PLASMA EXPANSION IN A LOW-POWER MPD THRUSTER WITH VARIABLE MAGNETIC NOZZLE

T. M. York* and H. Kamhawi**
The Ohio State University
Columbus, Ohio 43210

ABSTRACT

The expansion and acceleration of plasmas generated in a 50-135 kW arc thruster with applied magnetic nozzles has been studied experimentally. Earlier studies of this device did not allow magnetic field penetration into the thrust chamber. Nozzle field generation for 1 sec. allowed for evaluation of power input and acceleration processes with variable field penetration. Modest changes in the nozzle shape and gradients resulted in significantly different electric current between coaxial electrodes, with the plasma being accelerated by electrothermal forces and the interaction of the discharge current with a self induced and/or externally applied magnetic field. Three different thrust mechanisms are simultaneously present: ordinary gas dynamic pressure on the physical (anode) nozzle, pressure on a magnetic nozzle; direct plasma acceleration in magnetic blowing (j_x x B_y) and magnetic pumping (j_x x B_z); and the electromagnetic interaction of azimuthal current with the radial field of an applied magnetic nozzle. The addition of magnetic nozzle fields generating an expansion field geometry has been seen to produce: enhanced thrust, improved thrust efficiency, and reduction of electrode erosion.

INTRODUCTION

The magnetoplasmaodynamic (MPD) thruster has potential propulsion application for earth orbit transfer, maneuvering of large space systems, and interplanetary missions. In the MPD thruster, plasma is generated by a direct discharge of electric current between coaxial electrodes, with the plasma being accelerated by electrothermal forces and the interaction of the discharge current with a self induced and/or externally applied magnetic field. Three different thrust mechanisms are simultaneously present: ordinary gas dynamic pressure on the physical (anode) nozzle, pressure on a magnetic nozzle; direct plasma acceleration in magnetic blowing (j_x x B_y) and magnetic pumping (j_x x B_z); and the electromagnetic interaction of azimuthal current with the radial field of an applied magnetic nozzle. The addition of magnetic nozzle fields generating an expansion field geometry has been seen to produce: enhanced thrust, improved thrust efficiency, and reduction of electrode erosion.

Applied Field Thrusters

In 1977, Kimura and Arakawa reported experimental results showed that thrust increased with the strength of the applied magnetic field. In 1987, Tahara, Yasui, and Kagaya studied the effects of an axial field on the performance of a quasi-steady (0.6 ms) MPD arcjet; the anode was segmented. The field application lowered discharge voltages and greater thrust than the case with no applied field. In 1988, Tanaka and Kimura showed that increasing the magnetic field strength reduced the current concentration at the cathode tip and thus reduced cathode erosion. In 1992, Sasoh and Arakawa derived a thrust formula which included applied-field acceleration from the energy conservation equation. This enabled one to estimate the contribution of the respective acceleration mechanisms.

In 1989, Mantenieks, Sovey, Myers, Haag, Raitano and Parkes evaluated a 100-KW class, steady state, applied-field MPD thruster. The thruster variables were mass flow rate, arc current, and magnetic field strength. The geometry of the anode was varied and a number of propellants were used. Applied magnetic fields caused higher thrust and improved thrust efficiency and specific impulse. This present work is a continuation of earlier efforts to understand the mechanisms inherent in magnetic nozzle effects on plasma flow. York, Zakrzewski, and Souls had evaluated the performance of a low power (50-150 kW) quasi-steady arc thruster. The thruster pulse-forming network delivered currents up to 2.30 kA for 0.5 ms, while the magnetic nozzle network delivered currents up to 2.7 kA for 0.4 ms. Results showed that the applied field caused the electron temperature and density, thrust, and specific impulse to increase, the latter by a factor of 1.6. This was for a case where there was no penetration of the arc chamber by the applied fields.

Objective and Scope of Work

The primary objective of the present work was to determine the effects of variable magnetic field geometry on the processes occurring in a magnetic nozzle. As the earlier work did not allow arc-chamber penetration, the effect of variable field penetration into the discharge chamber of a 1/4 scale device was also studied. Along with the diagnostics used earlier, there has been the unique application of Thomson scattering diagnostics. This laser-based, non-intrusive diagnostic has been used here to verify the accuracy of the Langmuir probe indications of electron density and temperature in the application with strong magnetic fields.

