EFFECTS OF NOZZLE GEOMETRY ON PLUME EXPANSION FOR SMALL THRUSTERS

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Abstract

The Direct Simulation Monte Carlo method is being applied to study flowfields of small thrusters. Accurate prediction of both the internal and external nozzle flow is important in determining plume expansion which, in turn, bears directly on impingement and contamination effects. The expanding plumes from three nozzle geometries including conical, trumpet, and bell-shaped contours are characterized and compared. The effects of gas species on plume expansion are also considered by comparing helium flow with that of nitrogen through a conical nozzle. The results from the DSMC simulation are compared with those obtained from a widely used plume impingement model and significant differences exist.

Introduction

Satellites require small on-board propulsion systems for stationkeeping and attitude control. An important consideration in placing electric thrusters on spacecraft is the interaction between the widely expanding nozzle exhaust and spacecraft surfaces. Plume impingement effects range from contamination; heating and electrical charging of solar arrays, instrumentation and other subsystems; to induction of destabilizing forces and torques. These effects may seriously reduce satellite lifetime. Assessment of such interactions between the spacecraft and thruster plumes requires an accurate description of the expanding flow from a thruster nozzle.

A research program is in progress at NASA-Lewis Research Center to develop methods to predict plume impingement effects on satellites. The goal is to establish an engineering tool that determines the optimum design and placement of on-board propulsion systems such that impingement impacts are minimized without compromising performance. The current work is specifically directed at applying gas dynamics on the molecular level to the phenomena associated with viscous flows in nozzles and plumes of small electrothermal thrusters, such as arcjets and high-temperature resistojets, designed for satellite stationkeeping and attitude control. Of particular interest is the prediction of thruster-plume expansion, especially in the off-axis region where the plume may impinge on spacecraft surfaces. The problem is approached by numerically modeling the nozzle flow and plume on both continuum and molecular levels, with continuum results used to scale the grid and provide inflow conditions for the molecular model.

The particle model used in this study is the direct-simulation Monte Carlo (DSMC) method pioneered by Bird. DSMC is based on a stochastic model of kinetic theory and is an important technique used in the analysis of rarefied, expanding gas flows. This method is applicable to all flow regimes, from continuum to free molecular, where conventional continuum CFD assumptions break down as the flow becomes rarefied and depart from equilibrium. What makes DSMC such a powerful analysis tool is that complex thermochemical nonequilibrium and gas-surface interactions can readily be simulated.

In prior work, the flow of nitrogen in and from a nozzle was computed with two numerical techniques: one based on continuum theory that numerically solved the Navier-Stokes equations for compressible flow, and one that used the DSMC method. Each was applied to the solution of a low-density, viscous gas flow in a converging-diverging nozzle of conical shape that simulated flow in a resistojet. This work demonstrated that the numerically intensive DSMC technique could be applied readily to a

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low-density nozzle flow, where the flow varied from continuum at the throat to rarefied at the exit plane, and also demonstrated that results from the DSMC method matched well with experimental measurements of Pitot pressure and flow angle made in the plume.

As a first step in the prediction of plume impingement on spacecraft surfaces, the highly vectorized DSMC code developed by Boyd is employed to assess the effects of nozzle wall contour on plume expansion and surface impingement parameters. Nozzles of conical, bell, and trumpet shapes are considered with a flow rate of nitrogen at 6 mg/s. Helium flow through the conical nozzle at a rate of 2.4 mg/s is also examined to determine the effects of molecular weight. The extent of plume expansion for each configuration is quantified and compared by calculating normalized density and momentum profiles at various locations in the plume. These parameters are important in the prediction of impingement effects such as contamination and torques. To demonstrate the utility of DSMC to predict potential plume impacts and to assess the validity of a semi-empirical model commonly used for impingement analysis, comparisons are made of density contours produced by the DSMC code with those obtained from Simons' method.

**Nozzle Dimensions and Flow Conditions**

A typical conical arcjet nozzle is used as the reference model for the current problem with trumpet and bell shapes as variations. Table 1 contains the dimensions of each nozzle, and Table 2 the flow conditions. All three nozzles have the same exit-to-throat area ratio of 225:1 and throat diameter of 0.635 mm. The conical and trumpet-shaped nozzles both have lengths of 12.3 mm. The bell contour is derived from the classical Rao design methodology for optimal nozzle expansion, and is 30% longer than the conical and trumpet nozzles, at 16 mm. For the nitrogen flow simulations, the nominal flow rate is 6 mg/s with a total pressure, $P_o$, of 9100 Pa and a total temperature, $T_o$, of 294 K. These conditions produce a throat Reynolds number of 700, a typical value for high-temperature resistojets and arcjet thrusters, and a nominal thrust of 5 mN. For the helium flow to produce the same thrust level as nitrogen through the conical nozzle, a flow rate of 2.4 mg/s is used at $P_o = 10350$ Pa and $T_o = 297$ K. These conditions give a throat Reynolds number of 250. The wall temperature is held constant at 298 K in all simulations, and the flows exhaust into vacuum.

