Investigation of Electron Energy Distributions in an MPG Arc Jet Flow with Electrostatic Probes

Harald A. Habiger, Monika Auweter-Kurtz, Helmut Kurtz

Institut für Raumfahrtsysteme, Universität Stuttgart
Pfaffenwaldring 31, 70550 Stuttgart, Germany

Abstract

For the development of electric propulsion devices, the investigation of fundamental plasma parameters is essential in order to improve the knowledge of the physical mechanisms. The electron temperatures, electron densities, plasma potentials and electron energy distribution functions of plasma jets generated by magnetoplasmadynamic devices and thermal arcjets can be obtained by cylindrical electrostatic single-, double- and triple-probes. The evaluation of electrostatic probe data is normally based on the assumption of a Maxwell electron energy distribution. If the distribution is not Maxwellian, the logarithmic single probe characteristic evaluation will not give correct results of the electron temperature. Evidence for deviations from the Maxwell energy distribution has been found in arc heated plasmas generated by magnetoplasmadynamic plasma generators (MPG) and thermal arc jets. Information about the nature of the electron energy distribution function is included in the second derivative of the single probe characteristic. This paper will report first efforts to investigate the electron energy distribution function in an MPG air plasma flow by means of an numerical derivation.

Introduction

At the Institut für Raumfahrtsysteme of the University of Stuttgart, stationary thermal arcjet thrusters and MPD-self-field thrusters are under development and investigation. Based on the experience with MPD thrusters, a nozzle type MPD device was modified for operation as a plasma generator (MPG) for reentry simulation in two plasma wind tunnels [1]. The properties of the plasma plumes of the thrusters and of a certain plasma condition generated by the MPG, are experimentally investigated by optical means [2, 3], mechanical probe methods [3], mass spectroscopy [4] and a variety of electrostatic probe methods [5, 6] as well as by numerical calculations of the plasma conditions. Special interest is focussed on MPG plasma conditions, which simulate the conditions which were calculated to result in a maximum temperature at the leading edge for the re-entry of the former European spaceplane project HERMES [7]. The parameters for the flight environment are: a velocity of 7452 m/s at an altitude of 81.3 km, which corresponds to a specific free stream enthalpy of 28 MJ/kg, a specific mass flow rate of 0.117 kg/(m²-s) and a total pressure of 4 mbar. The investigations of the corresponding arc heated MPG air flow conditions are performed in the PWK2-IRS. The air plasma flow was generated by the MPG plasma source RD5 which is shown in Fig. 1.

Fig. 1: Plasma generator RD5 - of the PWK2-IRS.

With an arc-current of 1200 A, a total mass flux of 2.0 g/s air and an ambient pressure of 2.9 mbar, the required specific free stream enthalpy of about 28 MJ/kg and a total stagnation point pressure of about 4 mbar at an axial distance of 467 mm to the plasma source are obtained.
For the determination of electron temperatures and electron densities, different types of electrostatic probes with cylindrical electrodes, as shown in Fig. 2 for a single probe, are used.

Fig 2: Electrostatic single probe in sensorhead

The single probe electrode is made of tungsten wire, variable in diameter between 0.1 to 1 mm but typically 0.4 mm, and a length \( l \) is in the range of 10 to 20 mm. The isolation of the probe electrode is made of alumina tubes. The electrode respectively the alumina tubes are mounted in a cylindrical sensorhead made of brass. The connection to the data acquisition is performed by heat resistant and isolated copper wires which are pinched to the electrode. The complete sensorhead can be mounted to a water cooled probe support which is again mounted on the position system in the PWK. The voltage between the probe electrode and the reference electrode, which is the support system in contact with the plasma, is applied by an external power supply of ±30 V at up to 10 kHz sweeping rate. The probe current is measured as the potential drop over a high precision shunt. A transient recorder with a high input impedance records the probe current-voltage characteristic.

