PLASMA PARTICLE SIMULATION IN CUSPED ION THRUSTERS

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Abstract

A two-dimensional particle-in-cell (PIC) code was developed to calculate the orbits of charged particles in magnetic cusps. The simulation was performed in order to understand the ion loss mechanisms near magnets in the discharge chamber of cusped ion thrusters. This enables one to calculate the spatial distributions of plasma properties such as particle densities, particle flux, and space potentials. Calculated density distributions qualitatively agree with experimental data, which supports the validity of this simulation. In addition, this code can be used to examine the effects of the magnetic field and wall potential on the ion confinement and further to obtain information for improving ion confinement in the cusp region.

Nomenclature

\( A \) : magnetic vector potential
\( B_0 \) : maximum magnetic field strength
\( E \) : electric field
\( e \) : electronic charge
\( k \) : Boltzman constant
\( L \) : characteristic length of the magnetic field configuration
\( N \) : total number of ions
\( n \) : number density
\( n_i \) : mean ion density
\( m \) : mass
\( R \) : mirror ratio
\( r \) : position vector
\( r_c \) : Larmor radius
\( r_H \) : hybrid Larmor radius
\( S \) : weighting function; interpolation function
\( T \) : temperature
\( v_0 \) : canonical momentum
\( x \) : distance from the center line of a magnet surface
\( y \) : distance from the chamber wall
\( Z \) : number of charge of a particle
\( z \) : position along with the center line of a magnet surface
\( W \) : width of simulation zone
\( w_H \) : ion leakage half-width
\( \alpha \) : ratio of electron to ion flux to the wall
\( \Delta N_{i,w} \) : number of ions lost to the wall
\( \Delta t \) : time step
\( \Delta x \) : x-direction grid width
\( \Delta y \) : y-direction grid width
\( \varepsilon_0 \) : permittivity in the vacuum
\( \Gamma \) : total number of ions lost to the wall per unit time
\( \lambda_D \) : Debye length
\( \mu \) : ratio of ion to electron mass
\( \nu \) : ratio of electron cyclotron frequency to plasma frequency
\( \tau \) : ion confinement time
\( \varphi \) : space potential
\( \omega_{ie} \) : electron cyclotron frequency on the magnet surface
\( \omega_{pe} \) : plasma frequency in the center of the cusp

Subscripts

\( e \) : electron
\( i \) : ion, particle number
\( j \) : grid number

Superscripts

\( (') \) : differentiated value with respect to time
\( (') \) : nondimensional value

Introduction

Ion thrusters based on magnetic cusp plasma confinement are being considered for on-orbit propulsion functions and orbit transfer propulsion for large space systems. Since thruster performance has a strong impact on the trip time, payload, and power requirements of such missions, it is necessary to maximize the discharge performance while minimizing the ion production cost. According to Brophy's discharge performance model, the ion loss to the chamber walls has a significant influence on the discharge performance. On the basis of this model, a numerical code was developed in our laboratory for cusped ion thrusters. Using this code, however, it was found that the ion loss fraction to the wall was greater than the experimental results. Thus, it is required to reexamine this model.

In a cusped ion thruster, the ion loss flux to the wall is concentrated at the magnets. This is due to the fact that there is a resistance to the motion of charged particles across magnetic field lines, while along field lines, charged particles move almost freely. Near the magnets, strong magnetic fields induce charge separation between electrons and ions due to the difference between their Larmor radii. This causes strong electrostatic fields, which influence particle motion and particle confinement...
significantly. Nevertheless, the previous numerical code was based on a diffusion plasma model which assumed charge neutrality over the whole discharge chamber. As a result, it was thought that a quantitative difference in the discharge performance between calculation and experiment has originated in this assumption and that it is necessary to develop a new model including charge separation effects in the cusp region in order to more accurately calculate the ion loss flux.

In the field of nuclear fusion, many theoretical works have been performed for plasma confinement utilizing magnetic cusps. Among them, a particle-in-cell (PIC) method was successfully applied to investigate the physical phenomena in a magnetic cusp device called a 'picket fence.' However, in that simulation it was assumed that all charged particles were generated in the cusp region, while in cusped ion thrusters, the majority of the ions are considered to be produced outside the cusp region by ionization collisions of neutral atoms and primary or thermal electrons. For this reason, the conditions and assumptions should be modified in order to apply this method to ion thrusters.

