THE EFFECT OF SOLAR ARRAY DEGRADATION IN ORBIT-RAISING WITH ELECTRIC PROPULSION

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Abstract
A computer model incorporating data on the performance of silicon solar cells subjected to Van Allen belt radiation is used to calculate the trip times of a solar powered electrically propelled vehicle performing coplanar, equatorial orbit-raising between low Earth orbit (LEO) and geosynchronous orbit (GEO). The maximum delivery rate, which is the ratio of the payload fraction delivered to the total vehicle trip time, is calculated as a function of specific impulse, array coverglass thickness, and number of roundtrips performed. Optimum array coverglass thickness to maximize vehicle delivery rate is determined by the designer's choice of engine specific impulse and the number of roundtrips.

Nomenclature

A = Array area
a₀ = Initial acceleration
α₀ = Return trip initial acceleration
α = Specific mass, kg/kW
C = Exhaust Velocity = I₂p * 9.8 m/s²
D = Maximum normalized power output of irradiated cells
δcumulative = Cumulative delivery rate
F = Solar constant = 1.397 kW/m²
I₂p = Specific impulse
M_array = Array mass
M_f = Final mass
M_L = Payload mass
M₀ = Initial mass in LEO
M_pp = Mass of power plant
M_prop = Propellant mass
η_cell = Solar cell efficiency
η_t = Thruster efficiency
P = Electrical power
ρ_cell = Solar cell density
ρ_shield = Coverglass density
t_cell = Solar cell thickness
t_shield = Coverglass thickness

T = Thrust
τ₀ = Burn time
τ_l = Trip time

Introduction
Electric propulsion (EP) technology has long held the promise of improved payload performance over chemical propulsion due to its much higher specific impulse. This promise is often dismissed due to the extremely low thrust levels (10⁵-10⁷ times smaller than chemical rockets), which imply lengthy transfer times (days or months as opposed to hours). However, an increase in the payload fraction delivered to GEO due to the high specific impulse of EP can significantly reduce launch costs by allowing smaller launch vehicles to be used. Lower propellant fractions would also allow the use of an EP orbit transfer vehicle as a roundtrip 'space tug', capable of returning to LEO to acquire a new payload and propellant load for another trip to GEO.

The high electrical power requirement of EP mandates either a nuclear or solar power source; the only power system available for near-term application is the solar array. However, a solar power system operating in Earth orbit will degrade due to radiation damage incurred by exposure to the Van Allen belts. This paper will address the effect of solar array degradation on the design of an electrically propelled orbit transfer vehicle which delivers payload from LEO (300km) to GEO. The performance of the vehicle is a function of the time the vehicle spends in the radiation belts, the radiation resistance of the solar cell, and the amount of coverglass shielding the cells. Performance is measured by the delivery rate, which is the total payload fraction delivered to orbit divided by the total trip time.

Maximum Payload Fraction at Optimum Exhaust Velocity

The design requirement of a transfer vehicle is to maximize the cumulative delivery rate, δ:

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\[
\delta_{\text{cumulative}} = \sum_{i=1}^{n} \frac{[M_i]}{[M_{\text{i.o.,b}}]}
\]
\[
= \frac{\text{Total Payload Delivered}}{\text{Total Time in Orbit}}
\]

where \( n \) = number of roundtrips, units = days\(^{-1}\)

To derive the maximum payload fraction of an EP vehicle, the initial mass of a space vehicle, \( M_0 \), is broken down into three significant groupings, \( M_{\text{pp}} \), powerplant mass, \( M_{\text{prop}} \), propellant mass, and \( M_L \), payload mass. The mass of the powerplant, \( M_{\text{pp}} \), includes the mass of structure, mechanisms, and shielding for the generation, conversion, and distribution of electrical power. In this derivation, the mass of the payload, \( M_L \), includes non-power related structure such as propellant tankage and vehicle structure.

\[
M_0 = M_{\text{pp}} + M_{\text{prop}} + M_L
\]

\[
M_L = M_{\text{payload}} + M_{\text{structure}}
\]