**DSMC Method**

The DSMC technique simulates real gas flows by following the movement of a much smaller set of particles through a network of cells which represent physical space in real time. During each time step, the particles advance in space based on their individual velocities without interaction with other particles. Collision partners are then chosen within each cell on a probabilistic basis and new velocity distributions are obtained from the resulting energy exchange. Boundary interactions are also simulated. Once the flowfield reaches a steady state, samples of the velocity distributions are recorded from which macroscopic flow properties are determined.

A highly efficient DSMC code is used in the present study and simulates the flowfield in two-dimensional, axisymmetric coordinates. It utilizes the Variable Hard Sphere (VHS) gas model for the determination of collisional cross-sections. Rotational energy exchange is computed using the probability model of Boyd. The flows are assumed to be both chemically and vibrationally frozen. The nozzle wall is modeled by assuming diffuse reflection, full thermal accommodation, and is held at a constant temperature.

The computational grids used in the present work were constructed based on flowfield solutions computed by RKRPLUS, a continuum CFD code. A study was conducted to determine the necessary grid spacing with the DSMC code for this type of problem because large gradients exist in both the axial and radial directions within the nozzle. Grid-independence of the computed results was achieved when the cell sizes were approximately five times the local mean free path in both directions. No significant changes were observed with the use of a finer grid. In general, meshes consisting of approximately 22,000 cells are used in the nozzle flow simulations. For example, 250x88 cells are used in the nitrogen flow simulation through the conical nozzle. Each simulation employs approximately 800,000 particles. On the Cray/YMP, the simulation with the vectorized code uses 1.08 microseconds/particle/time step of CPU. The nozzle flow simulation begins just downstream of the throat and uses the continuum results for the inflow surface.

The nozzle and plume flows are simulated separately due to the large variation in density between the two regions. An inverse-squared relation between density and distance from the nozzle exit plane is used to scale the cell dimensions in the plume simulations. A mesh consisting of 130x96 cells is used to simulate the
conical nozzle plume flow, with approximately 48 particles per cell. The same vectorized code is used for the plume simulation on an IBM RS/6000 workstation and requires 14.4 μseconds/particle/time step on the scalar machine, which is a substantially higher execution time than on the Cray. This loss of CPU performance between the Cray and IBM workstation is partially due to the code not being optimized for the IBM RS/6000.

Results

Nitrogen Nozzle Flow Comparisons

The internal nozzle flows are simulated separately and used to establish inflow conditions for the plume simulations. For qualitative comparisons, isograms of Mach number of the internal flow of nitrogen for each of the three nozzle shapes are given in Fig. 1. The origin of each plot is located on the centerline at the throat. The figure illustrates the distinct flow structure of each configuration. The conical nozzle expands the flow to the highest Mach number, greater than 6 along the axis at the exit plane, where the trumpet nozzle only expands to a Mach number of about 5.5. Viscous effects are dominant in these nozzle flows, evidenced by the relatively gradual gradient in Mach number in the radial direction in the flow. In the bell contour, the viscous layer grows to fill the entire divergent section, causing a deceleration in the flow from a maximum Mach number of 6 in the core flow to a Mach number of about 5 at the exit plane.

Nitrogen Plume Flow Comparisons

To quantify differences in plume expansion for the various nozzle shapes and gas types, density contours along arcs of various radii originating from the center of the exit plane are presented. Figure 2 shows a diagram of the radial arc location with respect to the nozzle exit plane. The curves are normalized by the density on the plume axis (θ=0°) for each case. For reference, number density along the plume axis is given for each nozzle configuration in Fig. 3. Differences in number density are evident near the exit plane, with the bell contour producing the highest values. The differences then diminish with distance along the axis. In Fig. 4a, normalized density along an arc of one nozzle diameter is plotted for each nozzle. The trumpet nozzle has a higher normalized density than both the bell and conical nozzles indicating that the expansion from the trumpet is less collimated, as expected, causing more particles to expand off-axis than the other two shapes. At an angle of 20° off the centerline, the density in the plume of the trumpet nozzle decreases to 41% of its centerline value. For the conical and bell-shaped nozzles, the density decreases even more rapidly to values of 24% and 16.5%, respectively. At 50°, the trumpet plume density is 6% of its centerline value, with the conical nozzle at 3.35% and the bell-shaped nozzle at less than 2%.

