THEORETICAL MODELING OF MAGNETOPLASMA DYNAMIC ARCJETS

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ABSTRACT

Theoretical modeling of both self field and applied field magnetoplasmadynamic (MPD) thrusters is performed by utilizing the MACH2 code, a time dependent 2½-dimensional simulation tool. MACH2 is a single fluid, multi-temperature, nonideal radiation, magnetohydrodynamics code, which utilizes the SESAME Equation-of-State Library for standard thermodynamic quantities: fractional ionization state, and transport coefficients in tabular formats. Phenomenological models for anomalous transport coefficients based on plasma microinstabilities are also used, as local plasma discharge conditions require. The original MACH2 code has been modified for the present studies to add the full viscous stress tensor, and self-consistent, in-plane magnetic field boundary conditions for steady state. Several experimental data obtained from a variety of MPD thruster geometries are compared with MACH2 simulations. These are the Princeton University quasi-steady self-field extended and half-scale flared-anode thrusters, and the NASA Lewis Research Center steady state applied-field MPD thruster. Comparisons of thrust for the self-field thruster show excellent agreement even at operating conditions beyond onset. MACH2 voltage predictions are offset from the experimental values indicating the effects of electrode fall voltages not included in the MHD calculations. MACH2 predictions of the enclosed current contours for the flared-anode thruster are quite comparable with the experimental ones. The applied field simulations confirm the linear dependence of specific impulse and voltage on the applied magnetic field strength.

I. INTRODUCTION

Magnetoplasmadynamic (MPD) thrusters are potentially useful electric propulsion devices for a multiplicity of space applications. Research over the last thirty years, however, has proceeded in a largely empirical fashion. Such empirical study is only able to address the particular geometries, operating conditions, and protocols of a very limited number of thrusters. It is generally unsuitable for evaluation of the true limiting values of MPD performance (e.g., efficiency).

The complicated physical interactions that take place in these devices make analytical progress in modeling the complete flow extraordinarily difficult. Consequently, a large number of theoretical investigations have concentrated in isolating specific physical processes. Only recently has numerical modeling of MPD thruster flowfields become a subject of significant activity.

LaPointe6,7 has applied a steady state, two-dimensional, axisymmetric, single fluid, two temperature model to a variety of geometric configurations and discharge conditions. The plasma is assumed fully ionized. Thrust has been successfully predicted by the model and voltage trends have been reproduced for discharge currents below the experimentally measured onset values. The latter however, underestimate the experimentally measured voltage since the model excludes electrode effects. Calculated enclosed current contours do not resemble the experimentally observed ones; this is maybe due to exclusion of electrode effects and the fact that a steady state code was used to model quasi-steady onsets.
thrusters. A time dependent, two-dimensional, axisymmetric, single fluid, two temperature model has also been developed at the University of Stuttgart to study a cylindrical self-field MPD thruster. Similar trends are evident, but the calculated voltage underpredicts the experiment, and current distributions between the model and the experiment are not comparable. Other attempts to examine applied-field MPD thrusters have been limited to uniform axial magnetic fields and higher mass flow rate values than would be appropriate for space applications.

This paper presents comparisons of simulations performed with the MACH2 code, with experimental results for both self-field and applied-field thrusters on route to improving insight into MPD thruster operation, and proposing new experimental approaches.

II. THE MACH2 CODE

MACH2 is a time dependent, two-dimensional axisymmetric, single fluid, multi-temperature, non-ideal radiation, magnetohydrodynamics (MHD) code that has been used to model a variety of laboratory plasma experiments. These include z-pinches and plasma liner implosions for high energy radiation production and x-ray laser applications; laser target interactions for inertial confinement fusion and high altitude nuclear effect simulations; compact toroids; solid liner implosions; ion diodes: rail gun plasma armatures; and deflagration-mode plasma accelerators. In all cases the use of the code has allowed better understanding of the physical phenomena involved, and had great success in developing ways to improve experimental performance.

