CALCULATION OF A NOZZLE TYPE MPD THRUSTER AND COMPARISON WITH MEASUREMENTS

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Abstract

For several years experimental investigations with magnetoplasmodynamic (MPD) thrusters have been conducted at the Institute for Space Systems (IRS). In the past these experimental investigations were concentrated on measurements of integral values like thrust and voltage or plasma parameter distributions outside the thruster. Due to the necessary interest in the details of the internal flow of the MPD thrusters, a thruster with two optical windows have been built. The windows are positioned at the end of the nozzle throat and in the expansion part at the beginning of the anode. These are determined as critical regions for the beginning of unstable behavior of these thrusters. So, the measured data in these regions inside the thruster are of great importance for the development of high power MPD thrusters. In order to achieve a better understanding of the investigated thrusters, numerical procedures to calculate these MPD accelerators have been applied. A theoretical model for the calculation of the current, electron temperature and flow distribution in self-field MPD thrusters has been developed. The extended Ohm's law is used to calculate the current contour lines; the electron temperature distribution is solved by including the electron energy equation, whereas a two-dimensional flow code is used to obtain the velocity, pressure and heavy particles temperature distributions. With both temperature distributions, a thermal nonequilibrium in the plasma flow is obtained. The calculations are done first with the high enthalpy flow solver HEFLOS, which is a finite volume code on structured grid, and second with a finite volume code on unstructured grid, which is being developed at the IRS. The propellant used in both codes and the experiments is argon. For the verification of a numerical code a comparison of plasma parameters inside the thruster is of importance. The correspondence between the numerical and measured values gives an idea of the quality of the numerical results and, especially inside the thruster, of the quality of the applied boundary conditions.

Nomenclature

\( B \)  magnetic induction  \( e \)  electronic charge
\( \varepsilon \)  specific energy  \( \mathcal{E} \)  electric field strength
\( F \)  spatial derivatives  \( \mathcal{G} \)  spatial derivatives
\( h \)  Planck's constant  \( \mathcal{H} \)  source vector
\( I \)  electric current  \( \mathcal{J} \)  current density
\( k \)  Boltzmann constant  \( l \)  thickness
\( m \)  mass  \( \mathcal{M} \)  mass flow rate
\( \mathcal{m}_{\text{IV}} \)  reduced mass  \( n \)  particle density
\( \zeta \)  quantum number  \( \mathcal{N} \)  normal vector
\( P \)  pressure  \( \mathcal{O} \)  flow variables
\( Q \)  cross section  \( \mathcal{P} \)  radial coordinate
\( \mathcal{R} \)  dissipative derivatives  \( \mathcal{S} \)  dissipative derivatives
\( \mathcal{T} \)  thrust  \( t \)  time
\( T \)  temperature  \( \nu \)  velocity
\( \psi \)  voltage  \( \xi \)  relaxation time
\( \zeta \)  axial coordinate  \( \zeta_i \)  charge number
\( a \)  heat transfer coefficient  \( \beta \)  plasma parameter
\( \mathcal{a} \)  ionization energy  \( \zeta \)  dissipative factor
\( \mathcal{a} \)  curvilinear coordinate  \( \lambda \)  thermal conductivity
\( \mathcal{a} \)  curvilinear coordinate  \( \mu \)  permeability in vacuum
\( \mathcal{a} \)  electric conductivity  \( \rho \)  mass density
\( \Phi \)  stream function  \( \omega \)  electron gyrofrequency

Subscripts

\( e \)  electron  \( h \)  heavy particles
\( i \)  ions  \( k \)  excitation state
\( r \)  radial  \( rct \)  reaction state
\( s \)  specific  \( z \)  axial
\( \nu \)  paricles  \( \theta \)  azimithal

1 Introduction

For planetary space missions, like the human mission to Mars, specific impulses of about 2000 s and high thrust levels of some 10 N will be required for the thruster system\(^1\). MPD thrusters are considered for such missions due to their high thrust level and simplicity. But much effort is still required to increase the specific enthalpy and thrust efficiency.
Experimental investigations on self-field magnetoplasmadynamic (MPD) thrusters are conducted at the Institute for Space Systems (IRS). To achieve a better understanding of the investigated thrusters, numerical procedures to calculate these MPD accelerators have been applied.