Figure 4b gives the normalized density profiles at 2 nozzle diameters from the exit plane for each shape. In general, the normalized density in each plume decreases more rapidly from the centerline value than at 1 exit diameter. At 20°, both the bell and cone give densities less than 20% of their centerline values. The trumpet expands more particles off-axis than the other two contours with a normalized density of 30% at the same location.

At an arc radius of 5 exit diameters, the differences in the plume shapes between the three nozzle contours become less distinct, as shown in Fig. 4c. The bell and trumpet contours produce very similar results, where the conical nozzle gives a slightly lower density profile. Figure 4d compares normalized density profiles at 10 exit diameters, and the results appear to be similar to the profiles at the previous location. However, the rate of decay decreases for all nozzle profiles beyond 2 nozzle diameters from the exit plane.

Plume impingement effects such as contamination and torque depend primarily on the momentum transfer between the impinging particles and the spacecraft surfaces. Figure 5 compares profiles of normalized momentum at a distance of 1 exit diameter for all three nozzle contours. The momentum is calculated in the radial direction of the arc extending from the exit plane on the centerline. The results are very similar to the normalized density profiles at the same location as nitrogen quickly reaches its terminal velocity in the plume. Therefore, the curves in Figs. 4b through 4d also illustrate momentum profiles, with the trumpet plume producing the highest momentum away from the nozzle axis.

Nitrogen - Helium Plume Flow Comparisons

Of interest is the effect of molecular weight on plume expansion. Helium, with 1/7th the molecular weight of nitrogen, would be expected to expand off-axis to a greater extent than nitrogen given its higher thermal speed and fewer number of collisions in the lower-density plume. This result is indeed verified in
Fig. 6a, which compares the normalized density profiles in the plumes of helium and nitrogen flow from the conical nozzle at 1 exit diameter. At an angle of 20°, the density of helium is 47% of its centerline value, where the nitrogen flow is 24%. Figure 6a also compares the density profiles at 2 nozzle diameters from the exit plane. The difference between the plume expansion of the two gases is still significant.

The density profiles of the nitrogen and helium plumes at arc radii of 5 and 10 exit diameters are given in Fig. 6b. The normalized density in the helium plume is consistently higher than for nitrogen, but the differences diminish with increasing distance from the exit plane. At 10 exit diameters the normalized density is generally higher over the arc compared to the values at 5 exit diameters, similar to the nitrogen plumes shown in Fig. 4.

Comparisons of DSMC and Simons’ Model

Comparisons are made between the results from the DSMC simulation and those calculated with the inviscid part of the Simons' plume model, a semi-empirical analysis of plume density as a function of distance from the nozzle exit plane. Only a first-order comparison is made between the DSMC method and the Simons' model, as the parameters for the Simons' model are derived from one-dimensional isentropic gas dynamic relations, and do not include the boundary-layer corrections available in the model. Future comparisons will include the viscous part of the Simons' model. For this comparison, the density within the expansion of the nozzle core flow is described by

$$\frac{\rho (r, \theta)}{\rho^*} = A_p \left( \frac{r^*}{r} \right)^2 \cdot f(\theta) \quad (1)$$

with

$$f(\theta) = \cos \gamma - 1 \left( \frac{\pi}{2} \cdot \frac{\theta}{\theta_{lim}} \right) \quad (2)$$

and

$$\theta_{lim} = \nu (\infty) - \nu (\gamma_E) + \theta_E; \quad \theta \leq \theta_0 \quad (3)$$

$A_p$ is the so-called plume constant, $\gamma$ is the ratio of specific heats, $\theta_{lim}$ is the maximum turning angle of the streamline at the nozzle edge for inviscid flow, $\nu$ is the Prandtl Meyer angle, $\gamma_E$ is the isentropic nozzle exit Mach number based on area ratio, $\rho^*$ is the density at the nozzle throat, and $r^*$ is the throat radius.