The applicable-probe theory had to be chosen with respect to the Debye-length \( \lambda_D \), the mean free paths of the charged plasma particles \( \lambda_{ei} \) and the probe radius \( r \). For plasmas in which the thickness of the sheath forming around the electrodes, given by the Debye length, is smaller than the mean free path of the charged particles, the particle movement can be assumed collisionless (\( \lambda >> \lambda_D \)). Considering charged particle densities in the range of \( n_e = 10^{17} \) to \( 10^{20} \) m\(^{-3} \) and electron temperatures \( T_e \) of 1000 to 20000 K in the investigated plasma flow region covering axial distances from 117 to 600 mm, this results in Debye-lengths of \( \lambda_D < 40 \mu m \). The Coulomb mean free paths are in the range of \( \lambda_{ei} \geq 1 \) mm and therefore the plasma sheath can be assumed collisionless (\( \lambda >> \lambda_D \)) [8, 9]. These estimations are based on the assumption of only single ionized particles of mass \( m_i \) and the quasi-neutrality \( n_i = n_e \) of the investigated plasmas. Since it is assumed that only dissociated nitrogen and oxygen is ionized, the plasma parameters are calculated using the atomic mass of nitrogen which is close to the mass of oxygen. It has been experimentally verified [10] that the collisionless theory of Laframboise [11] for cylindrical probes in a Maxwellian plasma is valid, if the probe is aligned with the plasma flow and if the probe is long enough to ensure "End effect" parameters [12] of \( \tau_1 \geq 50 \).

In order to verify the assumption of Maxwell distributions of the electron energies in the investigated plasma conditions, electrostatic single probes must be used due to their sensitive response to electron currents [9, 13].

Single probe theory

Electrostatic single-probes, also known as classical Langmuir probes, are most commonly used for plasma diagnostics. The current \( I \) to a probe is measured as a function of the applied potential \( V_S \). For large negative probe voltages \( U = V_S - V_P \), all electrons are rejected by the probe, only ions are drawn by the probe, resulting in an ion saturation current. For less negative probe voltages in the electron retarding region of the probe characteristic, an increasing amount of electrons contributes to the net current drawn by the probe while the ion current part decreases. At the floating potential \( V_F \) no net current is drawn since the ion current \( I_i \) equals the electron current \( I_e \). At probe voltages positive with respect to the plasma potential \( V_P \), the ions are rejected and only electrons contribute to the probe current. The values of the electron saturation current are much higher than that of the ion saturation current due to the higher thermal motion of the electrons. The shape of the probe characteristic especially in the electron retarding region, where the transition from ion to electron current occurs, is governed by the electron energy distribution function.

For a Maxwellian electron energy distribution function in a quasineutral, flowing plasma, the electron current density \( j_e \) at a probe voltage \( U \) with respect to the plasma potential is given by [9]:

\[
j_e = j_{eo} \exp\left(\frac{eU}{kT_e}\right)
\] (1)
Here $j_{eo}$ is the electron current density at the plasma potential with $U=0$ and given by:

$$j_{eo} = e_n e_0 \sqrt{\frac{kT_e}{2\pi m_i}}.$$  \hspace{1cm} (2)

Here $m_i$ is the ion mass, $n_e$ is the electron density, $e$ the electron charge and $k$ the Boltzmann constant. Taking the logarithm of Eq. 1 yields

$$\ln j_e = \ln j_{eo} + \frac{eU}{kT_e}.$$  \hspace{1cm} (3)

and by differentiating one obtains the formula:

$$\frac{d \ln (j_e/j_{eo})}{dU} = \frac{e}{kT_e}.$$  \hspace{1cm} (4)

If the electron energy distribution function is Maxwellian, the electron temperature $T_e$ can be obtained by plotting the slope of the logarithmic electron current in the retarding region versus the probe potential. This is only applicable if the electron diffusion current to the probe is not altered by electron-neutral collisions. Since the neutral mean free paths $\lambda_n$ are larger than the electron mean free paths $\lambda_{ee}$, this can be neglected. By the theory of Laframboise [11], the ion current density $j_i$ to a single-probe aligned with the plasma is given by

$$j_i = e n_e \sqrt{\frac{kT_e}{2\pi m_i}} i_i (\chi_{p}, \frac{T_i}{T_e}, \frac{r}{\lambda_D}).$$  \hspace{1cm} (5)

