A DOUBLE PENDULUM PRECISION THRUST MEASUREMENT BALANCE

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ABSTRACT

A highly sensitive and precision thrust measuring double pendulum balance has been developed. It can measure a force, an impulse or thrust as low as 0.1 mN (milli-Newton) free of mechanical noise, electrical and magnetic pick-ups. The Double Pendulum system consists of two parallel conducting plates. One or both of the plates can be suspended by needles. The needle suspended plate (or plates) can swing freely with negligible friction because of the sharp points of the needles. When one of the plates is impacted by an impulse it will swing relative to the fixed plate or other movable plate. The capacitance between the plates changes as a result of such a motion. The change of capacitance as a function of time is recorded as an oscillating voltage signal. The amplitude of such a voltage signal is proportional to the impacting force or impulse. The proportional factor can be calibrated. Therefore the forces can be read out from the recorded value of the voltage. The equation of motion for the pendulum system has been solved analytically. The circuit equation for the electronic measurement system has been formulated and solved numerically. The analytical solution of the overall characteristics agrees greatly with the measurement. The thrust of a Tandem Mirror plasma thruster has been measured and is also well in agreement with predictions.

1 Introduction

There is a need to develop an accurate and sensitive target balance to measure the momentum transfer in a plasma system, such as an electric propulsion device, plasma propulsion systems and in the divertor of a tokamak fusion reactor. In electric propulsion research the common method to measure the thrust is a balance consisting of the suspended thruster and a displacement sensor. The thrust is determined from the displacement of the thruster when it recoils at the firing of the thruster. This method has several drawbacks: The thruster is heavy. It is tied to heavy electric power cables and gas feed line and therefore there has a very high motion resistance. There are frictional losses due to the suspension of the heavy thruster and the transfer of displacement through pulleys to the sensor. In the future, the header will probably need to be cooled for high power operation. The motion resistance will be much higher with added cooling lines and heavier power cables. These factors will reduce the sensitivity and accuracy.

For the purpose of studying the momentum transfer in a plasma system there are two known methods. One is to attach the target through a long shaft to the diaphragm of a commercial pressure transducer. The shaft is suspended by two strings. The suspended target and the shaft system behaves like a pendulum and the pressure transducer serves as a displacement sensor. The other method is similar to the suspended target and shaft assembly whereas the sensor consists of a magnet and a pick-up coil. The force can be determined from the voltage induced on the pick-up coil from the motion of the magnet. The sensor unit has to be housed in a magnetically shielded box. The long arm of these two systems will make them difficult to be installed in the plasma stream in a closed system like tokamak.

None of the systems mentioned above has provided adequate means to deal with the problem of mechanical vibrations, electric and magnetic interferences.

The target balance system developed in this work consists of two identical plates suspended by needle points. The device is compact, light and is symmetric with respect to the equilibrium position as either or both plates impinged by plasma from opposite directions. The system can, therefore, be inserted in the plasma stream in a closed system like tokamak. The mechanical vibrations can be eliminated as a common mode to the two identical moving pendulums. The electric and magnetic pick-ups have been reduced to a negligible level by the use of differential amplifiers, shieldings and dynamic bandpass filters. The detailed developmental process of the pendulum system, electronic circuit, the analysis and measurement are described in the following sections.

2 The Balance Development

The purpose of this work is to find a suitable method to measure the thrust in the Tandem Mirror Plasma propulsion device [1] [2] [3] [4]. At present the device is operated in the low power level and at high specific impulse (about 13,000 sec ). The thrust level is very low and is estimated to be at about 100 mN. To measure such a low thrust a sensor must have a sensitivity and accuracy of less than 1 mN. The balance has to be very compact so that it can be mounted in the limited space of the exhaust chamber.

The propulsion system is presently operated at pulsed mode and plasma is created by microwave radiation and heated to high temperature by rf power. The magnetic impulse and mechanical vibrations, rf pick-ups, electric field, electromagnetic noises from the power supply systems are many orders of magnitude stronger than the anticipated thrust level. Undoubtedly they must be eliminated. At the beginning we were hoping to adapt the known methods. As described in the introduction none of them are adequate to our system. The possibility of using torsion wire and strain gauge has also been carefully examined and found not desirable either.
In order to keep the system as simple and compact as possible, we have given the simple pendulum serious consideration. However, it loses its simplicity and compactness when the position sensing system is incorporated. After weighing all the odds we decided a system consisting of double plates as our best bet. We were encouraged by the success of a pendulum and fixed plate system as shown in Figure 1. Both the moving and fixed plates are identical in size and shape and are made of aluminium. The displacement of the pendulum is translated into variation of capacitance of the plates. The capacitance measuring method will be described later. The signals for the balance in the air and in the vacuum chamber are shown in Figure 2 (a) and (b) respectively. The oscillations in the air were produced by mechanical shock. The oscillation in the vacuum is produced by the pulsing of the 

