ELECTROMAGNETIC WAVES RADIATED BY ION BEAM INJECTED TO IONOSPHERE

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1. Introduction.

Plasma fluxes and beams of charged particles draw the investigators attention to use them for amplification and generation of electromagnetic waves. The principles of excitation, amplification and generation of microwaves in electron fluxes are well studied in such devices as TWT, EWT, magnetron types of devices etc [1].

In ion injection when neutralizing the ion beam charge electrons from a cathode-compensator are accelerated by potential difference from several to tens volts between the beam and cathode, then inside the initial region of ion beams we have the electron movement with the velocity \( v_{be} \gg v_{bi} \) \( (v_{be} \) and \( v_{bi} \) are the velocities of electron and ion beams respectively). Such beam similar to plasma-beam system that is unstable in the sense of excitation of plasma waves. [2], [3], [4].

The waves excited are one of the main reasons of electromagnetic noises generation. They also influence on efficiency of excitation from external generator of electromagnetic oscillations and can influence on electrodynamic characteristic of beam movement in ionosphere plasma. In the latter case we have the complicated multi-component system with several fluxes. The analysis of such systems is rather difficult so in this paper the wave properties of cylindrical plasma flux infinite in longitudinal direction are analyzed and the efficiency of such plasma waveguide excited by the
external generator is excited. Since the problems of matching are of great importance the analysis of these problems on the basis of developed equivalent scheme is given. The excitation of extremely low frequency waves (Alfvén ones and whistlers) in ionosphere by the ion beam is considered in the cold plasma approximation [5].

2. Plasma waves in a coaxial plasma stream.

The object of this paper is to investigate plasma waves propagation in a coaxial cylinder with vacuum boundaries.

2.1. Formulation of the problem

An axial symmetrical system is considered \((d/d\varphi=0)\). The problem is treated in the linear kinetic approach by means of the following integro-differential system:

\[
\frac{d^2 \Sigma}{dr^2} - \frac{1}{r} \frac{d \Sigma}{dr} + \frac{-2 \Sigma}{R^2} = \frac{J_z}{\omega c}\quad (1)
\]

\[
\sum_{\alpha} \frac{df_{\alpha}}{dt} + \bar{V}_{\alpha} \frac{df_{\alpha}}{dr} + \frac{e_{\alpha}}{m_{\alpha}} \bar{E} \frac{df_{\alpha}}{dr} = 0\quad (2)
\]

\[
\delta_{\Sigma} = \sum_{\alpha} e_{\alpha} \int_{-w}^{w} \int_{r}^{\infty} f_{\alpha} \bar{V}_{\alpha} d\Sigma dV_{r}\quad (3)
\]

Where \(\Sigma=\Sigma(0,0,\Pi_{z})\) is an electrical polarisation potential (Hertz's vector) related to the e.m. field by the expression \(\bar{E}=(\text{grad} \bar{V} + \bar{R}^2)\bar{\Pi}\) and \(\bar{E}=i\text{rot} \bar{\Pi}\). \(f_{\alpha}\) - is the disturbed part of the distribution function \((f_{0\alpha})\). \(\bar{r}=(r,\varphi,\Pi_{z})\).

The summation is carried out for electrons \((\alpha=e)\) and ions \((\alpha=\text{ions})\). Boundary conditions have to be added to equations (1-3). The mirror reflection condition [2] is chosen for \(f_{\alpha}\).
\[ f^+_\alpha (R, V_r, V_z) = f^-_{\alpha, V_z} (R, V_r, V_z) \]
\[ f^-_{\alpha, V_z} (a, V_r, V_z) = f^-_{\alpha, V_z} (a, V_r, V_z) \]  \hspace{1cm} (4)

Where \( V_r \) and \( V_z \) are velocity components along \( r \) and \( z \) respectively and
\[ f_{\alpha} = f^+_\alpha + f^-_{\alpha}. \]

The field conduction on the plasma-vacuum
\[ E_z^{p}(R) = E_z^{v}(R), \quad H^{p}(R) = H^{v}(R) \] \hspace{1cm} (5)
\[ E_z^p(a) = E_z^v(a), \quad H^p(a) = H^v(a) \] \hspace{1cm} (6)

Where indices \( p \) and \( v \) denote the plasma and vacuum.