Axisymmetry implies that the code includes spatial variation in two dimensions. All three components of velocity and magnetic field, are computed however, as functions of the two spatial coordinates. The code provides computation in either planar or cylindrical geometry and can manage almost any geometric configuration without any code modification. This broad geometric class of domains is handled by division of the physical domain into appropriate block-like regions which in turn are transformed into logical rectangular blocks that make up the computational domain.

Initial conditions require the input of mass density, temperature(s), and applied magnetic field for each physical block. A variety of distributions is available for each of the variables without code modification. The code computes the evolution of the variables and presents the results in numerical output, a set of vector and contour plots, a set of slice plots, and a set of movies as desired.

MHD EQUATIONS

The single fluid MHD equations used in MACH2 include the continuity equation, the momentum equation in three vector form, a set of energy equations, and the magnetic field transport equation. (Summation is indicated by repeated indices.)

Mass continuity:

\[ \frac{\partial \rho}{\partial t} = \nabla \cdot \left( \rho \mathbf{v} \right) \quad (1) \]

represents conservation of mass, where \( \rho \) is the mass density and \( \mathbf{v} \) the velocity.

Momentum:

\[ \rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla P + \rho \mathbf{v} \times \mathbf{B} + \mathbf{Q} - \mathbf{\tau} \quad (2) \]

represents conservation of momentum, where \( P \) is the pressure, \( Q \) is numerical viscosity, \( \sigma \) is the stress tensor, \( \mathbf{B} \) is the magnetic induction. Surface forces consist of pressure and viscous forces, the latter represented by the divergence of the stress tensor can have two different representations in MACH2, depending on the problem at hand:

Stress Tensor:

\[ \text{Elastic:} \quad \frac{\partial \sigma_{ij}}{\partial t} = 2G \delta_{ij} - \mathbf{v} \cdot \nabla \sigma_{ij} \]

\[ \text{Viscous:} \quad \sigma_{ij} = \mu \left( \mathbf{v}_{ij} \frac{\partial \mathbf{v}}{\partial t} - \frac{2}{3} \delta_{ij} \mathbf{v} \cdot \nabla \right) \quad (3) \]

where \( G \) is the shear modulus, \( \mu \) is the viscosity coefficient, and \( \delta \) is the Kronecker delta. The elastic model allows the user to handle material strength, while the viscous isotropic fluid model transforms the code into a Navier-Stokes equation solver. The body forces in the momentum equation are the electromagnetic forces due to the motion of free charge; (displacement current and forces due to polarization and magnetization of the plasma are assumed negligible).
Electron Specific Internal Energy:

\[
\frac{\partial \epsilon_e}{\partial t} = -\rho \nu \cdot \nabla \epsilon_e - P_e g \frac{\partial}{\partial \nu} \nabla_{\nu} + \eta \left( \nabla \times \mathbf{B} \right)^2 - \rho \frac{\partial}{\partial t}
\]

where the rate of change of electron specific internal energy is balanced by the flow work of the electron gas, Joule heating due to diffusion of magnetic field, diffusive transport of energy, radiation cooling, and energy exchange due to collisions with the ions as scaled by the classical equipartition time. \(^{12}\)

Ion Specific Internal Energy:

\[
\rho \frac{\partial \epsilon_i}{\partial t} = -\rho \nu \cdot \nabla \epsilon_i + (P_i + Q) g \frac{\partial}{\partial \nu} \nabla_{\nu} + \eta \left( \nabla \times \mathbf{B} \right)^2 + \frac{\partial}{\partial t}
\]

where the rate of change of ion specific internal energy is equal to the contributions from ion flow work, heat conduction, and the rate of thermal equilibration from collisions with the electrons. In equations 4 and 5, \(\epsilon\) is the specific internal energy, \(\eta\) is the electrical resistivity, \(\chi\) is the thermal conductivity, \(T\) is the temperature, \(\alpha\) is the Stefan's constant, \(\chi\) is the mean opacity, and \(c\) is the speed of light.

