ON THE EFFECTS OF SWIRL IN ARCJET THRUSTER FLOWS
V. BABU, S. AITHAL, AND V. V. SUBRAMANIAM
Department of Mechanical Engineering
The Ohio State University
Columbus, Ohio 43210

ABSTRACT
Swirl has long been thought to be necessary to sustain arcs in high-speed flows. In this paper, we explore the effect of injected swirl on both the cold flow (i.e., non-ionized and non-reacting) and the plasma flow in arcjet thrusters. The fully implicit Linearized Block Implicit (LBI) scheme of Briley & McDonald[1] is used to develop highly accurate solutions to internal, viscous, supersonic flows of the type encountered in arcjet thrusters. In the case of cold flow, it is found that the injected swirl persists downstream for low mass flow rates (on the order of mg/s) and is actually enhanced at the constrictor entrance, near the cathode tip. However, for higher mass flow rates (on the order of g/s), the injected swirl is overcome by the torque due to viscous forces (both in the flow as well as at the walls) and fails to persist far enough downstream. Consequently, swirl stabilization of the flow in arcjets can only succeed under certain operating conditions. The actual plasma flow is also investigated in the presence of swirl, but our present simulations show continuous decay of injected swirl. As we show in this paper, this is an artefact and limitation of our present simulation. Real arcjet flows should experience the effects we observe in our cold flow simulations. This effect has important ramifications for the design and locations of the swirl injectors and overall arcjet performance. On-going improvements to our simulation to rectify this problem are discussed.

Nomenclature

\[ \begin{align*}
B_0 & \quad \text{magnetic induction (T)} \\
\epsilon & \quad \text{internal energy per unit volume (J/m}^3) \\
j_r & \quad \text{radial component of current density (A/m}^2) \\
j_z & \quad \text{axial component of current density (A/m}^2) \\
k & \quad \text{thermal conductivity (W/m}^2 \text{K}) \\
k_B & \quad \text{Boltzmann's constant (J/K)} \\
m_a & \quad \text{mass of an argon atom (kg)} \\
n_a & \quad \text{number density of atoms (particle/m}^3) \\
n_e & \quad \text{number density of electrons (particle/m}^3) \\
p & \quad \text{pressure (N/m}^2) \\
r & \quad \text{radial coordinate (m)} \\
t & \quad \text{time (s)} \\
T & \quad \text{temperature (K)} \\
u & \quad \text{radial velocity (m/s)} \\
v & \quad \text{swirl velocity (m/s)} \\
w & \quad \text{axial velocity (m/s)} \\
z & \quad \text{axial coordinate (m)} \\
\epsilon_i & \quad \text{ionization potential (J)} \\
\mu & \quad \text{coefficient of viscosity (Ns/m}^2) \\
\mu_0 & \quad \text{permeability of free space (Tm/A)} \\
\rho & \quad \text{mass density (kg/m}^3) \\
\psi & \quad \text{magnetic stream function (Tm)} \\
\sigma & \quad \text{electrical conductivity (mho/m)}
\end{align*} \]

1 Introduction

Despite the near flight-readiness of arcjet thrusters, the plasma flow in these devices is only beginning to be understood. Unlike its distant cousin, the Magnetoplasma-dynamic (MPD) thruster, the arcjet is basically an electrothermal device. The propellant (either monatomic or molecular) stream is heated by an electric arc and expanded through a converging-diverging nozzle to supersonic speeds. Although the device is simple in structure, the phenomena that govern flow in the device are quite complicated and are yet to deciphered. Among these is the question of whether or not injected swirl is necessary for effective operation, and whether this swirl plays any role in arcjet fluid dynamics in the first place. Swirl in the flow-field forms the central focus of this paper.

The compressible Navier-Stokes equations
govern the physics of flow fields in arcjets. These flows are often times further complicated by the effects of departure from thermodynamic equilibrium of the internal modes of molecular motion. This disequilibrium between internal modes (vibration, rotation and electronic excitation) and external modes i.e. translational molecular motions gives rise to complicated chemistry. Many numerical methods have been devised to solve the compressible, Navier-Stokes equations, the most popular of these being the MacCormack scheme [2], the Beam-Warming scheme [3] and the Linearized Block Implicit (LBI) scheme of Briley & McDonald [1]. Our ultimate aim is to study arcjet flows including the effects of finite-rate chemistry and internal mode disequilibrium. We have chosen the LBI method for this purpose. This method has the advantages of being able to handle large numbers of species as well as being extendable from quasi-1D through 2-D, to fully 3-D with relative ease.