The dependence of the fields, \( f_{\alpha} \) and \( V_z \) is \( \exp(ikr-\omega t) \), where \( k_3 \) and \( k \) are the longitudinal wave number for the plasma and free space respectively.

### 2.2 Dispersion relations.

Solving equation (2) with the boundary conditions (4) and substituting the electrical field components \( E_r \) and \( E_z \) by \( \Pi \), we may find the perturbation function, \( f_{\alpha} \) and therefore the current has the following form

\[ J_z = \sum_{n=0}^{\infty} \frac{\cos \alpha n r \cdot A}{(r^2 - \alpha^2_n); (R-a)} \left[ \Pi(\xi) \cos \alpha_n \xi d\xi \right] \]

\[ - B \sum_{n=0}^{\infty} \frac{\cos \alpha n r \cdot A}{(r^2 - \alpha^2_n); (R-a)} \left[ \Pi(\xi) \cos \alpha_n \xi d\xi \right] \] \hspace{1cm} (7)

Where
\[ A = \sum_{\alpha} \frac{2 t_{\alpha} \cdot dV}{m_{\alpha} \cdot \int_{-\infty}^{z} \frac{r}{V_r} \frac{df_{\alpha}}{dV_r} dV_r} \]
If the result (7) is substituted into expression (1) it will be difficult to solve equation obtained. However, the equation may be easily solved if we accept that $\gamma^2 > \alpha^2_n$; that is accomplished easily for $V_z > V_r$ or when $(R-a)$ is great and $n$ is small. Such assumption is quite true for the electron beam moving in the plasma along axis $z$.

Then if we expand expression $1/\gamma^2 - \alpha^2_n$ in the series and retain only the first terms of the expansion we shall obtain the following equation:

$$
\frac{d^2 \Pi}{dr^2} + \frac{C}{r} \frac{d \Pi}{dr} + \tau^2 \Pi = 0
$$

(8)

Where

$$
C = 1 + \frac{(K^2 + K_3^2)A}{j\omega_o \gamma^2} + \frac{KB}{e_o \omega \gamma^2}
$$

$$
\tau^2 = \left[ (K^2 - \gamma^2) - \frac{(K^2 - K_3^2)}{j\omega_o \gamma^2} \right] C
$$

This equation is easy to solve in terms of cylindrical functions. To obtain the dispersion relation, it is necessary to use the boundary conditions (5) and (6). We consider plasma cylindrical layer at the inside and outside surfaces is bounded by vacuum. The dispersion relation for this case will be:

$$
\frac{T^2 H_o(KR)}{K^2 H_o(KR)} = \frac{D_{\nu}^r(\tau) - Y_{\nu}^r(\tau)}{D_{\nu-1}^r(\tau) - Y_{\nu-1}^r(\tau)}
$$

(9)

Where
The equilibrium electron and ion distribution functions were chosen in order to analyse the equation (9) and assuming the ion and electron thermal velocities to be respectively less and more than the phase velocities of the low frequency waves and the phase velocities of the high frequency plasma waves more than thermal velocities, it is easy to calculate the A and B integrals and, therefore, the $T^2$ and $\nu$ parametres. If the electron stream having the $U$ velocity is moving in the plasma, the expression of $T^2$ parameter for the high frequency plasma waves is

$$T^2 = (K^2_0 - K^2_3)^2 \left[ 1 - \sum_\beta \frac{2 \omega e \beta}{(\omega')^2} - \frac{3 \omega^2 e \beta K^2_3}{(\omega')^4} \right]$$

$$+ \left[ 1 - \sum_\beta \frac{3 \omega^2 e \beta (K^2_0 - K^2_3)}{r^2_3 v_T e} + \frac{\omega e \beta}{(\omega')^2 K^2_3} \right]$$

(13)

Where indices $f$ and $\beta=b$ denote the plasma and the electron stream respectively.
\[ \omega_{e\beta} = \sqrt{\frac{e^2 \beta}{mc^2}} \] - the plasma frequency, \( \omega' = \omega - \omega_{e\beta} \) for summation over \( \beta = \mu \) and \( \omega' = \omega - \omega_{e\beta} \) for summation over \( \beta = \mu \); \( U \) - the electron stream velocity along axis \( z \), \( V_{Te} = (2KTe/m)^{1/2} \) - the electron thermal velocity, \( T_e \) - the electron temperature, \( K^2 = k_0^2 - k_3^2 \); \( K_0 \) - the wave number in the free space.