EXPERIMENTAL APPARATUS

Thruster and Support Systems

The arc thruster used in this experiment was manufactured...
was a scaled version of one tested at the Air Force Astronautics Laboratory (AFAL); a schematic is given in Fig.1. The anode was made of solid copper, with an inner diameter of 2.5 cm and a outer diameter of 7.4 cm. The cathode was made of 2% thoriated tungsten with an outer diameter of 0.5 cm. Both the anode and cathode had a length of 1.25 cm. Boron nitride was used to insulate the anode and cathode and served as a back plate for the two electrodes. Nitrogen was injected through sixteen orifices drilled into the boron nitride back plate. The assembly was mounted onto a plexiglass plate which was in turn mounted onto one end of the vacuum chamber. The coils used to generate the applied magnetic fields were positioned outside the vacuum vessel housing the thruster, were physically independent, and had their own electrical circuitry.

The gas feed system had three major components: a reserve plenum, a thruster plenum, and an electrical valve connecting the two plena. The reserve plenum was constructed from a cylindrical copper pipe which was 8 in. long and had a diameter of 1.5 in. Openings in the boron nitride back plate served as the outlet of the thruster plenum. A Skinner valve (Model VS208-2100) with a 3/8 in. orifice connected the two plena. In all experiments, gas entered at a mass flow rate of 0.135 g/sec was generated by a reserve plenum pressure of 295 Torr. This flow rate allowed the thruster to function smoothly without mass starvation effects.

**Electrical Discharge System (Thruster)**

Two independent circuits supplied current to the thruster and to the coil generating the magnetic nozzle fields. The components of the thruster electrical discharge system were a pulse forming network (PFN), a charging circuit, a switching circuit, and a timing system. The thruster PFN was designed to deliver a constant current pulse of 0.7 to 2.3 kA for 200 μsec. The switching circuit employed spark gaps. The timing circuit controlled the triggering sequence of the PFN switching circuit, the gas feed valve, and the high current magnetic nozzle switch.

**Vacuum System**

For acceptable thruster operation, tank pressures of 5 mtorr or lower were required. The components of the experimental system included a T-shaped pyrex duct (6 in. i.d.), a vacuum pump (Welch Duo-Seal, Model No. 13978), and a 4 in. i.d. electropneumatic gate valve separating the vacuum pump from the T-shaped pyrex discharge chamber. The MPD thruster assembly was mounted onto one end of the T-shaped duct; diagnostic probes were mounted from the opposite end, and the Thomson scatter beam was directed through a window at 90 deg.

**Generation of Magnetic Nozzles**

Two applied field magnetic nozzle configurations were studied. The magnetic fields were generated by multi-turn coils whose axes were aligned along the axis of the thruster. The primary nozzle, A, was designed to generate magnetic field strengths and shapes similar to those generated for previous magnetic nozzle studies. The earlier work was done with pulsed currents (3000 A) whose fields did not penetrate the thrust chamber. In order to investigate the effects of variable penetration, as well as to insure full penetration, currents for durations of 1-3 sec were needed, but for that duration currents on the order of only 500 A were possible. Accordingly, new coil configurations had to be developed. Also it was of primary interest to investigate the magnetic nozzle and to examine the quantitative results on the expansion and exhaust flow of the plasma and accordingly a second nozzle geometry, B, was also developed.

To be able to predict the magnetic field strength and shape, a computer code was written using the differential form of the Biot-Savart integral. The input parameters included nozzle diameter, arrangement of wire coils (number of turns and layers), diameter of wire, and current. Predicted magnetic fields for nozzles A and B are shown in Figs. 1 and 2, respectively. Experimental tests were in close agreement with the final field configuration predicted for and generated by the coils.

Nozzle A was constructed with 5 layers of 11 wraps of 10 gauge stranded copper wire wound an 8.75 in. dia PVC pipe. Nozzle B was constructed in 3 parts, joined axially: 7 layers of 4 wraps, 1 layer of 15 wraps, and 7 layers of 4 wraps; all three parts were wrapped around an 9.75 in. dia PVC pipe. Each part was fitted on its ends by 3mm thick panels to provide axial rigidity.

With nozzle A, 500 A generated a magnetic field strength of 1.46 kG on-axis at the center of the coil when the face of the thruster was positioned 2.295 cm from the center of the coil. With nozzle B, 427 A generated 1.70 kG at the center of the coil when the face of the thruster was positioned 3.375 cm from the center of the coil.

**Switch for Magnetic Nozzle Current**

Current for the magnetic nozzles was supplied from a series of ten, 12 V batteries which were loaded beyond delivery specifications. Accordingly, precision switching of the circuit was needed, particularly at current termination, to avoid damage. A unique switch assembly was designed, fabricated, and tested. Definition of penetration times was accurate to within 1 msec.