The correction factor $i_i$ depends on the normalized plasma potential $\chi_{p} = e(V_{p}-V_S)/kT_e$, the Debye-ratio $r/\lambda_D$ and the temperature ratio $T_e/T_i$. The correction factor can be set to 1 in good approximation if the Debye-ratio is greater than 50, which can be achieved by choosing the corresponding probe radius $r$ in accordance with this requirement and if the ion current is obtained by the extrapolation of the saturation current region to values of small normalized probe potentials $\chi_{p}$ [11]. The plasma potential $V_{p}$ can be obtained by the shape of the logarithmic electron current to a single-probe. The plasma potential is given by the extrapolation of the linear electron retarding region and the saturation region of the $\ln(I_e)$ [9]. From the measured ion current, which is $I_i = j_i A$, the electron density is calculated for a known probe surface area $A$.

In the case of deviations from the Maxwellian electron energy distribution, i.e. one has another isotropic energy distribution function, this distribution function can be obtained from the probe characteristic in the retarding region at $V_S < V_{PL}$. The electron current $I_e$ to a probe electrode of area $A$ includes the electron current density $j_e$, which is basically determined by the electron energy distribution function. Thus

$$I_e = -A e \pi \int_{0}^{\infty} f \left( \frac{1}{2} \frac{m_e}{m_i} v_e^2 - eU \right) v_e^3 dv_e.$$  \hspace{1cm} (6)

[9]. The function $f$ depends on the electron velocity $v_e$ on the probe surface and the probe voltage $U = V_S - V_{PL}$. The term in brackets is the electron energy $E$ in case of a collisionless movement of an electron from the plasma to the probe surface. With the following relations

$$v_e^2 = \frac{2(E+eU)}{m_e},$$  \hspace{1cm} (7)

and

$$v_e dv_e = \frac{dE}{m_e},$$  \hspace{1cm} (8)

Eq. 6 can be rewritten to

$$I_e = -A e \frac{2 \pi e}{m_e} \int_{-eU}^{\infty} f(E) (E+eU) dE.$$  \hspace{1cm} (9)

If now the electron current is differentiated twice with respect to $U$ and the resulting expression resolved for the velocity distribution function $f(E)$, one yields

$$[f(E)]_{E=-eU} = -\frac{1}{A 2 \pi e} \left( \frac{m_e}{e} \right)^2 \frac{d^2 I_e}{dU^2}; \hspace{0.2cm} U < 0$$  \hspace{1cm} (10)

To convert the velocity distribution function $f(E)$ to an energy distribution function $F(E)$, the relation

$$F(E) dE = 4 \pi v_e^2 f(E) dv_e.$$  \hspace{1cm} (11)
can be used [9]. The left hand side of Eq. 11 is the number of electrons per unit volume in the energy range \( E+\Delta E \) and \( E = 0.5 \, m_e \, v_e^2 \) [3]. The energy distribution function of the electrons is thus given by

\[
F(E)_{E=-eU} = \frac{4}{\pi e^2} \sqrt{\frac{-m_e \, U \, d^2 I_e}{2e \, dU^2}} .
\] (12)

The shape of the second derivative of the electron current \( I_e \) to a single-probe, with respect to the probe voltage \( U \) is therefore proportional to the shape of the energy distribution function of the electrons which can be numerically fitted. To do this, the plasma potential \( V_p \) must be known since the probe potential must be measured with respect to the plasma potential.

**Determination of the second derivative**

Different ways exist to evaluate the second derivative of the electron current flowing to a negative probe electrode. In the electron retarding region of the probe characteristic, the change of probe current is mainly driven by the change in the electron current part which is much greater than the change in the ion current. Thus it can be assumed that the second derivative of the total probe current is essentially the same as the second derivative of the electron current [9].

