag{29}$$

where, $T$ is the thrust and proportional to the square of discharge current. The difference of the current dependence of discharge voltage from the ideal model in higher current region, shown in Fig.9, arises from the fact that the significant fraction of input power is lost to thermalization of plasma in the case of non-ideal model considered in the present investigation.

The thrust density–current characteristics are shown in Fig.10. For 1T model, the curve of thrust density shows that $T/A$ is proportional to $J_{0}^3$, as anticipated in the ideal model and it seems that this characteristic is not affected so much by thermalization and ionization. For 2T model, this current dependence of the thrust density also holds in the region of discharge current higher than 2.5kA, but for current levels lower than 2.5kA, the thrust density takes smaller values than that in the case of the ideal or 1T model. This decrease of thrust density may be caused by the relatively large percentage of energy deposit to ionization process.

In the electromagnetic mode, because the contribution from pressure gradient to thrust density is very small compared to that from Lorentz force, the effect of thermalization or ionization on the thrust density does not appear obviously. But the discharge voltage is significantly influenced by thermalization and ionization, because the MPD choking condition, Eq.(15), is the function of not only the magnetic field but also the species' temperatures and degree of ionization.

5 Conclusions

The characteristics of one-dimensional MPD flows in thermal and ionization nonequilibrium were investigated, based on the numerical solutions of a two-temperature model and those of an one-temperature model with the same geometry and mass flow rate. Several important features of self-field accelerated flows have been identified:

1) The onset phenomenon in the case of a two-temperature model was also caused by the back-EMF approaching to the applied electric field somewhere in the middle part of channel, and the onset discharge current in the case of an one-temperature model was larger than that in the case of a two-temperature model, in the range of mass flow rate, 0.01~1g/s.

2) The variation of $H$ (distance between plane electrodes) under a given discharge current does not cause any change in the flow characteristics, when $\dot{F}_0$ (mass flux) is adjusted to keep a fixed value, and the variation of $L$ (channel length), under usual MPD thruster flow conditions, does not cause noticeable change in the value of onset current, for given values of $H$ and $\dot{m}$ (mass flow rate).

3) In the case of the two-temperature model the sonic point located closer to the inlet of the thruster than that in the case of the one-temperature model, due to a sharp increase of the electron temperature and current density in the immediate downstream region of the inlet.

4) The mode of utilization of input power depends on the magnitude of discharge current under a fixed mass flow rate; the input power is spent mainly on ionization process in lower current levels and on acceleration process in higher current levels in the range of electromagnetic mode operation, although the percentage of input power spent on ionization is generally larger in the case of the two-temperature model.
5) Although the discharge voltage–current characteristic for the two-temperature model did not show a large difference from the characteristic for the one-temperature model, the thrust density in the case of the two-temperature model was lower under the influence of high degree of ionization, especially in the region of lower current levels.

References


Fig. 1. Configuration of a one-dimensional MPD thruster with plane electrodes.

Fig. 2. Velocity and Mach number profiles along the longitudinal direction of the thruster for one- and two-temperature models (symbol * indicates sonic point).

Fig. 3. Temperature profiles for one- and two-temperature models (symbol * indicates sonic point).
Fig. 4. Distributions of current density for one- and two-temperature models (symbol • indicates sonic point).

Fig. 5. Distributions of degree of ionization for one- and two-temperature models (symbol • indicates sonic point).

Fig. 6. Mode of utilization of input power for discharge current (two-temperature model).

Fig. 7. Mode of utilization of input power for discharge current (one-temperature model).
Fig. 8. Onset discharge current versus mass flow rate for one- and two-temperature models.

Fig. 9. Discharge voltage-current characteristics for one- and two-temperature models.

Fig. 10. Thrust density-current characteristics for one- and two-temperature models.