**DIAGNOSTIC DEVICES:**

**Current and Voltage Probes**

The current and voltage of the MPD thruster arc were determined using a Rogowski loop and a Tektronix P6105 voltage probe, respectively. The Rogowski loop was a 22 in. long, 7/16 in. dia Tygon tube with 100 wraps of 18 gauge wire evenly spaced. An electronic integrator with an RC of 8.19 msec was used; the calibration was 1.93 x 10^5 V/A.

**Magnetic Field Probes**

The applied magnetic nozzle fields were mapped with Hall generator probes. Both the axial-field Hall probe, Model no. BH-203, 0.06x0.15 in. Hall plate, and the radial-field Hall probe, Model No. FH-301-040, 0.04x0.08 in. Hall plate (F. W. Bell, Orlando, FL) were placed inside pyrex tubes with fused ends. Coaxial cable connected the Hall probe to a x (100) probe amplifier and current supply circuit. The Hall probes were calibrated using a solenoid and a Helmholtz coil. The sensitivity of the Hall probe was 6.75 mV/(mA-kG), the radial/azimuthal probe was 135 mV/(mA-kG).
Impact Pressure Probe

An impact pressure probe design, reported earlier, was used. The output of the probe was matched with a follower and amplifier (×100) circuit, and the probe was calibrated being a reflected shock wave in a shock tube; a calibration factor of 3.45 V/PSI was obtained. In applied field operation, induced stresses generated oscillations which were suppressed with a tubular metal collar fitted over the outside of the insulating pyrex tube.

Langmuir Double Probe

Electron temperatures and electron number densities were determined using a Langmuir double probe. The probe elements were 0.127 mm tungsten wire, had an exposed length of 10 mm and were separated by a distance of 5 mm. The voltage bias across the elements was supplied by a capacitor in a floating probe circuit. In order to avoid distortion to the current drawn by the probe because of surface contamination, chemical cleaning was used periodically, while glow cleaning was used after each shot.

Thomson Scattering Diagnostic

The details of this system will be discussed in the context of data gathering.

ARC THRUSTER EXPERIMENTAL DATA

Introduction

Experimental data were taken with the arc thruster in the self-field ($B_{\text{self}}=0$), and the applied-field cases with nozzles A and B for different magnetic field penetration times into the thrust chamber. The applied magnetic field penetration times ranged from 20 msec to 500 msec; 0 msec corresponds to self-field operation. The experimental measurements included: interelectrode oscillations, current-voltage characteristics of the thruster, applied magnetic field mappings, impact pressure profiles, and electron temperature and electron number density profiles from Langmuir probes. Thomson scattering provided $T_e$ and $n_e$ at points for verification.

Prior to data taking, the vacuum pump lowered the pressure in the exhaust vessel to less than 5 mTorr. The pressure of the reserve plenum was maintained between 295 and 300 Torr. The capacitor bank of the thruster was charged to voltage (i.e., 7kV for 1.15kA case, and 10kV for 1.675kA case). The triggering circuit for the applied magnetic field coil was set for the proper magnetic field penetration time (i.e., 20, 50, 200, or 500 msec).

Thruster Current and Voltage Data

The Rogowski loop and voltage probe were applied; typical data are shown in Fig.4. The current and voltage records were read at 200 usec into the event. Fig.5 shows voltage versus current characteristics for nozzle A for various magnetic field penetration times; Fig.6 shows these for nozzle B.

Applied Magnetic Field Line Geometry

The magnetic field strength of the multi-turn coils in the axial and radial directions was determined at axial locations of 1, 3, 5, 9, and 14 cm from the face of the thruster and at radial locations of 0, 2, 4, and 6 cm from the axis. The experimental values agreed well with the computed results presented in Figs.2 and 3 for nozzles A and B. The time for initial magnetic field penetration of the thrust chamber was estimated to be 10 msec, from the experimental data.

Impact Pressure Profiles

The impact pressure probe was used to determine radial profiles for the thruster operating at current levels of 1.15 kA and 1.675 kA, with different magnetic field penetration times for both nozzle A and B. Typical impact pressure signals are shown in Fig.7. The oscillations are due to B induced stresses discussed earlier.

For magnetic nozzle A, with the thruster operated at 1.15 kA, impact pressure measurements were taken at axial locations of 5, 9, and 14 cm from the face of the thruster and at radial locations of 0, 1, 2, and 3 cm for different magnetic field penetration times (i.e., 0, 20, 50, 200, and 500 msec). Results are presented in Fig.8 for self-field operation and variable penetration time for applied-field operation. With the thruster operated at 1.675 kA, similar impact pressure measurements were taken.