The electrical power required by the thruster system is the power in the jet divided by the thruster efficiency. For the case of constant thrust, which we are assuming here, the mass flow of propellant can be expressed as the mass of the propellant divided by the burn time:

\[
P = \frac{1}{2}\eta_\text{t} \frac{\text{prop} c^2}{2\eta_\text{t} T_{\text{b}}}
\]

\[
= \frac{M_{\text{pp}} c^2}{2\eta_\text{t}}
\]

(3)

The performance of space power plants can be characterized by the specific mass of the system, \( \alpha \), which is the ratio of the power plant mass to the electrical power produced.

\[
\alpha = \frac{M_{\text{pp}}}{P}
\]

(4)

From the rocket equation, it is a well known result that:

\[
\frac{M_{\text{prop}}}{M_0} = 1 - e^{-\frac{\Delta v}{c}}
\]

(5)

Combining expressions (3), (4), and (5) into equation (2) and rearranging them yields the payload fraction, which is the ratio of \( M_L \) to the initial mass of the system:

\[
\frac{M_L}{M_0} = \frac{-\frac{c \Delta v}{2\eta_\text{c}} (1 - e^{-\frac{\Delta v}{c}})}{\left[1 - \frac{e^{-\frac{\Delta v}{c}}}{\eta_\text{c}}\right] + e^{-\frac{\Delta v}{c}}}
\]

(6)

For an electrically propelled vehicle, a maximum payload fraction for a one-way vehicle can be found by differentiating with respect to exhaust velocity:\

\[
\left[\frac{M_L}{M_{\text{o, max}}} = \frac{\Delta v}{2(1 - e^{-\frac{\Delta v}{c}}) + \frac{\Delta v}{c}}
\]

(7)

This expression is different for a roundtrip vehicle, one which carries enough fuel to return to LEO after boosting a payload to GEO. For a roundtrip vehicle, the payload for the return trip, \( M_{\text{L down}} \), is just the fixed structural mass of the vehicle, \( M_{\text{structure}} \) (i.e. tankage and non-power plant related structure). The initial mass of the vehicle on the trip up to GEO is:

\[
M_{\text{O up}} = M_{\text{pp}} + M_{\text{prop up}} + M_{\text{prop down}} + M_{\text{L up}}
\]

\[
= M_{\text{pp}} + M_{\text{O up}} (1 - e^{-\frac{\Delta v}{c}})
\]

\[
+ M_{\text{O down}} (1 - e^{-\frac{\Delta v}{c}}) + M_{\text{L up}}
\]

(8)

Given that:

\[
M_{\text{L up}} = M_{\text{payload}} + M_{\text{L down}}
\]

\[
M_{\text{O down}} = M_{\text{pp}} + M_{\text{prop down}} + M_{\text{L down}}
\]

\[
= M_{\text{pp}} + M_{\text{O down}} (1 - e^{-\frac{\Delta v}{c}}) + M_{\text{L down}}
\]

\[
= \frac{M_{\text{pp}} + M_{\text{L down}}}{e^{-\frac{\Delta v}{c}}}
\]

(9)
Assuming constant thrust, the expression (9) can be rewritten:

\[
\frac{M_{I_{up}}}{M_{O_{up}}} = e^{-\Delta v \frac{1}{c}} - \frac{\alpha c^2 \Delta v}{2 \eta_{b-up} + \frac{\alpha c^2}{2 \eta_{b-up}}} + \frac{M_{I_{down}}}{M_{O_{up}}} \left(1 - e^{-\Delta v \frac{1}{c}}\right)
\]

which results in the payload fraction for the LEO-GEO portion of the roundtrip. Differentiating with respect to \( c \), exhaust velocity, yields:

\[
\frac{d}{dc} \left[ \frac{M_{I_{up}}}{M_{O_{up}}} \right] = 0
\]

\[
\frac{2 \eta_{b-up}}{\alpha c^2} = \frac{\Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}}{\Delta v \frac{1}{c}} - \frac{2 \Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}}{\Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}} - \frac{M_{I_{down}}}{M_{O_{up}}} \frac{\Delta v \frac{1}{c}}{\Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}}
\]