![Figure 1: A thrust balance consisting of a simple pendulum and a fixed plates](image1)

magnetic field. As can be seen from these figures, the signal taken in the air damped quickly whereas there was almost no damping in the vacuum. This shows that the friction loss at the needle support is negligible. The magnitude of the magnetic impulse is very large and can be eliminated by dividing the plates into very thin strips.

It appears that the mechanical vibrations are the second order noise which arises when the magnetic impulse is removed. As shown in Figure 3 such noise is random and could not be reduced with suspension methods. To eliminate the mechanical noise, both plates are suspended and swing as an independent pendulum. Since both plates respond identically to the noise, no net change in the capacitance is produced and therefore no net signal is recorded from the noise.

In the plasma stream, the conducting plate acts as a big Langmuir probe which will detect the electric field and current. These electric fields are the third order effects which arise when the vibrational noise are cancelled out. As shown in Figure 4, these electric fields are still very large. To reduce the electric field, insulating materials such as ceramic or silicon are used on the surface facing the plasma. Further, a set of plates with conducting strips are placed above the sup-

![Figure 2: Signals from the thrust balance: (a) A damped oscillation from a mechanical shock in the air and (b) a nondamping oscillation produced by magnetic impulse in the vacuum.](image2)

![Figure 3: Noise signal from mechanical vibrations in vacuum.](image3)
port. The electromagnetic noise from the inverse pendulums can then be used to cancel the noise picked up by the regular pendulums below the support. The inverse pendulum above the support swings opposite to the regular pendulum below so that the signs of the voltages from these two sets of capacitor plates are opposite, whereas the signs of pick-up voltages from electric and magnetic noise are not. By using a differential amplifier, the pick-up signals are cancelled out and the real signals are summed. This system is shown in Figure 5.

Since the inverse pendulums are smaller than the regular ones, the noise cannot be removed completely. To further avoid the electric field effect due to plasma, the plate in contact with the plasma is made entirely with an insulator, i.e., no conductor strips even on the back face. As depicted in Figure 6, the device is now a quadruple pendulum system where there is a set of double capacitor plates above the support and another set of double capacitor plates below the impact plates which are insulators. The double capacitor plates consist of conductive metal strips used to monitor signals. These capacitor plates are enclosed in a conducting box to shield out the electric and magnetic pick-up. Any residue pick-up can be further eliminated with the use of differential amplifiers. This method effectively reduces the pick-up and enhances the sensitivity. The magnetic noise produced by the power supply is not eliminated by the method was reduced with a carefully designed filter.

The main difference between the DHTL and the FTL lies in the driving term for the radial feeders. This is different from the Full Turn Loop (FTL) geometry in which the contribution from the radial feeders cancel out (Since one leg is entering and the other is exiting, net $J$ is zero). The modelling of the driving term $\tilde{J}(k_z)$ for the DHTL is detailed in a full report in preparation.

The wave propagation of both types of antenna is quite similar. Figures 1 and 2 show the propagation characteristics of the ICRF $B_t$ and $E_t$ fields as the waves propagate from the launch point ($z=0$) towards the resonance at mid-plane. The results that are readily observable from this collisionless propagation of the $n=0$ mode towards resonance are given below:

3 The Signal Treatment

The simplified electronic circuit diagram is shown in Figure 7. To detect the small change in capacitance of the two pendulums, a carrier radio frequency voltage at 30 kHz is applied to plates. There are two pairs of signals, one pair from the top capacitor plates and the other from the bottom capacitor plates. The signals from the each pair of plates are carried by a pair of twisted wires to the inputs of a differential amplifier. Since the inverse pendulum plates on the
top swing in the opposite direction to the pendulum plates at the bottom. The signs of the signal from the output of the differential amplifiers are opposite. These outputs are again summed by the third differential amplifier. Therefore the overall signal is enhanced.