For magnetic nozzle B, with the thruster operating at 1.15 kA and 1.675 kA impact pressure measurements were taken at similar axial locations and at 0, 1, and 2 cm from the axis for penetration times of 20 msec and 200 msec. Results are presented in Fig.9 for 1.15 kA to show the differences between A and B nozzles. Data are also shown for 1.675 kA for comparison. Observation of the raw data showed that at the different radial locations the pressure signal included an initial blast wave that was observed in earlier work. For both nozzles A and B, impact pressure data were read at times of 175 usec into the signal and were read in a way to adjust for the high frequency oscillations.

Langmuir Double Probe Current-Voltage Characteristics

The Langmuir double probe current-voltage characteristics were used to determine the electron temperatures and number densities in the plasma plume. These curves were constructed at each axial location of interest. To obtain a data point, the probe elements were biased at a fixed voltage and a Tektronix P6021 Current Probe was used to record the current drawn by the electrodes as a function of time. This procedure was repeated for 6 different voltages between 0.5 V to 22 V. For the self-field ($B_{\text{self}}=0$) case, data were taken at axial locations of 5, 9, 14, and 20 cm for the thruster operating at 1.15 kA, and 1.675 kA, respectively. For the applied field case, results were not obtained at 5 cm, because of the probe elements' proximity to the thruster electrodes. The Langmuir double probe current-voltage characteristic curves were used to determine: the ion saturation current, $I_m$; slope of the current-voltage characteristic at zero current, $dV/dI|_{I=0}$; and the saturation voltage, $V_m$. $I_m$, $dV/dI|_{I=0}$, and $V_m$ were then used to determine the electron temperatures and number densities.

Thomson Scattering Apparatus

The laser source was an Apollo model 35 ruby laser which used an oscillator and a one-pass amplifier arrangement. This laser was manufactured by Apollo Lasers, (Los Angeles, California). It provided 7J into a 3-mrad divergence cone with a 3/4-in. diameter beam for a 25-nsec pulse. Three components of this diagnostic system were: the focusing optics, the detection optics, and the
The diagnostic beam was split into two paths by a beam splitter and they were subsequently detected by the photomultipliers. The filters had a 30-A full bandwidth at half maximum and performed the function of separating the scattered-wavelength spectrum into two components for temperature determination. One filter was set at normal incidence to the light and its wavelength passband centered on 6943 Å. This fixed the peak point of the scattered-wavelength spectrum when calibrated, this signal allowed electron density to be determined. The other filter was set at an angle $\psi$, to normal incidence.

The temperature was derived from the scattered-wavelength spectrum through a determination of the 1/e half width of that spectrum; the 1/e half width is given by the difference between the center wavelength of 6943 Å and the wavelength where the scattered signal is down by a factor of 1/e from that at the center of the spectrum at 6943 Å. The 1/e half width for a ruby laser with $\psi = 90$ degrees gives

$$\Delta \lambda_{1/e} = 19.7T^{1/2} \text{(Å)}$$

Accordingly, determination of the temperature involved resolution of the 1/e points of the scattered-wavelength spectrum.

Data were taken at $z = 20$, $r = 90$ cm for $I = 1.65$ kA with nozzle A, no field penetration into the discharge chamber.

The $T_e$ results are in good agreement with those derived from other diagnostics.

**ANALYSIS AND INTERPRETATION OF EXPERIMENTAL DATA**

**Thruster Characteristics**

The current-voltage data presented in Figs. 5 and 6 enable the calculation of the power input, $P = IV$, as follows:

<table>
<thead>
<tr>
<th>Thruster</th>
<th>$I$ (kA)</th>
<th>$V$ (V)</th>
<th>Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self</td>
<td>1.15</td>
<td>90.45</td>
<td>1.675</td>
</tr>
<tr>
<td>Noz. A, $tp=200$ msec</td>
<td>1.15</td>
<td>76.27</td>
<td></td>
</tr>
<tr>
<td>Noz. B, $tp=200$ msec</td>
<td>1.15</td>
<td>112.84</td>
<td></td>
</tr>
</tbody>
</table>

Further, the current-voltage graphs for self- and applied-field operation show a linear relationship. This indicates that electrothermal acceleration is the dominant mechanism. As reported earlier for short magnetic field penetration times (400 usec), there was no difference between the voltages across thruster electrodes for the self- and applied-field cases. Figures 5 and 6 confirm that the thruster voltage increased with the magnetic field penetration of the interelectrode gap. Magnetic fields fully diffused by $tp=200$ msec. Thruster voltages were higher for nozzle B than for nozzle A for a given thruster current. This behavior is clearly related to the local magnetic field values.