Despite the simplicity in design and power conditioning of the MPD thrusters, they are handicapped even today by the shortcomings of low efficiency and only moderate specific impulses. This is due to today's low development status of these thrusters. For steady state self-field MPD thrusters showing the longest life span, a maximal specific impulse of about 1500 s is achieved. For an improvement of the efficiency and specific impulse of these MPD thrusters the new experimental investigations mainly inside the thrusters and the theoretical investigations are very helpful to meet the requirements. However, it should be noted that the new investigations are also useful for the study of the effects such as the electrode instabilities (which were studied experimentally and analytically, and phase instabilities observed at high specific impulses (which were investigated by means of the dispersion relation).

Considering the higher ionization modes up to the sixth ionization level, the differential discharge equation can be calculated. The Navier–Stokes equations for the heavy particles flow are calculated. The electron energy equation is solved to determine the thermal non-equilibrium of the plasma flow. All equations applied to the iteratively solved high enthalpy flow are coupled by corresponding source terms which correlate the different physical processes and their corresponding conservation equations to one another.

In this paper a new two-dimensional axisymmetric program system HEFLOS is used for the determination of the calculated results of the nozzle type MPD thruster DT7. From a second numerical code on unstructured grid, which is currently under development at the IRS, only few example results are presented. The DT7 thruster shown in Fig. 1 is especially designed for inside optical diagnostics. A detailed parameter presentation of the numerical results of this MPD thruster is shown in this paper. Emission spectroscopic measurements were carried out inside the MPD thruster, using optical windows in the throat respectively anode region. The determined data from the spectra measurements are compared with the calculation results, in particular, the numerical and experimental radial distributions.

2 Experimental Investigations

A new MPD thruster DT7, a scheme of which is shown in Fig. 1, was built for detailed investigations by optical diagnostics inside the thruster itself. The thruster has the same geometry as the well investigated nozzle type thruster DT2. Two small slits, one in the throat region, the other between the anode and the first neutral segment, offer a free view through the plasma flow at both these positions over the whole cross section. The slit in the nozzle region is closed by an optical window, the other one is open. A lens system with fiber optics is fixed on a positioning system to perform radial scans. The emitted light is transmitted to a spectrometer and analysed by an OSMA (Optical Spectral Multichannel Analyser).

![Fig. 1: Nozzle type MPD thruster DT7.](image)

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![Fig. 2: Argon spectrum at 4000 A, from the center of the throat.](image)
The lines of highest intensity, also marked by arrows, are all emitted by second ionized argon. The spectral line data as energy of upper quantum level $\epsilon_i$, statistical weight factor $g_i$ and transition probabilities $A_{\nu i}$, taken from $^{15}$, are listed in Table 1.

For further interpretation of the spectral data, the plasma conditions should be checked. Griem $^{16}$ gives for hydrogen like species a necessary criterion of the electron density for obtaining a PLTE (partial local thermal equilibrium) plasma:

$$n_e = 7 \cdot 10^{18} \frac{z^2}{n_i^{17/2}} \sqrt{\frac{kT}{z^2 \epsilon_i}} ,$$

(1)

where $\epsilon_i$ is the ionization energy, $n_i$ the main quantum number and $z$ the ionization level. Assuming fully single ionization, the estimated electron number density in the throat region satisfies the criterion, so that the PLTE condition can be assumed.

The total line emission coefficient $\epsilon_L$ is a function of the density $n_k$ in the excited state $k$:

$$\epsilon_L = \frac{h \nu}{4\pi} A_{\nu i} n_k$$

(2)

Integrating the total emission coefficient along the line of sight, the intensity of a homogeneous plasma is given by

$$I_{ki} = \int \epsilon(z) dz = \frac{h \nu}{4\pi} A_{\nu i} n_k l$$

(3)

where $l$ is the thickness of the emitting region.