The objective of this work is to simulate charged particle motions in the cusp magnetic field and to understand the ion loss mechanisms near magnets in the discharge chamber of cusped ion thrusters. A further objective is to obtain information regarding the reduction of ion losses to the chamber walls.

**Simulation Model**

The simulation is performed on a rectangle domain $W_x \otimes W_y$ as shown in Fig. 1, where the magnetic cusp is formed by a magnet attached to the chamber wall surface. Here, $x$ denotes the distance along the wall surface measured from the center line of the magnet and $y$ is the distance normal to the wall surface. The cusp region is enclosed by the wall, the field-free plasma, and the intercusp regions through the wall surface, plasma boundary, and side boundaries, respectively.

![Fig. 1 Simulation region and magnetic field lines.](image)

This simulation code is based on the electrostatic approximation, i.e. the electric field is found self-consistently, while the magnetic field has a given form. It uses charged particles moving about due to the forces from their own electrostatic field and the applied magnetic field. For simplicity, the following assumptions are used:

1) magnetic and electric fields are two-dimensional,
2) charged particles are collision-free in the cusp region.

The magnetic vector potential $A$ in two-dimensional case can be written as

$$A = (0, 0, A_z).$$

The particle motion is governed by the Newton-Lorentz equation

$$m \ddot{r} + eZ \{E + \dot{r} \times (\nabla \times A)\},$$

where $Z$ is the number of charges on a particle, i.e., $Z= -1$ for electrons and $Z= 1$ for singly charged ions. Using eq. (1), the $z$-component of eq. (2) can be integrated with respect to time and arranged into the form

$$m \dot{z} + eZA_z = \text{constant},$$

which makes it unnecessary to compute the $z$-position of particles in each cycle of the PIC code.

The relationship between the particle position and the field quantities is made by first calculating the number density of particles on the grid. Once ion and electron densities are established on the grid, the electric potential is found from the Poisson's equation

$$\Delta \varphi = -\frac{e}{\varepsilon_0} (Zn_i - n_e).$$

Then, the electric field $E$ is calculated from

$$E = -\nabla \varphi.$$
\[
\mu = \frac{m_i}{m_e},
\]
\[
\nu = \frac{\omega_{pe}}{\omega_e},
\]
\[
\omega_{ce} = \frac{eB_0}{m_e},
\]
\[
\omega_{pe} = \frac{4\pi n_e e^2}{m_e}.
\]

Where \(n_o\) is the ion number density averaged over the cusp region and \(B_0\) is the field strength on the magnet surface. \(\nu\) is given as an input parameter. We also obtain the nondimensional canonical momenta for electrons and ions, \(v_{0,e}\) and \(v_{0,i}\), which are invariant during the particle motion.

\[
\tilde{v}_{0,e} = \tilde{z}_e - \nu \tilde{A}_z
\]
\[
\tilde{v}_{0,i} = \tilde{z}_i + \nu \tilde{A}_z.
\]

In addition, the Poisson's equation in dimensionless units is written as

\[
\Delta \varphi = -(Z \tilde{n}_i - \tilde{n}_e).
\]

**Structure of the PIC code**

In general, a PIC code has a computational cycle consisting of five steps:\textsuperscript{4}

- solving the field equations on a grid,
- adding new particles,
- interpolating the fields onto the particle positions,
- integrating the equations of motion,
- assigning charge and current densities to the grid.

The outline of a cycle of the PIC code used in this work is shown in Fig. 2. The individual modules are as follows.

- Preparatory phase: This step determines the volume of the cells, the magnetic field, boundary condition of the space potential, and the initial electric field at the grid points. In addition, test particles are scattered with given velocities on the simulation zone. Random numbers are used to give the initial positions and velocities to them.
- Interpolation: To apply the electric and magnetic forces at the particle, the fields should be interpolated from the grid point to the particle position. Using the interpolating function \(S\), the electric field \(E_i\) at the particle position \(r_i\) is obtained from the grid field \(E_j\), as

\[
E_i = \sum_j E_j S(r_j - r_i).
\]

\(S\) is designed so that charge on the grid is the same as the total particle charge, satisfying

\[
\sum_j S(r_j - r_i) \Delta x \Delta y = 1.
\]