Electron temperatures and plasma conductivities can be estimated from the I-V data. The voltage across the electrodes can be expressed from Ohm's law:

$$V_{TOTAL} = \int \left( \frac{\sigma}{\sigma_{|\sigma|}} I dZ + \frac{\sigma}{\sigma_{|\sigma|}} \int J dZ \right) dI + V_{FALL}$$

where $V_{TOTAL}$ is the total voltage drop, $\sigma$ is the plasma conductivity, $\Omega$ is the Hall parameter, and $V_{FALL}$ is the electrode fall voltage. Neglecting ion slip and Hall effect the third term drops out. Integrating the first term provides the Ohmic voltage contribution. The second integrand is known as the back-emf, and the fourth term is the fall voltage. The electrode fall voltage can be obtained from the extrapolation of the voltage data to zero thruster current; this includes electrode and sheath losses. Remembering

$$J = \frac{I}{2\pi z\sigma}$$

where $z_0$ is the channel length and $\mu_0$ is the permeability of free space. Equation (2) is solved for $\sigma$; the velocity $u_e$, is taken as sonic speed. The (V-I) indicated fall voltage for self-field operation is 32.41 V; for nozzle A is 51.59 V and for nozzle B is 64.67 V. Electron temperatures can be estimated using a conductivity model for a fully ionized gas with a Maxwellian velocity distribution. The resulting values are:

<table>
<thead>
<tr>
<th>Noz.</th>
<th>$T_e$ (eV)</th>
<th>$\lambda$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.922</td>
<td>1.675</td>
</tr>
<tr>
<td>B</td>
<td>0.859</td>
<td>1.15</td>
</tr>
<tr>
<td>C</td>
<td>0.859</td>
<td>1.15</td>
</tr>
</tbody>
</table>

**Impact Pressure Data**

Impact pressure data for the self-field case, Fig. 8, indicate that the plasma was ejected from the thruster and quickly expanded in all directions for both thruster currents; this behavior has been reported earlier. In contrast, with the magnetic nozzle fields, the plasma flow was clearly constrained in the radial direction and was restricted to flow along the axial direction.

With nozzle A applied, and thruster current of 1.15 kA, Fig. 9 shows that at an axial location of 5 cm the plasma was primarily within a 2 cm radius; at 9 and 14 cm the plasma was contained within a 3 to 3.5 cm radius. For thruster current of 1.675 kA, at 5, 9, and 14 cm the plasma was contained within 2, 2-3, and 2-3 cm radii. Thus, for higher thruster current the plasma was more confined; this discharge condition is compatible with higher plasma conductivity.

For magnetic nozzle B and thruster current of 1.15 kA, Fig. 11 shows that at axial locations of 5, 9, and 14 cm the plasma was contained within 3.5, 4.5, and 4.5 cm radii. For thruster current of 1.675 kA, at axial locations of 5, 9, and 14 cm, the plasma was contained within 2 to 3, 3.5, and 3.5 cm radii. Thus, for both nozzle A and B, higher thruster currents confined the plasma to smaller radii.

From the above results, it is evident that the magnitude of the impact pressure increased as the thruster power was increased and as the applied magnetic nozzle fields were increased.

For nozzle A, when the magnetic field penetration time was increased from 20 to 50 msec the impact pressure on the axis at 5 cm increased by a factor of 2, while it remained almost unchanged at the other radial and axial locations.
Increasing the magnetic field penetration time from 50 msec to 200 and 500 msec did not greatly change the magnitude of the impact pressures, and the impact pressure radial profile remained almost unchanged. As the magnetic field penetration increased from 0 to 20 to 50 msec, ions being markedly influenced by the magnetic fields in the interelectrode region.

For nozzle A, peak impact pressures occurred on axis, while for nozzle B, peak impact pressures occurred at a radius of 1 cm. Thus, changing the shape of the magnetic field lines did affect the distribution of the plasma in the exhaust plume at different axial locations. This effect could be internal to the thruster or external; it is yet not clear.

**Determination of \( T_e \) and Ne from Langmuir Probes**

Temperature and density were determined from the current collected with biased double probes. While corrections were necessary for Ne determination, simple collisionless analyses were first used to establish orders of magnitude for both properties.