For \( R \to \infty \) and \( a \to 0 \) eq (9) are transformed into the relation for unbounded plasma [3].

For \( a \to 0 \) the eq (9) are transformed into the dispersion relation obtained in paper [4].

Cylindrical function index \( \nu \) is near \( 1/2 \) with considerable changes of parameters. One can express the cylindrical function through elementary functions \( \sin \) and \( \cos \), then eq (10) for \( a = 0 \) is as follows:

\[ \frac{1}{TR} \cot TR = \frac{1}{K_0(KR)} \left( \frac{K_0(KR)}{K_0^2(K_1(KR))} \right) (11) \]

One can approximate the final expression of the dispersion relation (11) in the following way

\[ T \approx \frac{n\pi}{2R}, \quad n = 0, 1, 2, 3, \ldots \quad (12) \]

From equation (11) and (12) one can see the change of the plasma waves spectrum \( \nu_s \) the coaxial plasma cylinder dimensions (R) and this eq. may be easily solved.

As an example, some of dispersion curves for high frequency plasma waves are shown fig. 2 and 3. These dispersion curves were plotted after calculation of the equation (12). Influence coaxial cylinder dimension (R), thermal electron velocities, the velocity and the density of the electron stream and the phase velocity of e.m. waves are demonstrated by means of these curves.

The dispersion curves were calculated by the eq (12) for \( n = 2 \), \( \omega_s = (\omega_{e\beta} + \omega_{eb})^{1/2} \). \( V_F \) is the phase velocity of plasma waves.

The dispersion equation (11) determines the total spectrum (\( \omega, K_3 \)) of plasma waves and permits to estimate
antenna properties of plasma jet. Under the conditions of experiment the density of plasma stream on the injector cut is \( n_e = 10^{10} \text{ cm}^{-3} \), \( f_p = 10^9 \text{ Hz} \), the frequency of exciting signal is of order 50 MHz. From the calculations performed and taking into account spread in plasma stream parameters we have found that \( K_3 \) ranged in varies from 1 to 10 (m\(^{-1}\)). Knowing \( K_3 \), one can obtain the field distribution of surface wave. For large \( KR \) the distribution varies exponentially \( E_z = E_{zm} \exp(-K_3r) \). For \( K_3 = (1...10)\text{m}^{-1} \) the field intensity reduces in \( 10 \) times in the distance \( r=1..0.1\text{m} \). The size of string exciter must be within the same limits.

3. The conditions of wave excitation and matching of the exciter with HF generator.

To obtain maximum value of coefficient of transformation of generator oscillation into surface wave it is necessary that field structure formed by exciter is close to the field structure of cylindrical waveguide. In the conditions considered the excitation is made by the 20 cm diameter ring. This ring is placed into the volume around beam plasma with density \( n = 10^6...10^8 \text{ cm}^{-3} \) at a distance \( lm \) from injector. This the plasma antenna is a synthesis of metal ring, ion beam and the volume of near beam induced plasma. If we assume that the spatial distribution of critical plasma density is like cosine form at distance of about \( 1m \) from the injector one can find the efficiency of transformation of electromagnetic waves which are excited by this radiator.

The calculation show that for \( K_3 = 10\text{m}^{-1} \) and \( r=1m \) the efficiency of transformation is about 11% and for \( K_3 = 1\text{m}^{-1} \) the efficiency is about 7%. For the 6W generator the energy of surface wave will be not less than 0.6W. That is enough to receive the signal on the ground. However the
problem of generator and beam matching is very important. To determine the influence of matching one can use the equivalent electric scheme in the fig.4, where

- \( \text{II} \) - is the ion injector, \( E \) - exciting ring;
- \( C' \) - capacity between beam and \( E \);
- \( C'' \) - capacity between \( \text{II} \) and the space;
- \( C_{\text{in}} \) - input capacity of a coaxial cable;
- \( Z_1 \) - impedance between the injector and exciting ring;
- \( Z_{\text{PA}} \) - impedance of plasma antenna;
- \( r_i \) - internal resistance of the generator;
- \( U_g \) - output HF voltage of unloaded generator;
- \( U_{\text{PA}} \) - effective voltage at the input of plasma monovibrator.