Substituting the non-dimensional grouping (11) back into the expression for payload fraction (10):

\[
\frac{M_{I_{up}}}{M_{O_{up, max}}} = e^{-\Delta v \frac{1}{c}} + \frac{\Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}}{\Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}} - \frac{M_{I_{down}}}{M_{O_{up}}} \left(1 - e^{-\Delta v \frac{1}{c}}\right)
\]

This equation represents the maximum payload fraction a roundtrip vehicle can carry on its outbound trip, for a given \( \Delta v \) and optimum \( c \), assuming that the vehicle carries only its structural mass, \( M_{I_{down}} \), on the return trip. It is assumed that \( \Delta v \) and \( c \) will be the same for both trips.

**Effect of Solar Array Degradation on Trip Time**

Due to the lengthy trip times associated with low thrust, the calculation of the trip time for a solar powered vehicle must incorporate the effect of radiation damage on the vehicle's jet power. The jet power, and therefore the thrust of the vehicle, will be reduced as less electrical power is available, assuming that the engine efficiency is independent of power input. For the maximum payload fraction at optimum \( I_{sp} \), the resulting initial acceleration for a roundtrip vehicle is, from (3) and (12):

\[
a_o = \left[ e^{-\Delta v \frac{1}{c}} - \frac{M_I}{M_0} \frac{2 \eta_{thruster}}{\alpha c} \left(1 + e^{-\Delta v \frac{1}{c}}\right) \right]
\]

\[
a_o = \left( \frac{-\Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}}{\Delta v \frac{1}{c} - 2 + 2e^{-\Delta v \frac{1}{c}}} - \frac{M_{I_{down}}}{M_{O_{up}}} \left(1 - e^{-\Delta v \frac{1}{c}}\right) \right) \frac{2 \eta_{thruster}}{\alpha c}
\]

The initial acceleration is used as an input variable to the computer model in order to calculate trip time. On subsequent trips, the initial acceleration will be directly proportional to \( D \), the percent of maximum power a solar cell could produce as a result of the radiation damage previously incurred.

\[
P = \frac{T_c}{2 \eta_l} = \frac{M_I a_o c}{2 \eta_l}
\]

\[
a_o(trip n) = \frac{2 \eta_l P(trip n - 1)}{M_I c} = a_o(trip 1) D(trip n - 1)
\]

For the return leg (GEO-LEO) of a roundtrip, the vehicle is no longer burdened with a payload, and can therefore achieve higher acceleration due to its reduced mass. For constant thrust, the return trip initial acceleration is calculated by compensating the initial acceleration of the \( n \)th trip as follows, assuming that the \( \Delta v \) on the return trip is the same as for the outbound trip:

\[
T = M_l a_o(trip n) = (M_f - M_l) a_{return}(trip n)
\]

\[
a_{return}(trip n) = a_o(trip 1) D(trip n - 1) e^{-\Delta v \frac{1}{c}} - \frac{M_l}{M_o}
\]

Eqns. (14) and (15) state that at any point in a trajectory, the vehicle's acceleration can be calculated by knowing the initial acceleration and the value of \( D \) at that point. \( D \) is a function of the radiation resistance of the cell and the total radiation received by the cell.
To determine the total trip time, a computer model was built which iterated a low thrust trajectory while accounting for radiation exposure and loss of power due to array degradation. The model assumes an equatorial, co-planar transfer and includes circularization at apogee. Tada and Carter's model\(^3,4\) of the 1 MeV equivalent electron fluence (electrons/cm\(^2\)) is used to estimate the radiation received by a solar cell from the Van Allen belts as a function of orbital altitude, inclination and array shielding thickness. Their experimental data on the cell power output, \(D\), as a function of radiation degradation (up to \(10^{16}\) e/cm\(^2\) equivalent fluence) is used to estimate the degradation of the solar array. A curve fit was found to match the data given for radiation fluences less than \(10^{16}\) e/cm\(^2\) and was used to extrapolate cell power at higher fluence values. Radiation due to solar flare and increased solar activity, as well as the effect of the solar wind to compress the Van Allen belts on the sun-facing side of Earth was neglected in order to remove time constraints from the model. The reduction in light transmission to the cells due to coverglass darkening was also neglected as a second order effect.