The electron temperature was determined by applying a Maxwellian velocity distribution for the electrons, as

\[
T_e = \frac{eI_{sat}}{2k \frac{dV}{dt}} \quad (5)
\]

where \( I_{sat} \) is the ion saturation current and \( dV/dt \) is the slope of the 1-V characteristic. For self-fields, \( T_e \) at 1.675 kA were higher than those at 1.15 kA; input power was higher. For applied magnetic fields, in general, values were higher than those for self-field operation. \( T_e \) for nozzle A were higher than those for nozzle B in the expansion region (Fig.11). This indicates that nozzle B expanded the plasma flow differently than nozzle A. These applied-field temperatures are significantly lower than those presented in earlier work for an arc discharge without chamber penetration by the applied magnetic field into the inter-electrode region, even though the power input was higher in the present case.

To determine electron number densities, the probe regime for particle collection had to be determined. Three parameters of importance are: \( \lambda \), mean free path; \( \lambda_D \), Debye length; \( r_p = 6.35x10^6 \) m, probe size; three related ratios are: the Knudsen number, \( Kn = r_p/\lambda \), \( 1/\lambda_D \), and \( r_p/\lambda_D \). \( N_e \) values calculated from the collisionless model were used to give a preliminary indication of the probe's regime of operation, as

\[
N_e = \frac{2^{1/2} \cdot e^{-1/2}}{A_p \cdot 4 \cdot e} \quad (6)
\]

Knudsen numbers indicated for the different cases were between 48 and 500, \( 1/\lambda \), ratios were between 1116 and 4373, and \( r_p/\lambda_D \) were between 7.5 and 28. Thus, the probe was operating with moderate sheath thickness and collision effects.

A model that accounts for sheath effects in the Ne determination was developed by Kiel. Application indicated Knudsen numbers between 0.29 and 3.28, \( 1/\lambda_D \), ratios between 85 and 336, and \( r_p/\lambda_D \) between 97 and 367. Thus, collision effects were to be accounted for.

Peterson and Talbot analyzed weak collision effects for cylindrical probes. To calculate the electron number density an iteration is performed between equations expressing \( j_e \). Number density and electron temperature were put into the Saha equation and the degree of ionization was determined along with \( T_e \). The densities derived by this method are presented in Fig.12.

The presence of an applied magnetic field may also influence the electrostatic probe operation. If \( r_p/\lambda_D \), then the analysis of weak magnetic field is applicable. Using the \( T_e \) and \( n_e \), values, the ion and electron Larmor radii were calculated and \( r_i/r_p \) were between 64 and 82 while \( r_e/r_p \) were between 0.40 and 0.53. Since the ions were the species saturated in the double Langmuir probe, a weak magnetic field analysis is applicable.

**Evaluation of Ne using \( T_e \) (Langmuir) and Impact Pressure in a Gas Dynamic Expansion Model**

An independent technique for calculating Ne, which places stronger emphasis on pressure data, was also carried out. Knowledge of effective plasma radius and electron temperature at two axial locations were used to determine Mach numbers, electron number densities, and exhaust velocities, for comparison purposes.

With a normal shock wave assumed to form at the pressure probe, the expansion from 9 to 14 cm was assumed to occur at constant stagnation temperature. To calculate the Mach numbers \( M \) at 9 and 14 cm, a computer program was written to perform required iterations beginning with an assumed \( M = 9 \). By knowledge of the effective radius at 9 and 14 cm (from the impact pressure data) the corresponding effective areas were calculated. The area-Mach number relationship at 9 cm allowed the throat area, \( A_t \), to be calculated. With the area at 14 cm known, the Mach number can be found. Using \( T_e \) from Langmuir probe data, the stagnation temperatures at 9 and 14 cm were calculated. If the stagnation temperatures were not equal, the iteration process was repeated with a new guess of Mach number until a match was achieved.

The resulting electron densities were in good agreement with those derived from Langmuir probes. Mach numbers, and exhaust velocities were also used to predict the local mass flow rates and thrust values.

**Axial Profiles of Static Pressure**

Evaluation of plasma thermal pressure, \( P = \text{N/eV} \), where \( \text{N/eV} \), can provide indications of the extent of plasma expansion in the magnetic nozzles (Fig.13). For self- and applied-field operation, the pressure decreases with axial distance from the thruster. However, for applied-fields, pressures are at least five times higher than those for self-fields, indicating substantial confinement of the plasma.

**Evaluation of Thrust From Experimental Data**

Impact pressure profiles were integrated to calculate thrust at different axial locations, for self- and applied-fields, and for different magnetic field penetration times by
Thrust values for self-field operation at both arc current levels are presented in Fig. 14. Maximum values occur at an axial location of 5 cm, indicating that little or no electromagnetic force acted to confine the plasma in the radial direction. Thrust values at 1.675 kA are greater than those at 1.15 kA.