Under thermal equilibrium conditions, the density $n_\nu$ is described by the Boltzmann equation:

$$n_\nu = \frac{g_k}{U(T_{ex})} n_0 \exp \left( - \frac{\epsilon_k}{kT_{ex}} \right)$$

(4)

with the statistical weight $g$ and the partition function $U$.

Substituting $n$ in equation (3), the logarithmic equation can be obtained:

$$\ln \left( \frac{I_{ki}}{\nu A_{\nu i} g_k} \right) + \text{const.} = - \frac{\epsilon_k}{kT_{ex}}$$

(5)

The excitation temperature can now be determined by the relative line intensity method $^{16, 17}$ from the Boltzmann-plots, where $ln(I_{ki}/\nu A_{\nu i} g_k)$ is plotted versus $E$ for several emission lines belonging to the same species, the temperature $T$ can be determined from the inclinations. The Abel inversion technique $^{18}$ is used to determine the radial distribution of the plasma parameters from the scans through the cross sections.

### 3 Computational Model

The model for the numerical simulation of a self-field MPD accelerator was developed at the IRS$^{14, 19, 20}$. This model unites three different physical fields to one another. The parts of this model are the extended Ohm’s law for plasma to calculate the discharge, the electron energy equation to calculate the electron temperature and the Navier–Stokes equations to obtain the properties of the flow field.

In this two-fluid non-equilibrium model the viscosity, electron and heavy particles conductivity are included. The basic assumptions for this model is the rotational symmetry of a quasi-neutral plasma flow, no electrodes surface processes and boundary layer effects are included, and electron and heavy particles energy are linked only by elastic transfer. An additional assumption is the input of the ohmic heating in the electron energy equation, due to the higher mobility of the electrons. The transfer of this heat to the heavy particles is contained in the compensation between the electron and the heavy particles temperature.

#### 3.1 Discharge Code

The current distribution within a self-field MPD thruster is assumed to be two-dimensional, and no azimuthal current is to be expected. In order to calculate the current distribution of such an arc discharge, a two-dimensional computer code has been developed. The basic equation for the discharge is the extended Ohm’s law for plasmas:

$$\vec{j} = \sigma (\vec{E} + \vec{v} \times \vec{B}) - \frac{\omega_e \tau_e}{B} (\vec{J} \times \vec{B}) - \beta \nabla p_e$$

(6)

Here $\sigma$ is the electric conductivity, $\omega_e$ the electron cyclotronfrequency and $\tau_e$ the electron collision time. Rewriting the Ohm’s law by means of the Maxwell’s equations for steady state conditions, one obtains a vector equation for the magnetic induction field $\vec{B}$ in the form

$$0 = \frac{1}{\mu_0} (\nabla \times (\frac{1}{\sigma} \nabla \times \vec{B}) - (\nabla \times (\vec{v} \times \vec{B})))$$

$$+ \frac{1}{\mu_0} (\nabla \times (\beta (\nabla \times \vec{B}) \times \vec{B})) - \nabla \beta \nabla p_e$$

(7)
with \( \beta = \frac{\omega T}{B_0 \sigma} = \frac{1}{e n_e} \) \( (8) \)

The equation (7) yields with a stream function \( \Psi = r B_0 \) the elliptical, partial differential equation of 2nd order

\[
\frac{\partial^2 \Psi}{\partial r^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{r} \frac{\partial \Psi}{\partial r} \left( 1 + \frac{1}{\sigma} \frac{\partial \Psi}{\partial r} - \frac{\sigma \Psi \partial \beta}{r^2} + \sigma \mu_e v_r \right) - \frac{\partial \Psi}{\partial z} \left( 1 + \frac{\sigma \Psi \partial \beta}{r^2} - \frac{2 \sigma \Psi \partial \beta}{r^2} + \mu_e v_z \right) \] \( (9) \)

\[
= \mu_e \Psi \left( \frac{\partial v_r}{\partial r} - \frac{\partial v_z}{\partial z} \right) - \mu_e \Psi \left( \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) = 0
\]

The function \( \Psi(r, z) = \text{const} \) now represents a current contour line, since \( B = B_0 \) is proportional to \( \frac{I(r)}{r} \), where \( I(r) \) is the electric current carried through the cross sectional area \( \pi r^2 \).