Figure 1 depicts the model that was used to calculate the trip time as a function of initial acceleration, specific impulse, coverglass shielding thickness and cell type.

Figure 2 depicts the computed trip time of one trip from LEO to GEO as a function of initial acceleration and coverglass thickness.

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![Figure 1: Computer Model Flow Chart](image1)

![Figure 2: Trip time vs. Initial acceleration, Si 10 ohm-cm array](image2)
It would appear from Fig. 2 that more heavily shielded arrays result in shorter trip times due to the greater protection from radiation damage. A true comparison between the performance of a vehicle with a heavily shielded array relative to one with a lighter shield would incorporate the additional mass of the heavier shield into the initial acceleration. As will be shown below, increasing the shielding mass will result in a longer trip time, however, the more heavily shielded vehicle will result in a higher end of trip array power output.

The vehicle studied here is assumed to have no battery system for power through eclipse, so the specific mass of the power plant will be dominated by the weight of the array. This will allow us to characterize the difference in specific mass as a function of the shielding thickness:

\[
M_{\text{array}} = \rho_{\text{cell}} A (t_{\text{cell}} + 2t_{\text{shield}}) \\
A = \frac{P_{\text{elec}}}{\eta_{\text{cell}} F_{\text{solar}}} \\
\alpha = \frac{M_{\text{array}}}{P_{\text{elec}}} = \frac{\rho_{\text{cell}} (t_{\text{cell}} + 2t_{\text{shield}})}{\eta_{\text{cell}} F_{\text{solar}}} 
\] (16)

For simplicity, the density of both the cell and the shielding is assumed to be 2200 kg/m\(^3\), which is a good approximation for fused silica coverglass and silicon cells\(^5\). The shielding is the same thickness on the front and back of the cell. It is also assumed that the structural support mass of the array and the power conditioning mass is small compared to the mass of the cells and shielding thickness.

The silicon 10 ohm-cm base resistivity cells considered here are 3 mils thick; with a 6 mil shield on each side of the cell, the total cell thickness would be about 15 mils, and with a 20 mil shield, the total cell thickness would be 43 mils. From (13) and (16), the initial acceleration of a vehicle with a 20 mil shielded solar array would be 2.9 times slower than that with a 6 mil shielded solar array. If 0.008 m/s\(^2\) is arbitrarily taken as the initial acceleration of the 6 mil vehicle, then the 20 mil vehicle would have an initial acceleration of 0.0028 m/s\(^2\). When reexamining Fig. 2, the trip time for the 6 mil array vehicle is 9.06 days compared to a trip time of 25.34 days for a 20 mil array.

A roundtrip vehicle must maximize the power output of the array after a trip from LEO to GEO. In addition to minimizing trip time, the array must be able to produce enough power to survive the return trip to LEO as well as to perform subsequent trips. While it was shown above that extra shielding increases the trip time due to the addition of power plant mass, the thicker shielded array will result in a higher end-of-trip (EOT) power in spite of increased trip time. In Fig. 3, the EOT power is graphed as a function of initial acceleration. Note that a 1 mil shielded array with an initial acceleration below about 0.0015 m/s\(^2\) will be unable to complete the mission due to radiation damage.

![Figure 3. End-of-Trip Normalized Power Output vs. Initial Acceleration for Si 10 ohm-cm Solar Cells](image)

Using the example above, the 6 mil vehicle would have an EOT array power output of 0.71, or 71% of the initial undamaged cell power output. The 20 mil vehicle, however, would have a power output of 0.80, in spite of its much greater trip time. Higher EOT power is desirable for multiple trip vehicles but comes at the expense of slower trip times. These tradeoffs will be clarified when the cumulative delivery rate of a vehicle is examined.