One way to obtain the second derivative is to feed the signal twice through an electronic differentiating circuit. Another method, which is the most commonly used, is the superposition of an alternating potential on the probe voltage sweep. On this voltage sweep, an AC signal with a higher frequency and amplitudes of a few mV is superimposed. This superposition of a small AC signal leads to the occurrence of harmonics in the resulting probe current. The amplitude of the second harmonic of the current is then proportional to the second derivative of the probe current and therefore to the electron energy distribution function [13].

Both of the above described methods are applicable for rather smooth probe characteristics without fluctuations in the probe current. For the investigated air plasma flow, fluctuations in the current density in the saturation regions as well as in the interesting electron retarding region occur as can be seen in Fig. 3 and 4 in the following chapter. Having these fluctuations of large amplitudes, it is not possible to clearly resolve the second harmonic amplitudes of the superposition method without a Lock-in amplifier. Such a device was not available.

In order to handle the obtained probe characteristics, it is more efficient to treat the characteristics numerically using the opportunities offered by modern computer systems. In the following, the applied method to obtain the second derivative is explained.

The probe current is measured over a linear sawtooth voltage ramp starting from -25V to +25 V with a rising time of about 5 ms. The probe current is measured as potential drop over a 1 \( \Omega \) precision shunt. Applied voltage and the current signal are recorded by a fast transient recorder as current over the applied voltage. The probe characteristic is directly displayed and stored as ASCII-file. The data is then transferred to a fast personal computer for further evaluation.

As a first step, a special program is used to evaluate the electron temperature using Eq. 4 (semi-logarithmic method) and the electron density based on the assumption of a Maxwell electron energy distribution. A first validation of this assumption can be obtained from the linearity of the semi-log plot of the electron current.

A second program is used to evaluate the electron energy distribution function. First the fluctuations of the probe characteristic must be smoothed. This can be done by either calculating the average value over a window of certain data point, which is crucial, especially in the retarding region of the characteristic, or by fitting an analytical function to the probe characteristic. For a least square fit, the function

\[
I = A \arctan \left( \frac{U + B}{C} \right) + C
\] (13)

proves to be applicable for the investigated probe characteristics. An MS-Excel-solver is used to adapt the coefficients A,B and C for each probe characteristic [14]. As a next step, having the smoothed or fitted data values, a direct numerical derivation is applied. By using the formula

\[
\frac{\delta^2 I_\phi}{\delta U^2} = \frac{1}{12 \Delta U^2} \left( -I_2 + 16I_1 - 30I_0 + \ldots \right) + O(\Delta U^4) ,
\] (14)

\[
\ldots + 16I_{-1} - I_{-2} \right)
\]
the local second derivation of the data point \( I_0 \) can be calculated by including the two data points before (\( I_2 \) and \( I_{-1} \)) and after (\( I_2 \) and \( I_1 \)) into calculation [15]. The value \( \Delta U \) is the constant voltage difference between two data points. The last term on the right hand side represents higher orders and can be neglected for the calculation. In order to compare the obtained experimentally second derivation with a calculated energy distribution function, either Maxwell or any other isotropic function, it is necessary to determine the plasma potential in order to set the origin of the distribution function.

**Experimental results and discussion**

For discussion of the results, the axial distance to the MPG exit plane is notated as x-position, whereas the y-position refers to points lying perpendicular to the plasma flow centerline, which is the y=0 mm position. The measurements were performed with single probes of 12 mm electrode length and 0.19 mm radius at two x-positions. An erosions of the probe electrode and thus a change of the probe area \( A \) was taken into consideration, assuming a linear decrease to the final probe dimension after the test.

Fig. 3 shows two single probe characteristics on the centerline of the plasma flow at \( x = 117 \) mm and \( x = 467 \) mm. The later position corresponds to the required HERMES flight conditions.

As already mentioned, it can be seen, that fluctuations in the probe current occur. Without any clear ion saturation region, the shape of the characteristic at \( x = 117 \) mm differs remarkably from the one at \( x = 467 \) mm. Both characteristics were smoothed for evaluation, as described in the previous chapter. Regarding the two different current axis, the ion saturation current density increases by a factor of >75.