Recently, several groups have begun numerical simulations of arcjet flows[4, 5, 6, 7, 8, 9, 10]. Perhaps the earliest of these simulations were those of Keefer et. al.[4] and Butler et. al.[6, 7]. Both groups have identified $\sigma$, the electrical conductivity, as the single most important property that governs arcjet flow. This is because $\sigma$ varies by orders of magnitude between regions in the arc and regions near the colder electrode boundaries. $\sigma$ determines the current distribution in the gas, and hence the ohmic heating. The Ohmic heating in turn determines temperatures and ionization fraction. The latter has a profound influence on other properties, most notably the viscosity[11] so that the device performance is inevitably altered. Furthermore, the distribution of $\sigma$ profoundly affects the heat addition not only in the subsonic portion of the flow but also in the supersonic portion. Although the latter is unavoidable, it is important to minimize heating the supersonic flow as specific impulse will suffer. In addition, no existing work has systematically examined the effects of injected swirl on the flow-field or conductivity distribution in arcjets.

In this paper, we systematically examine the effects of injected swirl using an axi-symmetric 2-D model of a nominal 30 KW arcjet. Both cold-flow and flow including the arc are examined. We find that the key to obtaining solutions to these flows is the proper modelling of the boundary conditions and specification of the proper initial conditions so that the flow can accelerate smoothly from subsonic to supersonic speeds.

This paper is organized as follows. The governing equations and boundary conditions are presented first, followed by the numerical method. As brief illustration of code verification, sample solutions comparing against experiments solutions are presented for a converging-diverging nozzle similar but not identical to the nominal 30 KW arcjet geometry. Cold-flow and 2-D axisymmetric arcjet solutions with and without swirl are discussed next, followed by a summary and conclusions of this work.

2 Formulation

The equations that govern the cold (i.e. non-reacting) viscous, supersonic flow are the compressible Navier-Stokes equations. For the arcjet, these equations become considerably more complicated by the presence of a body force, heat generation, species diffusion, chemical reaction, ionization and recombination, and variable properties. However in this paper, we concern ourselves only with a monatomic propellant (Argon) and defer consideration of molecular propellants in a companion paper[12]. We further neglect species diffusion terms, assume quasi-neutrality, and take the electron and heavy-particle temperatures to be equal.
While significant departures from the latter are known to occur in the colder regions of the flow, we are mainly interested here in the flow away from the electrode-adjacent regions. The governing equations in cylindrical polar coordinates form are:

\[
\frac{\partial p}{\partial t} + \frac{1}{r} \frac{\partial (rp)}{\partial r} + \frac{\partial (pw)}{\partial z} = 0, \tag{1}
\]

\[
\frac{\partial (pu)}{\partial t} + \frac{1}{r} \frac{\partial (rp^2u)}{\partial r} + \frac{\partial (puw)}{\partial z} = -\frac{\partial p}{\partial r} + \mu \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right] \tag{2}
\]

\[
\frac{\partial (pv)}{\partial t} + \frac{1}{r} \frac{\partial (rpv)}{\partial r} + \frac{\partial (pvw)}{\partial z} = \mu \left[ \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right] + \frac{\partial \mu}{\partial z} \left[ \left( \frac{\partial u}{\partial r} - \frac{u}{r} \right) + \frac{\partial \mu}{\partial z} \frac{\partial v}{\partial z} \right], \tag{3}
\]

\[
\frac{\partial (pw)}{\partial t} + \frac{1}{r} \frac{\partial (rpw)}{\partial r} + \frac{\partial (pw^2)}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left[ \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right] \tag{4}
\]

\[
+ \frac{\partial \mu}{\partial z} \left[ \left( \frac{2}{3} \frac{\partial u}{\partial r} - \frac{2}{3} \frac{u}{r} + \frac{4}{3} \frac{\partial w}{\partial z} \right) + j_z B_\theta \right],
\]