We assume than \( Z_{\text{PA}} \) is the active impedance which equal 100...1600 Ohm, \( C_{\text{in}}=75 \text{pF} \), i.e. \( X_{\text{in}}=1/\omega C_{\text{in}} \approx 48 \text{ Ohm} \) for \( f=45 \text{ MHz} \); \( C''=11 \text{pF} \), \( X'=313 \text{ Ohm} \).

\[ Z_1=\frac{L}{s\sigma} \] - impedance of initial region of plasma beam where \( \sigma=(\varepsilon-1)i\omega \varepsilon_0 \) is the conductivity of plasma that are formed by the beam on the cut of the injector, \( \varepsilon \) is dielectric plasma permeability. \( Z_1=X_1=4 \text{ Ohm} \).

Taking above estimation one can write the expression for generator load at points 1-1

\[
Z_{11} = X_{\text{in}} \frac{r_{\text{PA}}^2 (X_1 - X'') - X' (r_{\text{PA}}^2 + (X_1 - X'')^2) - r_{\text{PA}} (X_1 - X'')^2}{r_{\text{PA}} (X_1 - X'')^2 + i \left[ r_{\text{PA}} (X_1 - X'') - (X' - X_{\text{in}}) \right] \left( r_{\text{PA}}^2 + (X_1 - X'')^2 \right)}
\]

Using above values for equivalent scheme one can obtain \( Z_{11}=-i45 \text{ Ohm} \), i.e. the generator operates with capacity load \( X'_{11}=45 \text{ Ohm} \). For \( r_{\text{PA}}>>X'' \) the voltage value \( U_{\text{PA}} \) can be defined by the following formula

\[
U_{\text{PA}} = \frac{X''-X_1}{X'+(X''-X_1)} \approx 0.1 U_{11}
\]
Where \( U_{11} = U_0 \frac{X_{11}}{\sqrt{r_1^2 - X_{11}}} = 26.5 \text{ V} \)

This is for the used excitation scheme of plasma antenna the efficiency of radio frequency generator is less than 10% for voltage and less 1% for power. In the case when \( Z_{pA} < 1600 \text{ Ohm} \) and taking into account the influence of near beam plasma the power of signal radiated by plasma antenna can increase by a factor of 10.

4. Oscillations in the circuits, gas discharge and beam of ion injector.

The electromagnetic processes in the ion injector can be divided into the internal and external ones. The internal problem includes the consideration of plasma instabilities in gas discharges, fluctuation in injector electric circuits and noises inside spacecraft. The external problem is connected with plasma instabilities in plumes, interactions of plasma jets with electronic equipment and environment.

Oscillation phenomena in gas discharge chambers play a substantial role in generation of electromagnetic fields in circuits, beam, jet and environment. They determine input excitations for injector plumes and experiments have shown that characteristic of fluctuations in the circuits and jets are closely interconnected. There are many gas discharge instabilities and oscillations. For example they can arise due to the processes of ionization \((5 \times 10^4 \text{ ... } 10^5 \text{ Hz})\), electron and ion drifts in crossed electric and magnetic fields \((10^5 \text{ ... } 10^7 \text{ Hz})\), cyclotron \((10^6 \text{ ... } 5 \times 10^6 \text{ Hz})\) and Langmuir plasma oscillations \((5 \times 10^7 \text{ ... } 10^{10} \text{ Hz})\) etc.

In spite of respectively low natural frequencies of
circuit filters the spectrum oscillations can be very wide one due to nonlinear performances of the electric circuit components and gas discharges. The oscillation processes can cause the discharge voltage $U_d$ and current $I_d$ modulation.

The measurements of spectral density of the electromagnetic field intensity of ion injector near electrical circuit wires were carried out in wave band $10^4...10^8$ Hz. These measurements demonstrated that magnetic components of electromagnetic LF and HF fields out of an ion injector are larger than electric fluctuation fields, which are located near wires with the discharge current. The spectral intensity of the magnetic fields are illustrated by the fig. 5. There are no electromagnetic fields larger than environment electromagnetic noise at the distances about 150m from injectors. The analysis of experimental results and analytical estimations have allowed to determine the spatial structure of electromagnetic fluctuation fields from injectors.

The theoretical and experimental studies of the plasma jets waves characteristics show that a slow potential and fast electromagnetic waves propagated [1,2,3,5]. Basically excitations of waves are connected both gas discharge and stream instabilities. Therefore we can consider gas discharge chamber as one of subsystems. Electrical circuits and energy system, plumes, onboard systems and ionosphere are other subsystems. All subsystems have different interaction between them. It was found by investigations that every subsystem has both a natural and correlated spectrum of oscillations which are defined by the other subsystem parameters.