Comparisons of nitrogen density contours for the conical nozzle from the DSMC method and the Simons' model for various angles off the plume axis are shown in Fig. 7. The calculations are made along a radial line extending from the centerline of the nozzle exit plane to 0.1 m into the plume. Along the centerline ($\theta=0^\circ$), Figure 7a, the Simons' model predicts a plume density of about an order of magnitude higher than the DSMC simulation. A difference in the density curve shape is also evident, as the DSMC results give a less rapid decay of density with radial distance in the near-field plume than does the Simons' model. Figure 7b compares the radial density profiles at an angle of 30° from the centerline. The results from both methods have a similar shape, but the Simons' model again predicts densities about an order of magnitude higher. As the Simons' model approaches its limiting angle, $\theta_{lim}$, the predicted density decreases substantially. Figure 7c gives a comparison between the methods at an angle of 45°, where the Simons' model results are lower than those of the DSMC method. At an angle beyond $\theta_{lim}$, e.g. $\theta=60^\circ$, the Simons' model predicts zero density, whereas DSMC shows a finite amount of gas in that region.

According to Eq. (1), for a given angle, $\theta$, the density varies inversely with the square of the distance from the nozzle exit plane such that $\rho(r)^* r^2 =$ constant. In Fig. 8, this product is plotted for the results from the DSMC simulation of nitrogen flow for $\theta=0^\circ$, and compared with the constant as computed by the Simons' model. The product from the DSMC results varies substantially in the region from the exit plane to approximately 2.5 nozzle diameters into the plume, where it is about 35% higher than the Simons' model constant. The DSMC result then decreases and reaches the Simons' model value at about 12 nozzle diameters. It appears that the DSMC result would approach an asymptotic value for this product further downstream in the plume, and follow the $1/r^2$ relationship.

Conclusions

The results for plume density from the DSMC simulation for nitrogen flows through nozzles of three distinctly different profiles show the effect of geometry on plume expansion is most significant in the near-field of the plume. The effect of geometry, however, diminishes with distance from the nozzle exit plane. The trumpet nozzle definitely expands more of the flow into the region away from the plume axis, with the effect on plume shape most pronounced near the exit plane. In future work, the momentum exchange with surfaces in
the flow will be computed for the different nozzle shapes to quantify actual plume impingement effects.

The influence of propellant molecular weight on plume expansion is significant for the case examined in this study, that is, helium and nitrogen flows in the conical nozzle producing comparable levels of thrust. The molecules of the much lighter helium are scattered to a greater extent off the plume axis, causing the plume to diffuse more rapidly. Unknown is the effect, if any, of the different degrees of freedom and thermal nonequilibrium between helium and nitrogen. Future studies will address this question by also considering flows of hydrogen.

The plume densities predicted by the DSMC method and the widely used Simons' model differ substantially for the case considered in this study. Some of the difference is likely attributable to not including the boundary-layer correction that is available with the Simons' model. The Simons' model is also more commonly applied to larger nozzles that are more aptly described as having a core flow and boundary layer, and are less viscous than those considered in this application. These comparisons should, therefore, be viewed as preliminary, and will be scrutinized further in continuation of this work.

Acknowledgment

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References


Fig. 1. Mach number contours of nitrogen flow through nozzles of various shapes
Table 1: Nozzle Geometries

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<thead>
<tr>
<th></th>
<th>Cone</th>
<th>Trumpet</th>
<th>Bell</th>
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<tr>
<td>Throat Diameter</td>
<td>0.635 mm</td>
<td>0.635 mm</td>
<td>0.635 mm</td>
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<tr>
<td>Exit Diameter</td>
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<td>9.526 mm</td>
<td>9.526 mm</td>
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<tr>
<td>Area Ratio, ((A/A^*))</td>
<td>225</td>
<td>225</td>
<td>225</td>
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<td>Exit Half-Angle</td>
<td>20°</td>
<td>40°</td>
<td>10°</td>
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<td>Nozzle Length</td>
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Table 2: Nozzle Flow Conditions

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<th>Propellant</th>
<th>Nitrogen</th>
<th>Helium</th>
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<td>Flow Rate</td>
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<td>(2.4 \times 10^{-6} \text{ kg/s})</td>
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<td>Total Pressure, (P_0)</td>
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<td>10360 Pa</td>
</tr>
<tr>
<td>Total Temperature, (T_0)</td>
<td>294 K</td>
<td>297 K</td>
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<tr>
<td>Wall Temperature</td>
<td>298 K</td>
<td>298 K</td>
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<tr>
<td>Throat Reynolds Number</td>
<td>700</td>
<td>250</td>
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Fig. 2. Diagram of arc radius with respect to the nozzle exit plane
Fig. 3. Number density of nitrogen flow along the plume centerline

Fig. 4. Normalized density vs. theta at various arc radii
Fig. 5. Normalized momentum vs. theta at arc radius = 1 exit diameter

Fig. 6. Normalized density vs. theta - He and N₂ at various arc radii
Fig. 7. Normalized density vs. radial distance at various angles from the centerline

Fig. 8. Plume density constant (number density * axial distance squared) along the centerline