\[
\frac{\partial}{\partial t} \left[ e + p + \frac{u^2 + v^2 + w^2}{2} \right] + \frac{\partial}{\partial r} \left[ e + p + \frac{u^2 + v^2 + w^2}{2} \right] = 1 - \frac{\partial}{\partial r} \left( k \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) \right) \tag{5}
\]

where, \( \Phi \) is the dissipation function given by,

\[
\Phi = 2 \left[ \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{u}{r} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial v}{\partial r} \right)^2 + \left( \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} \right)^2,
\]

and \( e \) is the internal energy per unit volume given by:

\[
e = \frac{3}{2} \left( 2n_e + n_a \right) k_B T + n_e \varepsilon_i. \tag{6}
\]

and the \( p \) is the pressure given by:

\[
p = \left( 2n_e + n_a \right) k_B T. \tag{7}
\]

In the above equations,

\[
\nabla \cdot \vec{A} = \frac{\partial }{\partial r} \left( r \vec{A}_r \right) + \vec{e}_z \frac{\partial A_z}{\partial z},
\]

and

\[
\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2},
\]

where, \( \vec{A} = A_r \vec{e}_r + A_\theta \vec{e}_\theta + A_z \vec{e}_z \) is any vector and \( \phi \) is a scalar.

In addition, we have the following species rate and magnetic diffusion equations:

\[
\frac{\partial n_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( n_e u \right) + \frac{\partial}{\partial z} \left( n_e w \right) = k_f n_e n_a - k_r n_e^3 \tag{8}
\]

\[
+ \mu \Phi + \frac{j_z^2 + j_z^2}{\sigma} + j_r w B_\theta - j_z u B_\theta,
\]

where, \( \Phi \) is the dissipation function given by,
Prescribing boundary conditions at the inlet and exit planes of the nozzle depends on whether the flow is subsonic or supersonic. All the solutions presented here consider subsonic inlets. Hence, following Briley & McDonald [1], the stagnation temperature and stagnation pressure at the inlet are specified. In addition, at the inlet, the radial component of velocity is set to zero and the swirl component of velocity is either zero or set to some prescribed distribution corresponding to injection. It is important to note here that the axial component of velocity which determines the mass flow rate through the nozzle, is not specified at the inlet but is determined from an implicit extrapolation of the interior values, as is $n_e$. In addition, the mass density at inlet is determined by using the isentropic relation between the stagnation and static pressures and temperatures there.

Along the exit plane, the flow is subsonic-supersonic. All the variables (including $n_e$) except mass density on the exit plane are evaluated by implicit extrapolation from the interior [1]. If the local Mach number $M$ defined as $w/a$, where $a^2 = \frac{5}{3}(1 + \frac{n_e}{n_a+n_e})k_BT/m_a$ is the frozen speed of sound is greater than one at a point on the exit plane, then the mass density is also evaluated by extrapolation. If, on the other hand, $M < 1$ at some point on the exit plane, then the exit static pressure is specified and the mass density is evaluated from the equation of state. The correct value of the exit static pressure would usually not be known, unless the plume is modelled as well. Care must be exercised in choosing a value for the exit static pressure, for, if it is too high, the boundary layer on the wall in the diverging section of the nozzle can separate. This is an important detail that often escapes mention in the literature.

The boundary conditions on $\psi$ could use improvement. In this work, we prescribe and hold $\psi$ fixed at a certain value based on the total current [4] on the anode wall from the inlet until the exit to the constrictor. Beyond this point,
\( \psi \) is varied linearly along the anode wall to zero at the exit. \( \psi \) is taken to be identically zero at the exit plane and at the centerline (i.e. \( r=0 \)). Along the inlet plane, \( \psi \) is varied linearly from its value on the anode wall to 0.99 times that value, and maintained at that value along the straight portion of the cathode as well as on the sloping face of the cathode until 5 grid points from the tip. For these 5 grid points near the cathode tip, \( \psi \) is decreased linearly from 0.99 times the maximum value to zero in order to force most of the current contours to approach the cathode axially.