The spectrum of interconnected oscillations is often very complicated and there are a lot of harmonics or spectrum modes. In laboratory investigations we have found that the intensity and spectrum of electromagnetic waves radiated into environment depend on both separate
subsystem parameters and whole complicated system. One can note that the spectrum of plasma oscillations in plumes is changed from gas discharge spectrum (f<10^8 Hz) not strong (see Fig. 5). In plasma streams the spectrum cutoff on the electron plasma frequency. The plasma density is decreased along streams and the spectrum becomes more narrow on large distance from the exhaust of an injector. Therefore plasma spectrum in plumes is determined by plasma gas discharge instabilities and natural instabilities of plasma plumes.

In ion injector one of the strong instabilities is the two stream one which arises due to the potential difference of voltage. The electrons from cathodes (compensators, neutralisators) go into plumes with velocities determined by these potential differences, i.e. the velocities of these electrons are larger than plasma streams ones. As result the stream instability arises. The features of stream instability in plasma plumes have been considered in the part two.

The instabilities as well as the generator of the EM oscillation can excite plasma plumes like waveguides or surface rode dielectric (plasma) antennas which radiate electromagnetic waves along axis of plumes or in any angles to axis.

The plasma instabilities are transformed into transverse electromagnetic wave on the plasma boundary or they induce alternative currents in conductive materials of the injector and spacecraft. These phenomena are the reasons of electromagnetic noise radiations which form interferences for electronic systems. To verify the theoretical and experimental laboratory experiments the onboard measurements of electromagnetic fields in operating conditions of plasma thrusters-SPT in near satellite environment (f=5...50 kHz, 0.5...1.2 MHz, 300...180...200 MHz) were carried out. Onboard
measurements are correlated to laboratory results. The
good example of this suggestion is the laboratory and
flight time performances of changes of HF waves intensity
(Fig 6)

5. The electromagnetic waves excitation by charged
particles beam and their propagation in ionosphere.

The conditions of wave excitation by the beam in the
extremely low frequency wave band for the orbit height
300. 400km can be analyzed by means of dispersion
equation for the interaction of infinite beam with
infinite plasma [4]. Even in this simply case the
dispersion equation solution for the wave propagation in
arbitrary direction in infinite ionosphere plasma in the
presence of the ion beam has the complicated character.
The relatively simple analytical equations for wave
frequency spectrum can be obtained in the limiting cases
of beam motion along and transverse to the geomagnetic
field. In the general case of the arbitrary direction of
the wave propagation for the longitudinal injection the
derivation of analytical solutions for dispersion
equations are rather difficult. The simply formulas can be
derived for the low frequency region \( \omega < \Omega_i \) (\( \Omega_i \) is the
ionosphere plasma frequency) for Alfvén and the
magnetogasdynamical wave spectrum. At the frequency band
\( \Omega_i < \omega < \Omega_e \) (\( \Omega_e \) is the ionosphere electron-cyclotron
frequency) the ion component of the beam excites the
electron whistlers. One can be concluded from estimations
that the ion beam excitation for Ar ion beam with 800 eV
energy at the low hybrid frequency is rather effective,
therewith these waves can propagate essentially transverse
to the geomagnetic field.

The comparison of every foregoing increment estimations for ion beam enables to draw a conclusion that
fast magnetosonic waves are excited most efficiently when
the beam injection direction is transverse to the geomagnetic field at the low hybrid resonance frequencies.

The analysis of whistler modes propagation shows, that for quasilongitudinal excitation the whistler modes can propagate to the ground while quasitransverse and transverse excitation the wave rays are captured by the ionosphere waveguide. The estimation show that the ground whistler signal reception must be conducted in the region with the width of ~100km, displaced approximately at 100 km from the underneath satellite point to the north.

6. Conclusions.

The injection of the ion beam into ionosphere is accompanied by the complex of radiophysical processes both in the injector subsystems (circuits, gas discharge, ion beam) and ionosphere. The given estimation for "Arfa" experiment conduction determine possible spectrum different wave excitation and allow to elaborate measurement devices and methods as well as methods of analysis of experimental results.

REFERENCES
