ELECTRIC ROCKET ENGINE ACCELERATED TEST CONCEPT

V. I. Baranov, A. I. Vasin, A. A. Kalyaev, V. A. Petrosov

The Scientific-Research Institute of Thermal Processes, Moscow, Russia

Abstract

The concept is based on the nonlinear regressive model of electric rocket engine lifetime prediction. The reduction of testing time is obtained by two methods: the use of the regressive model and engine boosting. The tests are subdivided into evaluation and check tests.

At the final stage of development, evaluation tests are carried out. In this case, the following problems are tackled:

1. Determination of the limiting admissible boosting.
2. Calculation of erosion regressive dependence on time in various functional modes.
3. Lifetime evaluation.
4. Determination of coefficients of rescaling from boosting modes to operating conditions.

At the evaluation testing stage, tests of the engine-leader in operating conditions are begun. In future it will ensure demonstration of lifetime and to refine rescaling coefficients, the critical value of erosion and effect of individual characteristics dispersion, and to control lifetime stability.

Check tests are conducted at the delivery stage. The objectives of testing are the following:

1. Check of lifetime compliance with requirements.
2. Check of lifetime stability.

When determining the check plan characteristics, the method of statistical hypothesis verification is used along with results of evaluation tests and statistical simulation.

For check tests of various time period the criterion of predicted lifetime compliance with the set requirements is derived. Then the dependence of probability of making decision on the lifetime compliance with these requirements on the actual lifetime is derived.

Introduction

Lifetime is one of the main parameters of electric rocket engines. It amounts to some thousand hours. Substantial expenditures of time and money are needed to confirm it by direct life tests. Thus, total running time equal to 4000 hours can be obtained during approximately 1.5 year of developmental testing.

In this context, the problem of accelerated test procedure development is acute. Acquisition of data on lifetime in such tests over a period of time, which is some factors lower than the lifetime, can be accomplished as a result of the use of models of parameter degradation which limit the lifetime and through electric engine boosting. It is evident that in the process of realization of such approach a number of problems arise, related to the adequacy of models, their accuracy, validity of extrapolation, coefficients of rescaling from boosting mode to nominal rating conditions, etc.

In order to obtain validated answers to these questions and to have reliable data on lifetime in reasonable terms, the present concept presumes the following:

1. Careful analysis of degradation physical processes.
2. Verification of model adequacy using a wide-scale statistical simulation.
3. Analysis of the life testing of analogue engines.
4. Demonstration of serviceability by life tests of the engine-leader.

Due to such approach, both time saving owing to accelerated testing and confidence of the obtained results are afforded. In this case, in the process of the electric engine development testing, as the bench running increases, the precision of lifetime estimates increases too.

At the delivery stage, the present concept provides for both the check of lifetime for compliance with the set requirements and control of its stability. The latter presumes determination of the extent to which the lifetime of delivered rocket engines complies with the engine-leader lifetime.

Tests are divided into two groups: evaluation tests, whose purpose is the estimate of lifetime and the extent of influence of various factors, and check tests that are carried out at the delivery stage with the purpose of lifetime check.

1. Evaluation Tests

These tests are carried out at the development stage. Therewith, the following problems are solved [1]:

1. Selection of the degrading parameters structure.
2. Development of physico-mathematical models of parameter temporal variation.
3. Evaluation of parameter critical values, i.e. values determining the lifetime.
5. Estimate of prediction error and factors influencing it, using methods of statistical simulation.
6. Development of a model and program of strenuous tests.
7. Determination of coefficients of rescaling from boosting mode to operating conditions.
8. Reliability evaluation based on test results.
The analysis of engines-prototypes life tests results suggests that as degradation parameters it is expedient to take parameters that characterize the technical condition of the most demanding engine elements and assemblies, such as the insulator erosion magnitude, the diameter of the cathode-compensator diaphragm hole, thickness of the "barrier layer" impeding diffusion of the casing material into the active emissive element (for LaB-cathodes), etc.

The solution of problems (see items 2-5) as applied to the insulator erosion is given in [2].

The procedure of strenuous tests may be based on the Sedyakin physical principle of reliability which resides in the fact that in some range of operating conditions the product reliability depends on the magnitude of parameters that have already been depleted and does not depend on the manner in which this lifetime was depleted [3]. The domain of operating modes, where this principle is realized, is called self-similarity. The boundary of this domain determines the ultimate permissible variation of operating mode, in particular, the limit of boosting.

In Fig. 1 the horizontal is the time axis, the relative variation of the degradation parameter $\omega = \delta / \delta_{l m}$ is plotted on the vertical, where $\delta_{l m}$ is the parameter limiting value corresponding to full depletion of lifetime.