2.2 Initial Conditions

Due to the wave nature of the governing equations, initial conditions must be prescribed with care. In general, arbitrary initial values for the dependent variables will not yield stable solutions for any numerical method. Furthermore, since it is impossible to simulate the breakdown process that initiates the discharge \( n_e \) cannot be expected to start from 0 (i.e. completely non-ionized state). In this work, we prescribe an upstream stagnation temperature (also equal to the reference temperature used for non-dimensionalization) of 10,000 K, an upstream stagnation pressure (which depends on the desired mass flow rate), \( u = 0, v = 0 \) (except at the inlet, if swirl is injected), \( w = 0.1a_0 \) where \( a_0 \) is the frozen speed of sound evaluated at the inlet stagnation temperature, and \( n_e = 0.1n_{ref} \) where \( n_{ref} \) is computed from the equations (except for the magnetic diffusion equation) are solved using odd-even point SOR with the relaxation factor \( \omega \) set to 1.0. Thus, the \( \psi \) field is solved for independently at each time step. The other coupled, linear algebraic equations resulting from

3 Numerical Method

Equations (1)-(5),(8) are solved a non-staggered non-uniform grid. This grid in the physical domain is transformed into a uniform grid in a computational domain using an analytical coordinate transformation. The transformation is such that the nozzle contour is transformed into a rectangle in the computational domain [14, section 5-6.1]. The governing equations together with boundary conditions are transformed to the computational domain and solved there.

The transformed forms of the governing equations (except for the magnetic diffusion equation) are solved using the LBI scheme of Briley & McDonald [1]. Since all the details of the scheme have already been reported elsewhere [1], they will not be given here. At each time step in the time-marching sequence, the magnetic diffusion equation is solved using odd-even point SOR with the relaxation factor \( \omega \) set to 1.0. Thus, the \( \psi \) field is solved for independently at each time step. The other coupled, linear algebraic equations resulting from
the other governing equations are then solved by using the Douglas-Gunn ADI method. The ADI method itself involves the solution of block tridiagonal systems in each coordinate direction, with block size equal to the number of dependent variables, which, in our case is eight. These systems can be solved using existing efficient LU decomposition methods [15]. Following Briley & McDonald [1], we add dissipation in the form a second derivative in the axial coordinate to the unsteady governing equations.

4 Cold Flow

4.1 C-D Nozzle

In this section, we present viscous 2-D axisymmetric solutions for flow in a C-D nozzle. These are for cold-flow (i.e. non-reacting and non-ionizing) of air ($\gamma = 1.4, \mu = 18 \times 10^{-6} \text{ Ns/m}^2, Pr = 0.701$) and are presented for the purposes of code verification. The nozzle geometry is identical to the geometry of the 30 kW Arcjet thruster (shown in Fig. 1) but without the inlet plenum and the constrictor. The transition from the converging to the diverging section is achieved within five grid points through the use of a quadratically varying profile for the outer radius. The exit to throat area ratio for this nozzle geometry is 25. Solutions presented here are obtained on a non-uniform grid with 256 points in the axial direction and 64 points in the radial direction. The most interesting feature of this flow is the presence of weak compression waves in the diverging section. Profiles of Mach number along the radial grid lines $j = 0$ and $j = 38$ are shown in Fig. 2 together with the Mach number profile from the corresponding frictionless, quasi 1-D solution. Here, the grid line corresponding to $j = 0$ represents the centerline. The profile along the $j = 38$ grid line is closest to the quasi 1-D profile. The maximum exit Mach number in the viscous case is about 4.6, which is less than the value of about 5.0 for the frictionless, quasi 1-D case. The profile along the centerline shows a sharp drop in the Mach number near a dimensionless axial coordinate of 8. One can see from the Mach number contours in Fig. 3, a change in the shapes of these contours near the same axial location ($z = 8$). In this figure, the sonic line ($M=1$) is shown as the thick line. The radial Mach number profiles have their maxima located on the centerline up until $z = 8$ and these maxima move to a location near the center of the channel once we cross this particular axial location. These contours look very similar to those obtained using the method of characteristics [16] for inviscid flow in a similar geometry. They also agree very well qualitatively with the experimental results presented in [17]. It was first shown by Darwell & Badham [16] that compression waves could be generated in C-D nozzle geometries from the location where the curved throat section meets the diverging conical section, if the radius of curvature of the nozzle wall profile is discontinuous at this location. These compression waves can coalesce at the centerline at a point downstream to form a weak oblique shock. Experiments conducted subsequently by Back et al. [18, 19] and Cuffel et al. [17] showed this to be indeed true. The sharp drop in the Mach number along the centerline in Fig. 2 is due the oblique shocks intersecting the centerline and the change in the shapes of the Mach number contours mentioned before is due to these incoming and reflected weak oblique shock waves. Further evidence of these waves can be seen in Fig. 4, where a plot of the radial velocity along the grid line $j = 8$. The radial velocity is negative in the converging section and then it rapidly turns around at the throat becoming positive in the diverging section. The radial velocity then decreases sharply in the neighbourhood of $z = 8$ and becomes negative as the flow traverses the incident oblique shock waves. The fluid is turned in a clockwise direction towards the centerline by
these waves, since they are right running. The radial velocity then increases just as rapidly, as the flow traverses the oblique shocks that are reflected from the centerline. These turn the fluid in an anti-clockwise direction away from the centerline, being left running waves. The radial velocity settles down to a constant value after about \( z = 14 \). The overturning of the fluid as it goes through the throat is evident from this figure when one compares the value of the radial velocity near \( z \approx 7 \) with the final value for \( z > 14 \).