The electric conductivity of a plasma is determined by \( (10) \)

\[
\sigma = \frac{3}{8} \sqrt{\frac{\pi}{2}} \frac{e^2 n_e}{\sum_{\nu}(\sigma_\nu k_{\nu} \sqrt{m_{\nu} k_{T\nu}})}
\]

Here \( m_{\nu} \) is the reduced mass and \( T_{\nu} \) the reduced temperature. With respect to the different ionized levels the Gvosdover cross sections follow by \( (11) \):

\[
Q_{\nu} = \frac{\pi}{4} \left( \frac{z_\nu e^2}{4\pi e k_{T\nu}} \right)^2 \ln \left( \frac{144\pi^2 z_\nu e^6 z_\nu^3 k_{T\nu}}{n_e e^6 (z_\nu^2 (z_\nu + 1))} \right)
\]

\( z_\nu \) stands for the charge number, which is calculated from the ionization level of the argon plasma. For \( Q_{\nu} \) being the cross section between electrons and atoms in equation 10 the Ramsauer cross section is used.\( (12) \) From these equations it is obvious that high ionization levels have a strong effect on the electric conductivity.

### 3.2 Electron Energy Equation

The electron temperature has a strong effect on the electrical and thermal conductivity and on the electron density, which again influences the discharge pattern. Therefore, a two-dimensional code for the electron temperature distribution, corresponding to the two-dimensional discharge code, was written. The electron temperature distribution is determined by the energy equation for the electron component of the plasma.

\[
T = \nabla (\lambda_e \nabla T_e) + \frac{5}{2} k_e \nabla T_e + p_e \nabla \cdot \vec{v} = \frac{\partial^2 T_e}{\partial r^2} - \sum_{\nu} n_{\nu} n_e \alpha_{\nu} (T_e - T_h) - \sigma \frac{\partial n_e}{\partial r} = 0 \] \( (11) \)

The subscript \( \nu \) represents the heavy particles. The first term on the left hand side represents the conductive heat flux in the electron gas, and the second term gives the convective heat flux of the electron gas due to the electron drift. The energy input due to ohmic heating is represented by the first term on the right hand side. The sum of losses due to the energy transfer from the electron gas to the heavy particles gas is calculated by the second term on the right hand side. The reaction losses are given by the last term, where \( \epsilon_i \) is given by Unsöld.\( (13) \) The heat transfer coefficient \( \alpha_{\nu} \) and the thermal conductivity \( \lambda_e \) depend on the electron temperature.\( (10) \)

\[
\alpha_{\nu} = \frac{8\sqrt{2}}{\sqrt{\pi}} Q_{\nu} \sqrt{\mu_{\nu} k_{T\nu}} \frac{k T_e}{m_e + m_{\nu}}
\]

and the thermal conductivity:

\[
\lambda_e = \frac{15}{8} \frac{\pi}{2} \frac{n_e k_{T\nu}^2 T_e}{\sum_{\nu}(\epsilon_\nu Q_{\nu} \sqrt{m_{\nu} k_{T\nu}})}
\]

In the sum of this conductivity equation \( \nu \) also includes the electron component. For \( \nu = e \) the collision cross section \( Q_{ee} \) can be set equal to the Gvosdover cross section.\( (14) \) If \( Q_{ee} \) be the cross section between the electrons and ions, the Gvosdover cross section is used with respect to the different ionization levels; and for \( Q_{ev} \) being the cross section between electrons and atoms the Ramsauer cross section is used.\( (15) \)