Magnetic Induction:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta \nabla \mathbf{B}) - \nabla \times \frac{1}{\epsilon \mu_0 n_e} (\nabla \times \mathbf{B} \times \mathbf{B})
\]

is derived from Maxwell's equations and a generalized Ohm's law. \(^{13}\) The rate of change of the magnetic induction is balanced by contributions due to convection and diffusion of the magnetic field and the Hall effect where \(n_e\) is the electron number density \(e\) is the electron charge. Along with the MHD approximation (displacement current is negligible when compared to the conduction current), the above equation assumes that ion slip and contributions from the electron pressure gradients are negligible.

The set of equations is closed with the addition of an equation of state and a caloric equation of state that prescribe the species' pressure and specific internal energy in terms of the species' number densities and temperatures. They can be either analytical or tabular, the analytical equations include the ideal gas model and a Grüneisen model \(^{14}\) (appropriate for solid materials). The tabular model \(^{15}\) in the code is the SESAME Equation-of-State Library \(^{16}\) generated and maintained by the T-4 group of the Theoretical Division at Los Alamos National Laboratory. Its data base includes the standard thermodynamic quantities, along with the fractional ionization-state based on local thermodynamic equilibrium (LTE) and transport coefficients. The tabular look-up aspect of the code is easily adapted to input tabular equation of state models supplied by the user.

**TRANSPORT COEFFICIENTS**

Relations for the transport properties can also be tabular or analytical. Tabular models in the SESAME Library are available for thermal and electrical conductivities and allow temporal and spatial variations.

The viscosity in the present calculations is based on a curve fit to graphical results \(^{17}\) (and may tend to overestimate the viscosity at the lower pressures within the thrusters).

The classical Spitzer-Härm \(^{12}\) models are implemented for the ion and electron thermal conductivities and include the components parallel and perpendicular to the magnetic field. They are given below (in MACH2 units; \(\text{J/m-sec-eV}\)):

\[
\begin{align*}
\kappa_{\|} &= \frac{3.103 \times 10^4 \left( 4.1 - \frac{15.5}{\text{lnA}} \right) \text{Tf}^{5/2}}{	ext{A}} \\
\kappa_{\perp} &= \frac{1.274 \times 10^3 \text{Tf}^{5/2}}{	ext{A}^{1/2} \text{InA}^{3/2}} \\
\kappa_{\text{e}} &= \frac{4.408 \times 10^{12} \left( 0.687 + \frac{0.3756}{0.2 + \text{lnA}} \right)}{\text{A}^{1/2} \text{B}^{2/3} \text{Tf}^{1/2}} \\
\kappa_{\text{J}} &= \frac{5.812 \times 10^{13} \text{r}^2 \text{lnA} \text{r}^2}{\text{A}^{3/2} \text{B}^{2/3} \text{Tf}^{1/2}}
\end{align*}
\]

where \(\zeta\) is the free electrons per ion, \(A\) is the atomic weight, and \(\text{lnA}\) is the Coulomb logarithm \(^{12}\).

The electrical resistivity models simulate both classical particle-particle interactions and anomalous particle-turbulent fields interactions. The classical Spitzer-Härm \(^{18}\) model used in MACH2 considers the tensor nature of the differing resistivities due to currents parallel and perpendicular to the magnetic field. It is given below in MACH2 units; \(\text{m}^2/\text{sec} (\text{mks units divided by } \mu_0)\):
where $m_e$ is the electron mass, $k$ the Boltzmann’s constant, $n_0$ the neutral number density, and $\sigma$ is a user-specified scattering cross-section. The two terms on the right hand side of the perpendicular component expression represent the contributions from electron-ion and electron-neutral collisions. There are three anomalous resistivity models available at present in MACH2. These add to the classical values in equation 8. They are (in m^2/sec),