Thrust values for magnetic nozzle A, both current levels, are presented for variable magnetic field penetration time into the discharge chamber (Fig.15). More important, thrust varies with axial distance (Fig.14); this again indicates that the applied magnetic fields are confining the plasma and guiding the plasma flow as desired; the plasma appears to expand primarily in the axial direction. Change in the magnetic field penetration time clearly did affect the thrust. With nozzle A the thrust at 1.15 kA was greater than at 1.675 kA.

Thrust values for nozzle B were also evaluated (Fig.14). At a current 1.15 kA, at 200 ms thrust was 70% higher than at tp=20 msec; this behavior was not observed 1.675 kA. Thrust values for nozzle B were higher than those for nozzle A. This evidently was due to the several differences between the field configurations. However, thruster power for nozzle B was higher than that for nozzle A.

Parametric Indication of Magnetic Nozzle Influence

The effects of applied magnetic fields are related to three parameters\(^\text{12}\). The Hall parameter,

\[ \alpha = \frac{g B^*}{m_n v_l e} \]

with values between 0.73 and 2.23. The ratio of fluid and magnetic pressure defines the parameter

\[ \beta = \frac{\Sigma f k T}{B^*/2 \mu_0} \]

with values between 0.028 and 0.098. The magnetic Reynolds's number describes the ratio of convection to diffusion

\[ R_s = \mu_s (U L) \]

In the evaluation of \( R_s \) and \( \sigma \), L of 0.05 m and Te at 9 cm were used, along with sonic flow values. Rem values for applied field conditions were 0.03-0.07, while for self-field conditions were 0.02-0.05.

The values of \( \alpha \) are indicative of a generally collision-dominated plasma, however the values \( \beta(1) \) point toward a tendency for stronger particle orbit effects. The \( \beta \) values are small, indicative that magnetic confinement with transport effects is the behavior to be expected. With the small values of \( R_m \), particle diffusion through the field lines would be expected to be significant. The major influence on \( R_m \) was thruster current, with values of 0.03 for 1.15 kA and 0.06 for 1.675 kA.

Mass Diffusion Effects in the Magnetic Nozzles

The primary function of a magnetic nozzle is to confine the flowing plasma and to allow it to expand with a controlled transfer of internal energy to kinetic energy. Accordingly, the efficiency of a magnetic nozzle is reduced by particle diffusion across field lines. Assuming fully ionized gas, Coulomb collisions give rise to a cross-field diffusion coefficient

\[ D = \frac{\eta f k T c^2}{\sigma B^2} \]

which is associated with a particle flux

\[ \Gamma = -D \n \]

where \( \n \) is the radial gradient in the number density (cm\(^{-3}\)).sec.

Langmuir probe results\(^\text{10}\) indicate that number density ratio drops to 1/e at a radius of 5 cm. Accordingly, based on that gradient, diffusion coefficients, radial flux, and radial mass flow rates were calculated. For nozzle A, 1.15 kA, 34% of the mass flow rate of 0.135 g/sec diffused radially; at 1.675 kA, 42% of the input diffused radially. For nozzle B, 1.15 kA, calculations indicate 31% was lost, while at 1.675 kA, 56% diffused radially. The loss rate for nozzle A is indicated to be higher than that for nozzle A. The loss rate for 1.15 kA is indicated to be less than that for 1.675 kA for both nozzles.

Flow Expansion in Magnetic Nozzles

The average flow velocity at an axial location in the expansion can be evaluated from where \( P_e \) is the exit pressure (10 mTorr), \( A_{eff} \) is the plasma plume effective area (from radial impact pressure profiles). Figure 16 presents exhaust velocities for self- and applied-field operation at different axial locations. In general, for self-fields, the velocity decreased with axial displacement; for applied-fields, the velocities increased, confined by the magnetic pressure of the magnetic nozzles. The magnitudes presented in Fig.16 do not account for any diffusive mass loss.

Thrust efficiency is defined as

\[ \eta = \frac{n^2}{n^2 + \eta} \]

where \( n = P_{out} \). Due to mass flow rate uncertainty, it is reasonable to define a comparative measure of thrust efficiency between self- and applied-field operation. An efficiency ratio is defined as

\[ \eta = \frac{n^2}{n^2 + \eta} \]

Values of this ratio for a thruster current of 1.15 kA, nozzles A and B are 2.25 and 2.6, respectively; for a thruster current of 1.675 kA, nozzles A and B the ratios are 0.62 and 1.13, respectively. Thus, nozzle B was more effective than nozzle A, and the ratio was higher at 1.15 kA than at 1.675 kA.