The plotted curves represent regressions approximations of parameter dependence on time that were obtained during tests in various modes of operation: $X_{oper}$ is operating conditions, $Y_2 > Y_2 > Y_1$ are boosting conditions. It is possible to find the limit of boosting and values of coefficients for rescaling one mode to another based on results of testing even two engines. To do this, the first engine is tested in the $X$ mode to the $\omega_0$ level, and then during the period of time $\Delta t_1$. The second engine is tested in the mode $Y_1 > X$ to the level $\omega_1$ and then in the $X$ mode for the period of time $\Delta t_1$. In doing so, let the levels $\omega_0$ and $\omega_1$, correspondingly, be reached. If the discrepancy between $\omega_1$ and $\omega_0$ is so great, that it cannot be explained by the scatter in experimental data, then the $Y_1$ mode exceeds the ultimate permissible one. Otherwise, the mode $Y_2 > Y_1$ is checked. For this purpose, the first engine is tested in the $Y_1$ mode to the level $\omega_2 = \max(\omega_1, \omega_0)$ and then over the period of time $\Delta t_2$. The second engine operates in the $Y_2$ mode to the level $\omega_2$ and then in the $Y_2$ mode over the period of time $\Delta t_2$. The raced levels $\omega_3$ are compared, etc.

Further, regressive dependences for various modes are refined that makes it possible to rescale from one mode to another. With sufficient boosting effect a strenuous check test may be conducted (boosting mode).

At the stage of evaluation tests, the tests of engine-leader are begun. It should be equipped with a sufficient number of precision measurement equipment ensuring evaluation of various degradation parameters. Below are the main tasks that are solved by engine-leader testing.

1. Lifetime demonstration.
2. Engine performance verification.
3. Detection of bottlenecks.
4. Revision of degradation parameter critical values.
5. Revision of models used in the accelerated test procedure.
6. Revision of procedure and models of strenuous tests.

2. Check Tests

Check tests are carried out at the delivery stage. The test goals are as follows:
1. Check of lifetime compliance with the required value.
2. Lifetime stability control.

At delivery, check-out of each electric rocket engine operation is made as well as performance evaluation by means of short-run tests, let them be called check and technological tests (CTT). From a product batch, having undergone CTT successfully, one sample is chosen for long-duration tests, the so called check and sampling tests (CST). During CST, compliance of performance with the requirements is checked, lifetime prediction is made based on one or several degrading parameters. In the case of CST being successful, the batch is accepted. Based on results of lifetime prediction, its compliance with the set requirements as well as its stability are checked. As the CST are conducted for shortened lifetime, then evaluation of the predicted lifetime has a certain dispersion. As a result, an erroneous decision might be made on availability of stability or compliance of the lifetime with the set requirements, when it does not correspond to the facts and vice versa. Let the probability of erroneous decision on conformity of lifetime to requirements or on stability
be called the user's risk (customer's risk) $\beta$, and the probability of erroneous decision on nonconformance of lifetime to requirements or stucture nonstability be called the manufacturer's risk (supplier's risk) $\alpha$.

The mathematical model of this concept is based on the method of statistical hypothesis verification. Let $T_p$ be the required lifetime value, $T_{le}$ be engine-leader lifetime and $T$ be lifetime in check and sampling tests that could be obtained if the CST were conducted for the full lifetime, $T_{pr}$ is the predicted lifetime value in CST.

First of all, the null hypothesis $H_0$ and the alternative hypothesis $H_a$ formulated and the significance level $\alpha_0$ is assigned which is the probability of null hypothesis deviation when it is true. The formulation of null and alternative hypotheses depends both on the type of check problem and which of the risks ($\alpha$ or $\beta$) is the most important to be provided at the specified level.

Given in the Table are formulae for $H_0$, $H_a$, and the critical domain which constitute the condition of null hypothesis deviation and adoption of an alternative, also the field of application and disadvantages of each variant. It is supposed that $\alpha_0=0.05...0.10$.

The lifetime critical values $T_1$ and $T_2$ are calculated in the following manner. Based on results of engine-leader or engines-prototypes testing, the regression coefficients are derived at which the lifetime is equal to the right side of inequalities given in the first column of Table and values of the regression $Y(t_i)$ in instants of time $t_i$, when the determining parameter values were determined during CST.

Then, pseudo-observations $Y(t_i)$ are simulated using the formula:

$$Y(t_i) = \hat{Y}(t_i) + \varepsilon_i,$$

where $\varepsilon_i$ is the residual error value in the point $t_i$ that was obtained during CST.