$$\eta_{\omega} = 8.1669 \times 10^{-5} \left( \frac{A}{\rho \zeta} \right)$$
$$\eta_{ch} = \frac{m_e}{e \mu_0} \left( \frac{0.7 A}{\rho \zeta} \right) \left( 1 - e^{-w_0/v_w} \right)$$
$$\eta_{\text{fed}} = \eta_{\text{ch}} \left( 1 + 0.3 \frac{v_d}{v_w} \right) \left( \frac{B^2}{B^2 + 6.154 \times 10^{-7} \rho /A} \right)$$

where $v_d$ is the electron drift speed and $v_a$ is the ion acoustic speed. The first model is proportional to the electron plasma frequency and has been used for several PIC simulations. The Chodura model is based on the saturated value of the ion acoustic speed, while the last model is based on the Lower-Hybrid Drift microinstability and enhances the previous model by directly considering magnetic field effects.

**BOUNDARY CONDITIONS**

Application of the boundary conditions in MACH2 is carried out in a unique fashion. The premise is that the conditions at a physical boundary are the limit of those in the fluid, thus they are related by, at most, simple instantaneous geometric expressions. This is accomplished by the ghost-cell technique, discussed in more detail later.

Magnetic field boundary conditions include appropriate expressions for idealized conductor and insulator surfaces, and the axis of cylindrical symmetry. The normal component of the magnetic field and the tangential component of the electric field must be continuous at the surface of a perfect conductor, which implies:

$$\hat{n} \cdot (\vec{B} - \vec{B}_c) = 0$$
$$\hat{n} \times \vec{E} = 0$$

where $\hat{n}$ is the unit vector normal to the surface, $B_c$ represents any field that may be embedded in the conductor during problems of longer time scale, and $E$ is the electric field. Eliminating $E$ from the second expression in (12) one obtains a toroidal current and magnetic field boundary condition,

$$j_b = 0, \quad \hat{n} \cdot \nabla (r B_b) = 0.$$ 

For steady state problems the poloidal magnetic filed boundary conditions have to allow evolution in response to azimuthal currents generated and sustained by the plasma in addition to any other applied fields generated by external coils. This is accomplished (in the version of MACH2 at the Ohio State University) by computing the poloidal field components at the boundaries of the computational region via the Biot-Savart law. Individual cells generated by the computational grid within the plasma are assumed to act as circular, current-carrying filaments that contribute to the total poloidal field at the calculational boundaries. The individual windings of the external field coil add their contributions in the same way. The magnetic field generated by a single current-carrying filament is given by

$$B_z(r,z) = \frac{\mu_0 I}{2 \pi a} \frac{1}{\sqrt{(1+R^2)+Z^2}} \left[ \frac{K + \frac{1}{1-R^2-Z^2}}{(1-R^2+Z^2)} E \right]$$

where

$$K(c) = \int_0^{\pi/2} d\phi / \sqrt{1-c^2 \sin^2 \phi}$$
$$E(c) = \int_0^{\pi/2} \sqrt{1-c^2 \sin^2 \phi} d\phi$$
$$c = \sin^{-1} \frac{4R}{(1+R^2+Z^2)^2}$$

are elliptic integrals of the first and second kind respectively, $I$ is the current and $a$ is the radius of the filament and $R = r/a$ and $Z = z/a$.

MACH2 also allows modeling of a perfect insulator, where the normal component of current
disappears. This condition produces a toroidal magnetic field boundary condition,

$$rB_\theta = \frac{\mu_0 I}{2\pi} \tag{14}$$

where 1 is the total current flowing between the boundary and the axis of symmetry.

Hydrodynamic boundary conditions can be either free slip or no slip while thermal conduction boundary conditions can also be of two types: no heat flux or conduction to a fixed temperature.

Modeling of inlets requires input of the inlet temperature(s), density, and velocity, while at an outlet the conditions inside the boundary determine the state outside the boundary:

$$\hat{n} \cdot \nabla \epsilon = \hat{n} \cdot \nabla \rho = \hat{n} \cdot \nabla \mathbf{u} = 0. \tag{15}$$

The axis of cylindrical symmetry should reflect that the conditions just outside the problem region are the mirror image of those just inside of it. Thus,

$$B_\theta = B_r = j_\theta = v_r = u_\theta = 0 \tag{16}$$

at the centerline, $r = 0$.