Theoretical Comparison with Thrust Evaluations

\[ \int P_{impact} dA = T_{exp} \]

\[ \eta = \frac{\int P_{impact} dA}{P_{impact}} \]

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A theoretical derivation of thrust for an applied-field MPO thruster has been reported. The model assumes that work done by the electromagnetic force is converted into kinetic energy. The interactions between the applied and induced currents, and the applied or self magnetic fields result in three thrust components:

1. The generalized Hall acceleration; it results in an axial force generated by the interaction between the azimuthal induced current and the radial applied field \((j_B)\).
2. The swirl acceleration; it results in an azimuthal force generated by the interaction of the radial applied discharge current and the axial induced field \((j_A)\).
3. The self-magnetic acceleration; it results in an axial force generated by the interaction of radial applied current and the azimuthal self-induced magnetic field \((j_B)\).

The different thrust components are summed as:

\[
T_{\text{tot}} = T_{\text{Hall}} + T_{\text{Swirl}} + T_{\text{Self}} \tag{16}
\]

The electrothermal thrust was calculated by:

\[
T_{\text{eth}} = \frac{m}{2C_p} \left(2C_p T_0\right)^{1/2} \tag{17}
\]

The theoretically calculated thrust values are presented in Table 1, along with the values derived from experiment. The results show that the generalized Hall acceleration contribution to the total thrust is negligible. The swirl acceleration is the largest contributor to the total thrust; the next is the electrothermal acceleration, and the self-magnetic acceleration. The theoretically calculated total thrust values decrease with axial distance from the anode face because the applied magnetic field strength decreases; this is in contradiction to the experimental results. In addition, the theoretical thrust values for nozzle B are greater than those for nozzle A because of the effect of the magnetic field shape. The total thrust values for 1.075 kA current operation are higher than those for 1.15 kA current operation because the swirl acceleration term is linearly proportional to thruster current and the self-magnetic acceleration is proportional to the square of the thruster current.

In general, there is reasonable agreement of the magnitude of the theoretical model predictions with the experimental results. The differences between the two provide a basis for further detailed studies of the acceleration mechanisms that are active.

REFERENCES

3. Kimura, I., and Arakawa, Y., "Effects of

Figure 1. Schematic of Thruster

Figure 2. Magnetic Field Contours for Nozzle A

Figure 3. Magnetic Field Contours for Nozzle B

Figure 4. Typical Current and Voltage Signals

Figure 5. I-V Characteristics for Nozzle A

Figure 6. I-V Characteristics for Nozzle B

Figure 7. Typical Impact Pressure Signals
Figure 8. Radial Profiles of Impact Pressure, Nozzle A, 1.15kA, a) Self-field, b) tp = 20 ms, c) tp = 200 ms

Figure 9. Radial Profiles of Impact Pressure, Nozzle B, a) 1.15kA, tp = 20 ms; b) 1.15kA, 200 ms; c) 1.675 kA, 200 ms

Figure 10. Schematic of Thomson Scatter Diagnostic

Figure 11. Axial Variation of Te for Self-field and Applied-field Nozzles A and B
Figure 12. Axial Variation of Ne for Self-field and Applied-field Nozzles A and B

Figure 13. Axial Variation of P(\text{mNkT}) for Self-field and Applied-field Nozzles A and B

Figure 14. Axial Variation of Experimental Thrust for Self-field and Nozzles A and B

Figure 15. Experimental Thrust as a Function of Magnetic Field Penetration Time (\text{t_p})

Table 1: Theoretical and Experimental Values of Thrust with Applied Magnetic Nozzles.

<table>
<thead>
<tr>
<th>Noz. Thruster</th>
<th>Z (cm)</th>
<th>T_{\text{th}} (N)</th>
<th>T_{\text{expt}} (N)</th>
<th>T_{\text{tot}} (N)</th>
<th>T_{\text{expt}} (N)</th>
<th>T_{\text{th}} (N)</th>
<th>T_{\text{tot}} (N)</th>
<th>T_{\text{expt}} (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.15</td>
<td>9 0.02  2.30 0.24</td>
<td>0.73 3.29 2.29</td>
<td>1.15  14 0.01  1.80 0.26</td>
<td>0.73</td>
<td>2.81 2.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.15</td>
<td>15 0.02  2.32 0.24</td>
<td>0.73 3.29 2.29</td>
<td>1.675 15 0.02  2.52 0.54</td>
<td>0.80</td>
<td>3.38 2.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.15</td>
<td>15 0.02  2.22 0.24</td>
<td>0.73 3.29 2.29</td>
<td>1.15  15 0.02  1.77 0.66</td>
<td>0.79</td>
<td>4.15 3.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 16. Axial Variation of Plasma Velocity for Self-field and Nozzles A and B