The exact mechanism by which these compression waves are generated is discussed in detail in [16, 19]. As the flow passes through the throat, the regions of the flow close to the wall are overturned so as to be parallel to the wall. Consequently, when the location where the discontinuous change in wall curvature occurs is reached, the flow must be turned back towards the centerline, i.e. away from the wall. This can only occur through a series of compression waves.

It is likely that such compression waves are generated in the constrictor region of the arcjet. Depending on the physical shape of the arc, it is possible that flow turning such as the type just discussed occur at the constrictor exit. Another way this effect can happen in arcjets is via actual changes in the contour of the nozzle (anode) wall due to electrode erosion. The amount of erosion needed to trigger such compression waves is rather small. Very subtle changes in the geometry can trigger these compression waves. When a discharge is first struck, the arc attaches either inside the constrictor or just at the exit of the constrictor. As the supersonic flow becomes established and steady state conditions are reached, the arc attachment on the anode shifts in the downstream direction. This shifting of the anode arc attachment is in reality accompanied by erosion. Generation of such compression waves with possible coalescence downstream into intersecting weak oblique shocks has very important consequences for any experimental measurements that are made at the exit plane.

### 4.2 30 kW Arcjet Geometry

Cold flow calculations similar to those described in the previous section, have been performed for the 30 kW arcjet geometry. No evidence of compression waves is seen in this geometry for the case of cold flow. However, the introduction of swirl at the inlet to the thruster leads to some interesting results. The profile of the inlet swirl velocity that we use is such that it is peaked very close to the anode wall, typically within 5 grid points (see Fig. 5). This type of swirl will be observed, for example, if the fluid were injected through ports peripheral to the anode wall. Shown in Figs. 6 and 7 are the contours of swirl velocities for mass flow rates of the order of a few grams per second and a few milligrams per second respectively. It can be seen that the introduced swirl is dissipated in the former while it is enhanced to values higher than the inlet values for the latter case. We find that for the swirl to enhance in the case of higher mass flow rates, the plenum should be shorter.

The phenomena governing the enhancement or decay of swirl can be explained from simple considerations of conservation of angular momentum. Injection of swirl at the inlet represents an influx of angular momentum. Thus for any station downstream, the eflux of angular momentum will equal the influx minus the total torque due to viscous forces. If this torque is sufficient to overcome the influx of angular momentum, then the injected swirl decays in the upstream plenum region itself. However, if this torque is smaller than the influx of angular-momentum, then the swirl will decrease but can persist until the beginning of the converging section. Once this converging section is reached, the decreasing area can if the condi-
tions are right, lead to an increase in the swirl component of the velocity from overall conservation of angular momentum. The total torque is composed of two quantities. The first is the torque due to the wall shear stresses (at anode and cathode), and the second is the torque due to the shear stress integrated across the flow volume between cathode and anode. Of these, the torque due to the wall shear at the anode is the most important.