The carrier frequency is removed from the signal by rectifiers. The residual noise are practically cancelled by the differential amplifiers. The magnetic ripples from the firing of SCRs in the power supplies are very stubborn. The frequency of these noise fallen in the range of the carrier frequency. These noise are filtered by a very carefully designed active bandpass filter. A high quality clean signal can be obtained.

4 Experimental

The two pendulums of the thrust balance are not exactly identical and, therefore, their natural frequencies are not equal. When a mechanical impulse is applied a beat as shown in Figure 8 is produced. The frequency of the beat is about one twentieth of the natural frequency or the period is 20 times longer. This beat frequency will become smaller when two natural frequencies is made to be closer. A great effort has been made to tune the two pendulums to make their natural frequencies match. However, perfect match can not be achieved and it can only make the beat period longer. The beat can not be eliminated completely. Currently the thruster is operated at pulsed mode and is fired in the first half cycle of the natural oscillation. The trace of the first cycle in Figure 8 is expended as is shown in Figure 9. This expended trace shows that the noise level is negligible. Figure 10 presents the relationship between the plasma density and temperature traces from a triple probe and the pendulum signal. This shot was made using the pendulum system shown in Figure 5 where the signal died out after one cycle.

It should continue to oscillate without damping and therefore it may not be the valid signal. In order to determine the validity of the measurement it is necessary to carry out detailed analysis which will be given in the next section. An exhaustive study of the residual noise from the power supply reveals that they can sometime cause misleading false signal even at very low level. The complete elimination of such noise is necessary and was accomplished with the use of dynamic bandpass filters.

Finally Figure 12 presents the signal obtained with the pendulum system shown in Figure 5 using the electronic circuit shown in Figure 7. The signal continued to oscillate without damping but was not symmetric with respect to the baseline. This requires solid explanation in order to convince ourself that it is in fact a correct signal. The amplitude of the signal rises from negative value to a constant symmetric oscillation in after about three cycles. This asymmetric property is found to be due to the electric circuit as demonstrated by the circuit analysis given in the next section.
As discussed above in order to validate the measurement it needs to carry out a complete analysis of the pendulum system and electronic circuit. Let us write down the equations of motion for a simple pendulum as shown in Figure 13. Assuming $\theta$ small, the equation of motion is

$$\frac{d^2\theta}{dt^2} = F_T - F_f - F_g$$  \hspace{1cm} (1)$$

where

- $F_T = \text{Thrust}$
- $F_f = A \frac{d\theta}{dt}$
- $F_g = mg$
- $A = \text{adamping factor}$

Let $\tau = \text{pulse length}$

$T_o = \sqrt{\frac{l}{g}}$

This equation has been solved for other systems. To understand the characteristics of this system the procedure for solving the equation of motion and results are presented in the following.

By apply Laplace Transformation

$$\Theta(S) = \frac{L[F_T]}{S^2 ml + SA + mg}$$  \hspace{1cm} (2)$$

Examine the solutions in two regimes:

1. $\tau << T_o$

   $$F_T(t) = F_T \delta(t), \ a \ \delta \ \text{function}$$
   $$L[F_T] = F_T$$

2. $\tau >> T_o$

   $$F_T = F_T \cdot 1(t), \ a \ \text{step function}$$
   $$L[F_T] = \frac{F_T}{S}$$

In each regime there are four different conditions:

1. $A = 0$, no damping
2. $0 < A < 4m2gl$, under damping
3. \( A = 4m^2gl \), critical damping

4. \( A > 4m^2gl \), over damping

In \( \tau << T \), regime, apply Inverse Laplace Transformation to the equations for each condition, we obtain

1. For \( A = 0 \)

\[
\theta(t) = \frac{F_T \tau}{m\sqrt{gL}} \sin \sqrt{g/lt} \quad (3)
\]

2. For \( 0 < A < 4m^2gl \),

\[
\theta(t) = \frac{2F_T \tau}{\sqrt{4m^2gl - A^2}} \exp(- \frac{A}{2ml} t) \sin \left( \sqrt{\frac{4m^2gl - A^2}{2ml}} t \right) \quad (4)
\]

3. For \( A = 4m^2gl \),

\[
\theta(t) = \frac{F_T \tau}{ml} t \exp(-\sqrt{g/lt}) \quad (5)
\]