**NUMERICAL SCHEME**

The physical model described in the previous sections is solved numerically by a time-split, time-marching algorithm. Time-splitting consists of the sequential application of separate portions of a system of equations, rather than the simultaneous application of the entire set. The thermal and equilibrium radiation diffusion, resistive diffusion, and the Lagrangian hydrodynamics are done with implicit time differencing while radiation cooling, coordinate system motion, convective transport, and the Hall effect are carried out with explicit differencing. MACH2 controls the time step such that the stability of the explicit differencing is maintained.

The finite volume approach is utilized for the spatial differencing formulas. The advantage of this scheme is that conservation laws involving the vector integral theorems are well respected by the differencing since that differencing is derived from those theorems.

The adaptive ideal coordinate system is generated by solving a variational problem on the block complex using a weight function selected from a standard family by the values of user specified parameters. Its mesh smoothness, desired concentration, and orthogonality can be easily manipulated via input parameters, thus allowing the user to dictate the mode of calculation for a given problem. The grid's subsequent motion utilizes an Arbitrary-Lagrangian-Eulerian (ALE) approach. This allows it to move in either Lagrangian, Eulerian, or arbitrary fashion with respect to the fluid. This method provides a means of avoiding negative aspects that can arise from purely Lagrangian (coordinate distortion) or purely Eulerian (numerical diffusion) calculations, while providing higher resolution in regions of physical interest and solution confirmation.

Application of the boundary conditions, and communication between neighboring blocks that make up the computational domain are carried out by the ghost-cell technique. Each side of a block's boundary is extended by one more row of cells, the ghost cells. Then the full difference equations may be applied to points on the region boundary just as to the interior points, since the region containing data is larger than the physical region. The principal advantage of this choice is that the boundary condition expressions need not be special boundary difference equations, and thus are simpler to derive and code.

**III. SELF-FIELD SIMULATIONS**

Two self-field MPD thrusters were simulated by MACH2 to compare with experimental results of thrust, voltage and current distribution. The Princeton University extended-anode thruster, the geometry of which is shown in Figure 1, provided a set of experimental data for thrust and voltage over a wide range of discharge currents. The Argon propellant is injected at a mass flow rate of 6 g/s through the backplate, with either all of it injected near the anode or half of it injected through an annulus near the cathode base and half through evenly spaced ports located 2 cm radially from the centerline. Onset of thruster instability was experimentally observed at 21 kA for the 50:50 propellant injection scheme.

MACH2 modeling of the thruster to date has assumed a uniform propellant injection at the backplate at 6 g/s instead of the 50:50 scheme. The computational region extends to 40 cm downstream of the backplate such that both discharge chamber and plume flows are modeled. The fluid is two-temperature, inviscid, with real gas equation of state. The no slip and adiabatic wall boundary conditions are imposed on the electrodes which are modeled as idealized conductors. The
downstream and side boundaries, the latter extending to 8 cm radially, are modeled as outlets. Transport coefficients are classical. Steady state is reached when overall variables, such as exhaust velocity and voltage, and more specific variables, such as temperatures, density, and velocity at the exit plane, display changes of less than 5%. Overlays of plots of the entire computational region at different times offer an additional confirmation of steady state.

Computation of exhaust velocity is performed on the downstream and side outlet boundaries as follows,

\[ U_{ex} = \frac{\sum_i \rho_i A_i v_{i,z}^2}{\sum_i \rho_i A_i v_{i,z}} \]  

where the subscript \( i \) denotes the cell number, \( A \) is the cell's area and \( v_{i,z} \) is the axial velocity. It then follows that thrust and specific impulse are given by,

\[ F = m U_{ex}, \quad I_{sp} = \frac{U_{ex}}{g} \]  

where \( m \) is the mass flow rate and \( g \) is the gravitational acceleration. Contributions to the thrust from the electromagnetic body force and pressure differences at the downstream and side boundaries are assumed negligible. Integration of the electric field along the inlet boundary provides the plasma voltage.