The critical parameter which determines whether or not the swirl persists and is amplified, is the propellant mass flow rate. At higher mass flow rates (and hence higher Reynolds numbers), the boundary layers are closer to the walls so that the wall shear stresses are high. Consequently, the torque due to viscous forces can overcome the influx of angular momentum and the swirl will perish long before it reaches the cathode tip. In contrast, at the lower mass flow rates (or lower Reynolds numbers), the boundary layers are thicker so that wall shear stresses are smaller leading to reduced torque due to viscous forces. Therefore, regardless of the magnitude of the viscosity there is some limiting mass flow rate beyond which the injected swirl will decay. Obviously, the higher the viscosity the smaller the limiting mass flow rate. Thus, this mechanism for swirl enhancement can indeed be operative at higher temperatures. The mass flow rate and inlet plenum length are thus key design parameters.

5 Arcjet Flow

In this section, we present results for a 30 kW arcjet geometry, current of 50 A, argon mass flow rate of 100 mg/s. We wish to illustrate the solutions to the one temperature model described in sections 2 and 3. We emphasize at the outset that no “limits” or “floors” are applied to the electrical conductivity as in [7, 6]. The inlet stagnation temperature is 10000 K and the inlet stagnation pressure is $10^4$ Pa. The initial current distribution is as shown in Fig. 8. A non-dimensional time step of $10^{-3}$ (dimensional value is $5 \times 10^{-5}$) is used and convergence to atleast three decimal places is achieved after 2500 time steps. The CPU time on the Cray Y-MP was 10 minutes.

Two cases were considered, one without swirl and one with the inlet swirl profile as shown in Fig. 5. Figs. 9 through 18, summarize the results obtained after convergence. It can be seen from Fig. 9 that the current contours have shifted downstream compared to the initially prescribed contours. This shift is caused by the varying conductivity as the supersonic flow becomes established. This reflects reality quite well as when the arc is first struck, it is usallt just downstream of the constrictor exit on the anode. As the steady supersonic conditions are reached, the anode attachment moves downstream. The density of current contours near $r=0$ suggests that the current densities and hence ohmic heating are highest in that region. As expected the temperature and electron number density contours follow this very well as can be seen from Figs. 11 and 12. The calculated temperatures upstream of the cathode tip are in our opinion unrealistically high. This is an inevitable consequence of having to prescribe a high inlet stagnation temperature in order to enable the gas to ionize. This is an artifact of our simulation and caused by the non-conservative treatment of the ohmic heating term in the energy equation. We are at present pursuing a conservative treatment of the ohmic heating term which should yield much lower temperatures in the inlet plenum section. The maximum temperature is 33000 K and occurs just downstream of the cathode as can be seen from Fig. 11.

Close examination of the Mach contours in Fig. 10 and the Mach profiles in Fig. 14, reveal that the flow reached supersonic speeds in the nozzle and becomes subsonic by the time...
the exit plane is reached. Heat addition to a supersonic flow is well known to slow it down. Examination of Figs. 10 and 14 shows some similarity to the cold flow solutions obtained for the C-D nozzle (see Figs. 2 and 3). The successive bumps in the Mach profiles of Fig. 14 suggest the generation of weak compression waves which coalesce at the centerline forming an oblique shock. This complex series of oblique shocks then reflect and exit the domain via the exit plane. The generation of such compression waves is due to the fact that once supersonic, the flow at the exit of the constrictor and beyond is forced to turn direction due to variations in ohmic heating and transport properties. Turning of supersonic flows as is well known is always accompanied by generation of oblique shocks or expansion fans.

Contours of pressure are shown in Fig. 13 and exhibit a decrease only until approximately midway in the nozzle. Although not shown in the figure, the pressure exhibits an increase near the exit plane indicating the passage of the flow through an oblique shock. Note the differences in the contours due different boundary conditions used here vs. [5]. Fig. 16 shows voltage as a function of streamwise distance. The power expended in the discharge is approximately 1 kW. Fig. 17 shows contours of swirl velocity indicating that the swirl decays and fails to survive for this case. The swirl is attenuated because the torque due to the shear stress at the anode is large. This is an artifact of our abnormally high inlet values of static temperature (approximately 10000 K) and ionization fractions (approximately 0.01). Elimination of the inlet plenum does not result in amplification of the swirl as can be seen in Fig. 18. Again this is due to the high temperatures and ionization levels encountered by the flow before the cathode tip. We must emphasize that we do not believe this to be a true representation of the arcjet. Had we treated the inlet boundary conditions accounting for ohmic heating, the temperature and ionization level would be far lower, thus decreasing the viscosity by orders of magnitude below the level for this case.