With respect to the rotational symmetry, equation 11 results in the following elliptical, partial differential equation of 2nd order:

\[
\lambda_e T_e^2 \frac{\partial^2 T_e}{\partial r^2} + \lambda_e T_e^2 \frac{\partial^2 T_e}{\partial z^2} + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial r} \right)^2 + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial z} \right)^2 + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial r} \frac{\partial T_e}{\partial z} \right) + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial r} \frac{\partial T_e}{\partial r} \right) + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial z} \frac{\partial T_e}{\partial z} \right) + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial r} \frac{\partial T_e}{\partial z} \right) + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial r} \frac{\partial T_e}{\partial r} \right) + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial z} \frac{\partial T_e}{\partial z} \right) + \frac{5}{2} \lambda_e T_e^2 \left( \frac{\partial T_e}{\partial r} \frac{\partial T_e}{\partial z} \right)
\]

Here \( j_r \) and \( j_z \) are the current densities in the cylindrical \( r- \) and \( z- \) directions.

### 3.3 Flow Field Code

For the description of the two-dimensional, axisymmetric flow, the following nonlinear hyperbolic sys-
System of differential equations with cylindrical coordinates is applied:

$$\frac{\partial}{\partial t} \vec{q} + \frac{\partial}{\partial z} \vec{F}(\vec{q}) + \frac{\partial}{\partial r} \vec{G}(\vec{q}) = \zeta \frac{\partial \vec{H}}{\partial z} + \zeta \frac{\partial \vec{S}}{\partial r} + \vec{H} \tag{15}$$

The terms on the left hand side are taken in the usual fluid dynamic manner, where $\vec{q}$ is the independent variables vector and $\vec{F}$ and $\vec{G}$ are the spatial derivatives vectors.\textsuperscript{25, 26} The first two terms on the right hand side are the dissipative derivatives. For the impulse equations $\zeta$ represents here the viscosity coefficient $\mu$, which is given by:

$$\mu = \frac{3}{8} \sqrt{\frac{\pi}{2}} \sum_{l} \frac{m_{l} n_{l} k_{B} T_{l}}{\sum_{l} \nu_{l} q_{l} \sqrt{m_{l} k_{B} T_{l}}} \tag{16}$$

Since the temperatures of the heavy particles are equal, $T_{l} = T_{e} = T_{h}$, and $m_{l}$ is the reduced mass for $l, \nu$ as heavy particles subscripts. For the energy equation $\zeta$ represents in the dissipative terms the heat conductivity coefficient $\lambda$.

$$\lambda = \frac{15}{8} \sqrt{\frac{\pi}{2}} \sum_{l} \frac{n_{l} k_{B}^{2} T_{l}}{\sum_{l} \nu_{l} q_{l} \sqrt{m_{l} k_{B} T_{l}}} \tag{17}$$

The transport coefficients are derived in a similar manner as in \textsuperscript{27} and more detailed derivations for the coefficients are described in \textsuperscript{28}. The cross sections $q_{l}$ for all coefficients are taken from the references \textsuperscript{21, 22}. Real gas effects are taken into account by a variable ratio of specific heats.\textsuperscript{9, 20, 28}

The source vector $\vec{H}$ is given as:

$$\vec{H} = \begin{bmatrix} 0 \\ j_{n} B_{n} \\ j_{e} B_{e} \\ \sum_{l} \nu_{l} n_{l} \alpha_{l}(T_{h} - T_{e}) \end{bmatrix} \tag{18}$$

The terms in the impulse equations represent the $\vec{j} \times \vec{B}$ forces. The transfer of Joule’s heat is contained in the temperature compensation between the electron and the heavy particles temperature, which is represented by the source term in the equation of energy conservation.