Comparisons of the experimental data with MACH2 predictions for thrust and voltage are shown in Figures 2 and 3 respectively. Predictions of thrust are in excellent agreement with the experiment even for discharge currents beyond the experimentally observed onset (21 kA). Voltage predictions exhibit the general experimental trend, however, since MACH2 does
not include electrode effects they are offset by what may be due to electrode fall voltage.

The half-scale flared-anode thruster was chosen for the second simulation by the code due to the existence of experimental enclosed current contours. A schematic of the thruster is shown in Figure 4. Argon propellant is injected in a 50:50 ratio at 3 g/s from an annulus at the cathode base and a ring of evenly spaced ports radially located at 1.5 cm from the centerline.

The case of 7.9 kA discharge current was simulated by MACH2. The propellant is again assumed to be uniformly injected through the backplate at a mass flow rate of 3 g/s. The fluid is two-temperature, viscous, with real gas equation of state. Classical electrical resistivity is enhanced by the anomalous Chodura model previously presented. Hydrodynamic boundary conditions are no slip, while the electrodes are assumed to be thermally conducting at a constant electron temperature of 0.5 eV. The computational region extends to 20 cm downstream of the backplate and 4.7 cm radially. These boundaries are modeled as outlets. MACH2 predictions of the enclosed current contours are depicted in Figure 5, and may be compared to the experimental distribution in Figure 6.

Figure 4. The geometry of the half-scale MPD thruster. Dimensions are in centimeters.

Figure 5. Predicted enclosed current contours for the half-scale flared anode thruster. A=86.7%, B=77.2%, C=67.7%, D=58.2%, E=48.7%, F=38%, G=28.5%, H=19%, and I=9.5%.

The predicted contours are quite comparable with

Figure 6. Experimental enclosed current contours for the half-scale flared anode thruster.

the experiment. The current density is lowest in
the flared section both in the calculation and the experiment. The comparisons give also an insight on the effect of the 50:50 mass injection scheme. Since there appears to be a discrepancy between experiment and theory of the current streamline near the backplate this may indicate the effect of the non-uniform injection scheme. This effect diminishes as we move downstream. The theory appears to have captured features such as the sloping of the current contours towards the exit anode corner, the degree of destention downstream of the exit plane, and the point of attachment at the anode face.

IV. APPLIED-FIELD SIMULATIONS

The NASA LeRC 4" diameter thruster\(^3\) was selected for applied-field MACH2 simulations due to the plethora of experimental data. The geometry is shown in Figure 7.

In the experiments for the present comparisons the thruster was operated with Argon propellant at 0.1 g/s and discharge current of 1000 A at varying applied magnetic field strengths. The propellant was injected through holes located near the midradius between anode and cathode and through an annulus at the cathode base. The external magnet coil consisted of 28 turns of 1.9 cm diameter tubing. The axial magnetic field strengths were measured at the cathode tip before injection of the propellant. The calibrated values yielded a slope of 8.48 x 10\(^{3}\) T/A as a function of the magnet current.

The MACH2 simulation assumed a mass injection of 0.1 g/s uniformly distributed over the insulating backplate. The discharge current was 1000 A. The fluid formulation was two-temperature, viscous, with real equation of state. No slip was assumed at the walls, and the cathode and anode were fixed at electron temperatures of 0.3 eV and 0.2 eV respectively. The Chodura anomalous resistivity model was used. The external applied field was modeled such that the user needs only to specify the magnet coil current and geometry in the input list. Three different cases of applied field strength were simulated; 0.03T, 0.04T, and 0.05T at the cathode tip (without plasma flow).

Steady state MACH2 predictions of specific impulse and voltage are compared to the experimental findings in Figures 8 and 9 respectively.