**Fig. 2** A cycle of the PIC code.

We have chosen the second-order spline as an interpolating function\textsuperscript{5} to obtain an accurate and stable solution, although the first-order function is more commonly used. Then, the magnetic field and vector potential at the particle position are calculated using the following equation\textsuperscript{6}

\[
A_z = -B_0 \frac{L_z}{\pi} \sin \left( \frac{\pi x}{L_x} \right) \exp \left( -\frac{\pi y}{L_y} \right).
\]

- Particle pusher: Each particle is moved independently with the others by means of the forces acting upon it.
- Particle addition: As particles which reach boundaries are lost from the simulation zone, new ones are supplied to it. An escaping ion is replaced by an incoming ion so that the total number of ions remains constant. The number of electrons to be added in a cycle, \(\Delta N_{ep}\), is determined from the relation as

\[
\Delta N_e = \Delta N_{ep} + \alpha \Delta N_{iw}.
\]

where \(\Delta N_{ep}\) is the number of electrons escaping to the intercusp regions or field-free plasma, \(\Delta N_{iw}\) is the number of ions lost to the wall, and \(\alpha\) is the ratio of electron to ion flux to the wall.
- Localization: In order to obtain the data necessary for the field calculations every particle must be localized.
inside the grid and interpolation weights must be assigned to the particles.

-Density: With the weights determined in the localization step, the number density on the grid is calculated by

\[ n_j = \sum_i n_i S(r_j - r_i) \quad (19) \]

using the same weighting function \( S \) as in eq. (15), where \( n_i \) is the density at position \( r_i \). If different weight functions are used in eqs. (15) and (19), a gravitation-like instability may occur.

-Potential: The electric potential is computed by solving the Poisson's equation using the Finite Element Method.

-Field: The electric field is determined from the potential by numerical differentiation.

In the PIC code, it is essential to calculate the induced electrostatic field that influences particle motions. In such a case, the spatial and temporal conditions

\[ \Delta x, \Delta y \leq \lambda_D \quad (20) \]
\[ \Delta t \leq \frac{1}{\omega_{pe}} \quad (21) \]

should be satisfied to obtain a precise and stable solution, where \( \Delta x, \Delta y \) are spatial grid widths and \( \Delta t \) is a time step. If either condition is violated, the solution diverges.

Results and Discussion

We choose \( W_z=W_z=8\lambda_D \) in order to reduce the computation time to a reasonable length and divide this zone to 16x16 cells to satisfy eq. (20), resulting in \( \Delta x=\Delta y=0.5\lambda_D \). The total number of ions, \( N \), is fixed at 5120. It was chosen so that large amplitude plasma oscillations would not occur during computation. \( \Delta t \) was chosen so that it satisfies eq. (21) and the cyclotron motion of electrons can be precisely followed.

We assume that, unless noted otherwise, 1) the wall is at the floating potential, which corresponds to \( \alpha=1 \), 2) there is no ionization in the cusp region, that is, all of the particles come from the field-free plasma with a uniform flow velocity added to their Maxwellian velocity. The flow velocity is self-consistently determined, 3) \( L_x=L_y=8\lambda_D \), which corresponds to the case that the mirror ratio, \( R \), is equal to 23.1. Here, \( R \) is defined as the ratio of the magnetic field strength at the wall surface to that at the plasma boundary on the \( y \)-axis and can be calculated from eq. (17), and 4) \( v=20 \). Moreover, in all cases, we adopted the conditions that 1) all of the ions are singly charged, 2) \( \mu =1024 \), and 3) \( T_e/T_i=0.1 \).

Figure 3 shows a typical distribution of the electron density, ion density, and space potential. As seen in the figure, both the electron and ion density distributions have their peaks at the center of the cusp. In the space potential distribution, there exists a potential valley at the center of the cusp. These trends qualitatively agree with the measured plasma properties reported in published papers.

Figure 4 shows the density distributions in case of a weaker magnetic field strength than that of Fig. 3. With such a weak magnetic field, both the electrons and ions are distributed more broadly and lost to the wall more easily.

In Fig. 5, the number of ions lost to the wall per unit of the nondimensional time are plotted as a function of \( \xi \), for two different mirror ratios with the same...
where $I$ is the total number of ions lost to the wall per unit time. Thus, an increase in the total number of ions lost to the wall corresponds to a decrease in the confinement time.