### 3.4 Boundary Conditions

The proper boundary conditions for the discharge equation follow from the geometry of the thruster walls and electrodes. At the insulator inside the thruster and at the inflow boundary $\Psi$ is set to $-\frac{\mu_{0}}{2} I$. For the electrodes the electric field is assumed to be normal to the surfaces $\vec{E}(\Psi) \cdot \vec{n} = 0$.

At the inflow boundary $T_{e}$ is set to a constant value of 6000 K in accordance with the measurements. The cathode is treated as thermal insulators. Therefore the boundary condition inside the thruster is given by $\nabla T_{e} \cdot \vec{n} = 0$, where $\vec{n}$ is the normal vector of the surfaces. Due to the axial symmetry, $\frac{\partial \mu}{\partial r} = 0$ on the axis. The boundary condition for the upper solid bodies of the thruster (the insulators and the anode) is adjusted by a minimum operator applied to the extrapolated electron temperature from the inside field, $\nabla T_{e} \cdot \vec{n} = 0$, and a constant electron temperature of $T_{e} = 20000 K$. At the outflow boundary $T_{h}$ is extrapolated from the calculated field.

For the flow field equations all values at the inflow boundary are determined from cold gas measurements and are fixed during the complete calculation. The outflow boundary is considered as a free stream boundary and either the independent flow variables are completely extrapolated for the case of supersonic conditions or the inner energy $e$ is calculated with the other extrapolated independent variables and a given pressure for the case of subsonic conditions. For all other boundaries, the solid body and symmetry axis, a no flux condition through the boundary is taken. This is done by an extrapolation of the density and inner energy, $\nabla \rho \cdot \vec{n} = 0$ and $\nabla e \cdot \vec{n} = 0$, a reflection of the tangential velocity and setting the normal velocity to zero, $\vec{v} \cdot \vec{n} = 0$.

### 4 Computational Method

For the calculation of the self-field MPD flow, a modular program system was developed which connects three different physical fields to one another. The three-program system parts described in the previous sections are the discharge, the electron energy equation and the heavy particles flow field code. All differential equations of this program system are transformed from cylindrical to general curvilinear coordinates\textsuperscript{9, 20} for the calculations using the high enthalpy flow solver HEFLOS on structured grid (shown in Fig. 3). The MPD flow calculation in general curvilinear coordinates enables a good adjustment of the DT7 thruster geometry even on a structured grid.

The three codes are connected iteratively in the following manner: for a given flow field and an electron temperature distribution, the current, and hence the magnetic field distribution, was determined. With the ohmic heating, as a result of the current distribution, an electron energy calculation follows and determines a new electron temperature distribution. With these results the flow field equations were integrated. In the next time step, this new flow field and the new electron temperature distribution were taken to calculate the new electromagnetic field distribution, and so on.
In the program system HEFLOS, the nonlinear, elliptical, partial differential equations for the discharge (9) and the electron energy (14) are solved with a finite difference method. The discretization yields a nonlinear equation which is iteratively solved with a modified Gauss-Seidel algorithm. The nonlinear, hyperbolic, partial differential equations for the flow field (15) are solved with the finite volume solver NSFLEX. Here this code was modified for a non-equilibrium state and for the electromagnetic source terms interaction. The subsystem NSFLEX uses a Godunov upwinding scheme for the flux discretization. With the Newton method an implicit formulation is achieved, and the relaxation is done by a point Gauss-Seidel procedure.

All equations used for the iteratively solved high enthalpy flow are coupled completely by corresponding source terms which correlate the different physical processes and their corresponding conservation equations to one another. The fact that the conservation equations are solved by three individual solution procedures does not reduce the coupling intensity among the different equations, it represents only a solution and modular program developing strategy. These individual solution procedures enable an easy implementation of different physical processes and a faster and more exact calculation for several processes. However, a faster calculation is possible, because not all processes need extensive calculation procedures, or the calculation step size need not be reduced for the required value of the slowest conservation equation. With an individual procedure the calculation and the boundary conditions can be adjusted more exactly for various problems.