**Results: Cumulative Delivery Rate**

With parameters such as \(\Delta V\), \(c\), \(\eta_{\text{i}}\), \(M_{\text{down}}/M_{\text{O}}\) and \(\alpha\) specified, an initial acceleration can be calculated from (13), and a trip time to GEO can
be estimated using the computer model. The maximum payload fraction is only a function of \( \Delta V, c, \) and \( M_{\text{down}}/M_{\text{O}} \), and is calculated from (12). The actual payload fraction delivered to GEO is the difference between the maximum payload fraction and \( M_{\text{down}}/M_{\text{O}} \), as determined in (8), and is the value used in the calculation of the delivery rate. The specific masses shown in Table 1 were calculated using Eqn.(16).

In Figures 4 and 5, the cumulative delivery rate is calculated for a roundtrip vehicle described in Table 1 which had performed 1, 3, and 5 roundtrips. It is assumed that the vehicle carries only enough propellant for one roundtrip and is replenished with fuel and payload in LEO.

<table>
<thead>
<tr>
<th>Solar Cell</th>
<th>Si 10 ohm-cm BSFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cell Efficiency</td>
<td>11.5%</td>
</tr>
<tr>
<td>Thruster efficiency</td>
<td>0.5</td>
</tr>
<tr>
<td>Structural mass fraction</td>
<td>0.1</td>
</tr>
<tr>
<td>( M_{\text{L down}}/M_{\text{O}} )</td>
<td></td>
</tr>
<tr>
<td>Mission ( \Delta V )</td>
<td>46.55 m/s</td>
</tr>
<tr>
<td>( \alpha ), 6 mil shielded array</td>
<td>5.2 kg/kW</td>
</tr>
<tr>
<td>( \alpha ), 20 mil shielded array</td>
<td>15.0 kg/kW</td>
</tr>
<tr>
<td>( \alpha ), 60 mil shielded array</td>
<td>42.8 kg/kW</td>
</tr>
</tbody>
</table>

**Table 1. Vehicle Properties for Figs. 4 and 5**

![Figure 4. Cumulative Delivery Rate, Trips 1 and 3](image1)

![Figure 5. Cumulative Delivery Rate, Trips 3 and 5](image2)
The effect of array shielding on the delivery rate is evident when the delivery rates for two vehicles with the same Isp and vehicle parameters but with different array shielding thicknesses are compared. After only one roundtrip, a 6 mil shielded array with an engine at an Isp of approximately 1600 sec will yield the maximum delivery rate for the parameters in Table 1. Yet after three roundtrips, the cumulative delivery rate for a vehicle with a 6 mil shielded array and an Isp greater than 2300s will be less than that for the same vehicle with a 20 mil shielded array. The benefit of heavier shielding at higher Isp and for multiple trips is apparent in Fig. 5. After five trips, at Isp less than 2300 sec, it is apparent that a 20 mil shielded array yields the greater cumulative delivery rate, whereas at Isp higher than 2300 sec, a 60 mil shield would be required to achieve the greater delivery rate. The delivery rate for a five roundtrip vehicle is maximized at an optimum array shielding of 20 mil and an Isp of 1400s.

Conclusions

Figs. 4 and 5 depict the impact of array degradation on the design of an EP transfer vehicle. The designer must consider the Isp of available engines and the required number of roundtrips in order to determine the amount of array shielding that will maximize the cumulative delivery rate for that particular vehicle. To optimize the cumulative delivery rate, an engine with an Isp in the 1400-1600 sec range would be required, which agrees with a similar conclusion drawn by Burton and Wassgren for the case of a constant power vehicle. Fig. 5 suggests that five roundtrips may be the practical limit of an EP transfer vehicle utilizing a solar array, due to the excessive trip times associated with a cumulative delivery rate below 0.001 days\(^{-1}\).

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References