Using the obtained sampling, lifetime bootstrap realizations are formed in accordance with the method set forth in [2], and also a nonparametric or parametric bootstrap distribution of the predicted lifetime.

Another variant of simulation is effected using the Monte-Carlo method. In this case $\varepsilon_i=\sigma_i\mu$, where $\sigma$ is the residual mean-root square error, which was obtained based on CST results, $\mu_i$ is a random magnitude which is distributed according to the normal $N(0.1)$ law.

Then a sampling of repeatedly simulated predicted lifetimes $T_{pr}$ is formed and the law of lifetime distribution is derived, based on the program that was developed by the authors.

The critical values $T_1$ and $T_2$ constitute quantities of bootstrap realizations, thus obtained, or distributions of simulated lifetimes.

$T_1 = T\alpha_0$ for the first line of Table; $T_2 = T_{le} - \alpha_0$

for the second line of Table.

In case of stability check:

$T_1 = T\alpha_0/2$, $T_2 = T_{le} - \alpha_0/2$.

Here, $T_{le}$ is quantile of the $\alpha$ level of predicted lifetime distribution.

To evaluate efficiency of the method the power function is calculated which constitutes the dependence of $H_0$ hypothesis deviation probability, i.e. acceptance of the $H_\alpha$ hypothesis, on the $T$ value for various durations of CST. For this purpose, the probability of entry into the critical domain at different $T$ is determined. Based on results of analysis of power functions, such duration of CST can be chosen, when the customer's and supplier's risks have reasonable values. Besides, the CST duration could be derived, based on comparison of losses, depending on the value of risks and the expenditure on test, related to the test duration.

As an example, we shall cite some results of calculation that were obtained in processing the data of electric engine lifetime testing lasting from 1.5 to 4.0 thousand hours. Fig. 2-5 show the distribution densities of predicted lifetime, that were obtained during simulation using the Monte-Carlo method, at acceleration coefficients $K=2...5$, $K=T/t$, where $T$ is lifetime, $t$ is test duration. In Figures, the critical domains are section-lined and lifetime critical values $T_1$ and $T_2$ are given. It can be seen that with the acceleration coefficient increase the lifetime distribution densities become more broad and asymmetrical, that is the result of prediction error increase and convexity of the regression function.

Fig. 6 shows the case when the acceptance condition is formulated as $T_{pr} \geq T_1$, i.e. for the test duration $t$ equal to 2000 hours the lifetime requirements ($T > 4000$)}
Fig. 3. Predicted lifetime distribution density
\[ f(t) \]

\[ T = 3, t_{\text{test}} = 1, K = 3, \alpha_0 = 0.15, T_1 = 2.588, T_2 = 3.487 \]

Fig. 4. Predicted lifetime distribution density
\[ f(t) \]

\[ T = 4, t_{\text{test}} = 1, K = 4, \alpha_0 = 0.05, T_1 = 3.295, T_2 = 4.898 \]

Fig. 5. Predicted lifetime distribution
\[ f(t) \]

\[ T = 5, t = 1, K = 5, \alpha_0 = 0.05, T_1 = 3.967, T_2 = 6.424 \]

Fig. 6. Probability of acceptance of hypothesis on lifetime inconsistency with set requirements (\( T \geq 4 \))
\[ H_0 : T \geq 4; H_0 : T < 4; \alpha_0 = 0.05 \]

Fig. 7. Probability of acceptance of hypothesis on lifetime consistency with set requirements (\( T > 4 \))
\[ H_0 : T \leq 4; H_0 : T > 4; \alpha_0 = 0.05 \]
Fig. 8 illustrates an example of stability check. When the lifetime is equal to the leader lifetime $T_L$, which in the present example amounts to 4000 hours, the probability of erroneous decision on instability at any test duration is equal to the significance level $c_0$, which is in the present example is 0.1. With discrepancy between lifetime $T$ and the $T_L$ value, there exists some probability of erroneous decision on $T = T_L$, which increases with the test duration reduction. Thus, at $T = 4500$ it will be regarded that $T = 4000$ hours with probability $-0.04$ for $t = 2000$ hours and with probability $-0.60$ for $t = 1000$ hours.

![Graph showing the probability of acceptance of hypothesis on instability](image)

**Fig. 8.** Probability of acceptance of hypothesis on instability

$H_0 : T = 4$  $H_0 : T \neq 4$  $c_0 = 0.10$

### Conclusion

The concept of electric rocket engine accelerated testing is proposed which makes estimation of lifetime possible in the acceptable period of time. The prediction accuracy is evaluated as a result of statistical simulation. With the engine life increase and engine-leader life testing the value of lifetime is revised. The method of lifetime and stability checking during delivery is proposed.

### References