As far as the effects of swirl are concerned, the present cold flow results appear to simulate real arcjet flows better. Amplification of swirl near the cathode tip would be beneficial in providing much needed cooling to confine and maintain the arc spot attachment area.

6 Summary & Conclusions

Both cold flow and ionizing flow in an arcjet thruster have been successfully simulated. These simulations provide insight as to the effects of boundary conditions on the results and serve to identify the behavior of the injected swirl at the inlet. It is clear that swirl injection (both magnitude and location of the maximum) is intimately tied to two key design parameters: the operating mass flow rate and the distance from the inlet plenum to the beginning of the converging section.

Improvements are certainly in adequately specifying the inlet boundary conditions, and conservative treatment of the ohmic heating certainly would be one. A two-temperature model along the lines of Miller et al. [5] could be an improvement for arcjets operating on monatomic propellants or hydrogen. However, the two-temperature treatment is suspect in the case of arcjets operating on ammonia or hydrazine due to the presence of large amounts of $N_2$ and other diatomic species [12]. Further work incorporating non-Maxwellian distributions of the EEDF would be essential for ammonia and hydrazine arcjets.

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References


Fig. 1: Schematic of the nominal 30-kW anode geometry. The C-D nozzle is the same as this geometry, but without the inlet plenum or the conductor.

Fig. 2: Mach number is shown here for j=0 (centerline), and j=38 (near the anode) versus the streamwise coordinate.
**Fig. 3:** Mach contours for the case of the high mass flow rate (g/s) in the C-D nozzle.

**Fig. 4:** Profile of the radial component of velocity versus the streamwise coordinate is shown here at $j=8$ (8 grid points above the centerline). Note the dramatic flow turning.
Fig. 5: Profile of the swirl (azimuthal) component of velocity introduced at the inlet.

Fig. 6: Contours of the swirl component of velocity for the 30 KW arcjet geometry for high mass flow rates (g/s). Note the rapid decay of the injected swirl.
Fig. 7: Contours of the swirl component of velocity for the 30 kW arcjet geometry for low mass flow rates (mg/s). Note the persistence of the swirl through the entire flow field, and its amplification in the constrictor region. Note also the presence of swirl at the exit plane.

Fig. 8: Contours of current initially prescribed.
Fig. 9: Contours of current after convergence. Note the significant effect of electrical conductivity, especially in shifting the current distribution further downstream along the anode, as well as in concentrating the current along the centerline in the constrictor. This case corresponds to Ar, 50 A, and a mass flow rate of ~100 mg/s.

Fig. 10: Contours of Mach number for case of Ar, 50 A, and ~100 mg/s. The highest Mach contour is the bubble in the nozzle section, with a value of 1.4.
Fig. 13: Contours of pressure normalized by $1.0 \times 10^4$ Pa. All the contours shown in this figure represent decreasing pressures in the streamwise direction.

Fig. 14: Profile of Mach number versus streamwise coordinate at various radial stations ($j$).
Fig. 15: Profile of temperature (normalized) versus streamwise coordinate at various radial stations ($i$).

Fig. 16: Profile of voltage versus streamwise coordinate (normalized by $L=0.01$ m). This profile was computed by integrating the electric field from the cathode tip, along the centerline to various locations along the anode.
Fig. 12: Contours of swirl (normalized) for Ar at 50 A, and -100 mg/s for the inlet swirl profile shown in Fig. 5. This is the case of the 30 kW arcjet geometry but with no inlet plenum.

Fig. 13: Contours of swirl (the swirl component of velocity is normalized by 1861 m/s) for Ar at 50 A, and -100 mg/s for the inlet swirl profile shown in Fig.