5 Discussion of Results

The following figures show the results for the nozzle type MPD thruster geometry DT7 (see Fig. 1). The structured grid used for the calculation with the program system HEFLOS is shown in Fig. 3 and the unstructured grid used for the calculation with the new finite volume code is shown in Fig. 4. In Fig. 3 the calculation grid is extended in the outflow direction up to \( z = 340 \text{ mm} \) at the symmetry axis. So, the total number contains \( 137 \times 21 \) grid points. The grid was created numerically, and the distortion of the grid is almost less than 1.1. The unstructured grid, in Fig. 4, is created by an advancing front generator. The triangles are as close to being equilateral as possible, so that a good convergence behavior was obtained. The total number of triangles is 6179, which contains 3249 grid points.

In accordance with the experiments the flow inlet boundary conditions were determined to coincide with an experimentally obtained cold gas thrust of 0.4 \( N \) at a mass flow rate of 0.8 \( g/s \). For a given current of 4 \( kA \) the computation yields the current density distribution as illustrated in Fig. 5. The calculated current contour distribution corresponds with the current contour distribution at a continuing mode if the cathode is hot glowing and is emitting electrons thermionically along its complete length.

The potential lines, resulting from the unstructured grid computation, within the discharge at a current level of 4 \( kA \) are plotted in Fig. 6. An integration across these lines yields the discharge voltage due to the plasma resistance. The total calculated voltage is 44.8 V, at an ionization voltage of 7.6 V for the first ionisation level of 0.8 \( g/s \) mass flow rate for argon. The measured voltage drop was 56.7 V. Since
the numerical model does not include electrode fall voltages and there is a large amount of highly ionized argon within the discharge region, the calculated number matches the experiment quite well.

The experimental data, which is determined by the Abel inversion from a measured argon spectrum, equals the excitation temperature for thermal equilibrium. Fig. 7 illustrates that this condition is approximately satisfied for the plasma flow at this region. Due to this approximation and the fact that the calculation model does not include heat losses to the thruster walls, the compared calculated electron temperature and the measured excitation temperature coincide fairly well.

Fig. 6: Potential lines within the discharge of 4 kA.

The heavy particle (top) and the electron temperature distribution (bottom) within and outside the DT7 thruster are shown in Fig. 7 for a given current of 4 kA. The maximum temperature value of the heavy particles occurs in front of the cathode on the symmetry axis. The maximum electron temperature value occurs also in front of the cathode.

Fig. 7: Heavy particles (top) and electron (bottom) temperature contours for 4 kA.

The resulting radial cross section of the electron temperature at the end of the nozzle throat compared with measurements at this position is presented in Fig. 8. The calculated electron temperature and measured excitation temperature at the nozzle throat of the DT7 thruster.

Fig. 8: Calculated electron temperature and measured excitation temperature at the nozzle throat of the DT7 thruster.

The ionization distribution of the argon plasma. This ionization number shown here is calculated with respect to the sum of the heavy particles $\alpha = n_e / \sum_n n_n$. After the inflow border the ionization grows up to 1 at the beginning of the nozzle throat region. In the nozzle throat region and in the beginning of the nozzle the ionization is greater than 2. Moving in front of the cathode to the symmetry axis, the ionization increases to the maximum value of higher than 3. This indicates that the temperature there is relatively high. Outside the thruster in radial direction out of the symmetry axis the ionization decreases slowly.

Fig. 9: Ionization distribution for 4 kA.
The optical measurements inside the thruster are of great importance to the achievement of a better understanding of the MPD flow. Despite the shortcomings of the numerical model, namely that the thermal losses to the thruster walls are not considered and the electrode fall voltages not included, the calculated values coincide fairly well with the measured data. Further plasma flow investigations and comparisons inside the thruster should provide the knowledge of the fundamental processes which restrict the operation of the self-field MPD thrusters at very high specific impulses.

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