4. For \( A > 4m^2gl \),

\[
\theta(t) = \frac{F_T \tau}{\sqrt{A^2 - 4m^2gl}} \left\{ \exp \left( \frac{A}{2ml} + \frac{\sqrt{A^2 - 4m^2gl}}{2ml} t \right) \right.
\] 
\[
- \exp \left( \frac{A}{2ml} - \frac{\sqrt{A^2 - 4m^2gl}}{2ml} t \right) \right\}
\]

These solutions are plotted in Figure 13. Comparing the analytical solution and the experimental result presented in Figure 11 it appears that the experimental result can be explained by the damped solution. But it has shown previously that the friction at the needle support is negligible. The damping can not come from the pressure built-up in the chamber during the shot because it has been shown that there is no observable damping in the chamber at mtorr pressure range. Therefore this is not a true result. The result shown in Figure 12 appears to be real and in agreement with the undamped solution except the asymmetric property in the first three cycle. We reason that this is an effect of the asymmetric relative motion of the capacitor plates and the electronic circuit. Because the relative motion of the pair of plates on the top is opposite to that of the pair of the bottom plates, the asymmetric effects nearly cancel out and are ignored for simplicity. The electronic circuit is very complicated and there are highly nonlinear elements. Fortunately, it can be represented by a drastically simplified diagram as shown in Figure 14 and the two simultaneous linear equations as follows:

\[
\begin{aligned}
\frac{dx}{dt} &= \frac{dy}{dt} + \frac{y}{T_1}; \\
\frac{dy}{dt} + \frac{y}{T_2} &= \frac{z}{T_2}.
\end{aligned}
\]

where \( T_1 = R_1C_1 \) and \( T_2 = R_2C_2 \), and \( y \) and \( z \) are the input and output signal respectively. The solution is shown in Figure 15. Except a slight phase different the analytical solution closely matches the experimental data. In any electric system phase shift is common and for this application we are not interested in its value. From this analysis we can confidently claim that the experimental result is the true measurement of the thrust free from mechanical, electrical and magnetic interferences.

6 Experimental Results and discussion

This balance has been used to measure the thrust from the Tandem Mirror propulsion device at MIT. The initial results are presented in Table I. The plasma temperature and density were monitored at a radial position two third of the plasma radius by a triple probe and the temperature at the
center is monitored by a spectrometer using Doppler broadening method. A Langmuir probe biased at +100 V is used to measure the electron saturation current.

The specific impulse is 12,852 sec corresponding to the plasma temperature of 172 eV. The measured thrust is 76 mN in agreement with prediction. The propulsive efficiency is 68%. This can be considered as a milestone of this research.

Detailed discussion of the propulsion device and experiment is not the subject of this paper. However, this demonstrates the successfulness and usefulness of the thrust balance as we set out to accomplish. This pendulum is made in the laboratory for testing purpose and is first of its kind. Therefore there is room for improvement. The plates are made from printed circuit board which consists of G-10 materials. G-10 material outgases during the plasma shot which degrades the plasma conditions. The new plates will be made from silicon wafers. For short pulse the particles can all be absorbed by the G-10 material. For long pulse the surface reflection has to be kept to a minimum and its effect has to be factored into the measurement.

There are two major advantages of this balance: (1) Since the plates are not parts of propulsion system and are simple and inexpensive to build; they can be made sacrificial for high power operation. (2) The balance is symmetric in both directions. It is particularly useful to study the pressure balance at the boundary layer between plasma and neutral gas in a gas divertor of tokamak reactors.

Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$T_e$</td>
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<tr>
<td>$T_i$</td>
<td>Electron temperature</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Plasma density</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Energy confinement time</td>
</tr>
<tr>
<td>$P_{\text{in}}$ (rf)</td>
<td>Input rf power</td>
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<tr>
<td>$P_{\text{in}}$ (microwave)</td>
<td>Input microwave power</td>
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<tr>
<td>$P_{\text{abs}}$ (ion)</td>
<td>Power absorbed by ions</td>
</tr>
<tr>
<td>$P_{\text{abs}}$ (electron)</td>
<td>Power absorbed by electrons</td>
</tr>
<tr>
<td>$P_{\text{exhaust}}$</td>
<td>Power into exhaust chambers</td>
</tr>
<tr>
<td>$\epsilon$ (coupling efficiency)</td>
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<td>$I_e$</td>
<td>Extracted specific impulse</td>
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<td>$m$</td>
<td>Measured mass flow-rate</td>
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7 Acknowledgement

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References