![Figure 7. The geometry of the NASA LeRC 4" diameter cylindrical thruster. Dimensions are in centimeters.](image)

![Figure 8. Specific Impulse Vs Applied Magnetic Field Strength (at cathode tip) for the NASA LeRC MPD thruster.](image)

Theoretical results confirm the monotonic increase of both specific impulse and voltage with increasing applied magnetic field strength. The linear dependence of voltage on magnetic field is contrary to the parabolic one expected with the Hall effect included. MACH2 predicts the experimentally observed slope by which specific impulse increases with applied field, however, the values are offset. This may be partly due to overestimated viscosity coefficient values at certain regions of low pressure that hinder plasma spinning. Discrepancies of the voltage predictions...
can be partly attributed to the exclusion of electrode effects in the code.

A basic description of the flowfield for the 0.05T case is presented in Figures 10 through 18.

![Figure 9. Voltage Vs Applied Magnetic Field Strength (at cathode tip) for the NASA LeRC MPD thruster.](image)

![Figure 11. Density distribution for the 0.05T applied field MACH2 simulation, (kg/m²).](image)

![Figure 12. Electron temperature distribution for the 0.05T applied field MACH2 simulation, (eV).](image)

The simulated flowfield depicts a region of very low density (two orders of magnitude less than the average values within the discharge chamber) and high temperatures off the cathode tip. The code does not include detailed modeling of flow interactions with electrodes (e.g., separation off the blunt end of the cathode) or particle models that may be more appropriate for the description of the low mass region. However, the questionable values predicted by the code in this region do not affect the basic mechanisms that produce the linear dependence of specific impulse and voltage on applied magnetic field strength. The enclosed current contours depict higher current density as we move upstream and no more than 10% of the current may be extending beyond the exit plane. This is partly due to inclusion of anomalous resistivity contributions. Previous simulations of the same case with classical electrical resistivity have shown excessive current destention downstream of the thruster's exit plane. The maximum resistivity ratio inside the thruster and at the anode face is 5.1 while the Hall parameter calculations (which include the anomalous resistivity values) yielded values that exceed 1 only at the cathode tip and the anode face.

Figure 17 shows the the magnitudes of plasma azimuthal velocity as a result of the
interaction of the radially applied current and the in-plane applied magnetic field. The kinetic energy

\[ T = 7.300E+04 \text{ CYCLE} = 64630 \]

**Figure 13.** Ion temperature distribution for the 0.05T applied field simulation, (eV).

\[ T = 7.300E+04 \text{ CYCLE} = 64630 \]

**Figure 14.** Enclosed current distribution for the 0.05T applied field MACH2 simulation, (A).

associated with this azimuthal motion is converted to exhaust kinetic energy and is one of the major thrust contributors.

Figure 18 displays the solenoidal magnetic field lines generated by the external magnet coil and the azimuthal plasma current as calculated by the Biot-Savart law.

**Figure 15.** Anomalous to classical electrical resistivity ratio for the 0.05T applied field MACH2 simulation.

**V. CONCLUSIONS**

Two self-field and an applied field MPD thruster geometries were simulated by the MHD computer code, MACH2. Self-field thrust comparisons of the theoretical predictions with experimental data have shown good agreement. The code's calculated enclosed current contours are comparable to the experimental ones reproducing most of the trends. Voltage predictions are offset suggesting the need for more detailed modeling that includes the effects of electrode fall voltages. Applied-field simulations of specific impulse and voltage have captured the monotonic increase with applied magnetic field strength observed in the experiments. The discrepancies however, support the need for more elaborate modeling.

As the modeling of the basic flowfield of both self-field and applied-field MPD thrusters continues, future goals will target electrode fall voltage modeling for appropriate calculation of thruster efficiency.

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Figure 16. Hall parameter distribution for the 0.05T applied field MACH2 simulation.

Figure 17. Azimuthal velocity distribution for the 0.05T applied field MACH2 simulation, (m/s).

Figure 18. Magnetic flux distribution for the 0.05T applied field MACH2 simulation, (webers).
REFERENCES