The 'leakage half-width' is defined as the width at one half the maximum height of the curve representing the flux of ions lost to the wall as a function of $\tilde{x}$. In Fig. 6 the half-width $w_H$ is plotted as a function of the mass ratio. The hybrid and ion Larmor radii are also plotted for comparison. $w_H$ is closer to the hybrid radius than to the ion Larmor radius, which agrees with the experimental data.\(^8\)

Figures 3-6 show results obtained when neither ions nor electrons are produced in the cusp region but flow through the plasma boundary into the cusp region from the field-free plasma. In Fig. 7, the electron and ion density distributions are shown for the case when the ion production rate is proportional to the electron density in the cusp region. This corresponds to the case when ions are produced by ionization collisions of Maxwellian electrons with neutral atoms. When a new pair of electron and ion is added, a random number is generated to determine their initial position from the cumulative distribution function for electron density. That function is reevaluated at each time step by taking into account the number of electrons in each cell. As seen in Fig. 7, both the distributions shift to the wall.

This simulation makes it possible to examine the effect of the wall potential, while the previous numerical code based on the diffusion model cannot. Figure 8 shows ion loss flux distributions with three different wall potentials. The wall potential can be changed along with the ratio of electron to ion flux to the wall, $\alpha$. Here, $\alpha=0, 1, 2$ were chosen to simulate the cases with a cathode potential, a floating potential, and an anode potential, respectively. This is based on the fact that when the wall potential is equal to the cathode potential, no electrons can approach to the wall and that when the

Fig. 4 Density distributions with weak magnetic field strength ($\nu=5$).

Fig. 5 Effect of mirror ratio on ion loss flux distribution.

maximum magnetic field strength. At the smaller mirror ratio, more ions are lost to the wall, particularly toward the center of the magnet. Define the ion confinement time $\tau$ by the equation

$$\tau = \frac{N}{I}. \tag{22}$$

Fig. 6 The effect of mass ratio on the leakage half-width.
The effect of interelectrodes can also be examined. The role of interelectrodes is described in Ref. 10. When a set of ring-shaped electrodes, biased positively with respect to the chamber wall, was installed in the region enclosed by the cusp fields, it was found that the ion density distribution becomes sharper as shown in Fig. 9, and that the discharge performance can be improved. To theoretically support this phenomenon, we performed a simulation for the case that interelectrodes are equipped. The interelectrodes, whose potential is set much higher than the wall potential, was located at the position that electrons can not flow freely into them. Figure 10 shows a comparison of the ion density distributions with interelectrodes and without the interelectrodes, where electrodes of a height $2\lambda_D$ are equipped with their center at $(-4\lambda_D, 2\lambda_D)$ and $(4\lambda_D, 2\lambda_D)$. It is seen that the existence of positively biased interelectrodes makes the ion density distribution sharper, and therefore reduces the ion loss flux to the wall. The results suggest that the ion confinement can be improved not only magnetically but also electrostatically without deterioration of the electron confinement. As the result, this code can be used to optimize the position of interelectrodes.

![Fig. 7 Density distributions with ionization in the cusp region.](image)

![Fig. 8 Effect of the wall potential on the ion loss flux distribution.](image)

wall acts as an anode, more electrons can arrive at the wall than ions. In this figure, it is shown that an increase in wall potential reduces the ion loss and makes the ion confinement more effective.

![Fig. 9 Measured ion density distributions.](image)

![Fig. 10 Calculated ion density distribution at $y=4\lambda_D$ ($\alpha=2$).](image)
Concluding Remarks

A particle simulation code based on the PIC method was developed to investigate the physical phenomena and ion loss mechanisms in the magnetic cusp of an ion thruster. Using this code, distributions of electron and ion densities and the space potential were obtained. Their trends qualitatively agree with those widely known, indicating the validity of this model. As the ion loss rate can be numerically estimated, this code can be used to obtain information for improving the ion confinement. However, some of the calculation conditions used here, for example, the values of mass ratio and cusp region area, are different with actual values of discharge chambers. This is due to the fact that this simulation is very time-consuming even with recent work stations (in most cases, CPU-time is more than several hours with HP730). Then, if it were not for restrictions in time and cost, this code would be able to determine the ion loss rate and ion confinement time quantitatively for actual thrusters, i.e. ring cusp thrusters utilizing argon/xenon propellants.

References