In Fig. 4, two characteristics at two different y-positions at \( x = 467 \) mm are shown. The centerline characteristic at \( y = 0 \) mm was fitted using Eq. 13. Both characteristics are of almost similar shape, resulting in only a small decrease in the electron temperature \( T_e \) of about 1500 K in y-direction, from the centerline to \( y = 60 \) mm. This temperature difference was also confirmed by electrostatic double probe measurements [3]. The floating potentials \( V_F \) are at about 2-3 V probe voltage.

In Fig. 5 and 6, the electron temperature \( T_e \) and the electron density \( n_e \) are shown as a function of the distance x to the MPG exit plane with standard deviation error bars. For validation of single probe results, also measurements with electrostatic double- and triple probes are shown [3, 6]. These figures also include the results of emission spectroscopy at positions close to the plasma source [3]. The positions of the electrostatic probe measurements correspond to the region of material tests in the plasma wind tunnel. The effect of the Doppler-linebroadening, measured with Fabry-Perot Interferometry (FPI), was used to determine the kinetic temperature \( T_k \) of the heavy particles and yielded a value of 10200 ± 1700 K in the centerline at a distance of \( x = 50 \) mm [2]. Together with the spectroscopically measured value of
Te = 13200 K, this results in a rather small degree of thermal nonequilibrium of Te/Tk = 1.3. The difference in the electron density ne, see Fig. 6, from \(x = 100\) mm, measured with emission spectroscopy, to \(x = 467\) mm is about 100. This compares well to the difference of >75 in the ion saturation current density, a sign for the quasineutrality of the plasma.

A first validation of a Maxwell electron energy distribution is possible by plotting the logarithmic electron current versus the probe voltage. For Maxwellian electrons, there should be a linear slope in the retarding region around the floating potential \(V_F\). The plasma potential \(V_{pl}\) is given by the crossing point of the extrapolated linear electron retarding region and the saturation region of the logarithmic electron current. According to Fig. 7, the electrons on the centerline at \(x = 117\) mm are non-Maxwellian, the plasma potential \(V_{pl}\) can be estimated at about 8-10 V probe voltage.

In Fig. 9 to 12, the experimentally obtained electron energy distribution functions are shown. For comparison, Fig. 9 shows also a theoretical Maxwell distribution function at an appropriate electron temperature \(T_e\). It can be clearly seen that for the positions \(x,y = 117, 60\) mm, \(x,y = 467, 60\) mm and \(x,y = 467, 60\) mm, the electrons do have a Maxwellian energy distribution. The maximum of the EEDF drops as the electron temperature increases, see Fig. 9 in comparison to Fig. 10.
centerline at $x = 117$ mm. High energy electrons are dominant. The shape of the distribution function requires further analytical and experimental investigations.

**Conclusions**

The electrostatic single probe proved to be an efficient method to check for the electron energy distribution function at different positions in a plasma flow. It is possible to obtain the necessary second derivative of the probe current by a solely numerical evaluation of the probe characteristic. This method was successfully applied to an arc heated air plasma flow generated by a magnetoplasmadynamic device, similar to MPD-thrusters. The method can easily be transferred to investigate electric propulsion devices.

For the investigated air plasma flow, there seems to be a deviation from the Maxwell electron energy distribution function at positions close to the plasma source on the plasma flow centerline. Further work is required to achieve an analytical expression for the measured distribution function.

**Acknowledgement**

The authors wish to thank Mr. Thomas Müller for his support in preparing this paper and all other coworkers of the electric propulsion and PWK simulation groups at the IRS for their assistance. This work was supported in part by the Sonderforschungsbereich SFB 259, "Hochtemperaturprobleme rückkehrrfähiger Raumtransportsysteme" of the Deutsche Forschungsgemeinschaft (DFG).
References


Maxwellian Plasma at Rest", UTIAS-Report 100, University of Toronto, 1966.


