NUMERICAL INVESTIGATION OF COMPRESSION FLOWS IN A PLASMA ACCELERATOR CHANNEL

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Abstract

The paper is a part of the cycle of researches on numerical simulation of physical processes in the high-current Quasi-Steady-state Plasma Accelerator (QSPA) [1,2]. The QSPA is based on the concept of a plasma accelerator offered by Prof. Morozov. It is the up-to-date plasma accelerator which as compared to preceding ones has more complicated design, another scales and parameters.

The QSPA was constructed and is comprehensively investigated in a number of the scientific institutes [1-6]. It is capable to generate high-power plasma streams, and so it is of great importance with the standpoint of its various applications including electric propulsions.

It is remarkable that in studying the QSPA one has to concern practically every section of plasma physics. This is due to that work of the QSPA is connected with a great number of various and insufficiently known plasma processes. Therefore, the QSPA exploration is of fundamental importance as well.

The present paper is devoted to an investigation of so-called compression plasma flows in a channel of the QSPA. The channel is formed by two coaxial electrodes (Fig.1). The electric current running between them produces an azimuthal magnetic field which accelerates the current-carrying plasma down the duct axis. In the case of the truncated inner electrode near its end the electric current has an appreciable axial component, and its magnetic field compresses and heats plasma in a narrow region at the channel axis. This region is called the compression region.

The discharge time is supposed to be much longer than the time of ion flight in the accelerating channel, and so the process may be considered as quasi-steady-state.

The compression phenomenon was first discovered by Prof. Brushlinsky as a result of the numerical investigations of plasma flows in channels of coaxial accelerators during the later sixties [7]. Theoretical analysis had been carried out by Prof. Morozov, and it led him to the concept of the magnetoplasma compressor [8]. At the same time the experimental evidence for this effect was obtained [9].

Our aim in this paper is in detail to study compression plasma flows in a geometry of up-to-date accelerators on the basis of magnetohydrodynamic (MHD) model with allowance for the most substantial physical processes using new computational methods. In particular, the finite conductivity, heat conduction, and radiation of the plasma and impurities [7, 10-13] are considered, with the cases of anisotropic and classical isotropic heat conduction being separately examined.

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The numerical method employed is based on the finite-difference TVD scheme of A. Harten [14]. The main advantage of this method is the high resolution of the shock waves and contact discontinuities, which is highly desirable in describing compression flows.

The steady-state flow regimes are obtained by the stabilization method. The calculations relate to the case of a fully ionized hydrogen plasma.

**Governing Equations and Computational Domain**

Solutions of the steady-state problem of axisymmetric plasma flow across the proper azimuthal magnetic field in the channel shown in Fig.1 were obtained in the process of numerically solving the unsteady MHD equations, which with allowance for the finite conductivity, heat conduction and radiation plasma we write in the form [7]:

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = 0, \quad \frac{\partial \rho \mathbf{v}_\alpha}{\partial t} + \frac{\partial \Pi_{\alpha\beta}}{\partial t} = 0,
\]

\[
\frac{\partial (\rho \mathbf{e})}{\partial t} + \text{div}(\rho \mathbf{e} \mathbf{v}) + \rho \text{div} \mathbf{v} = \text{div}(\kappa \nabla T) + \mathbf{v}^2 \cdot \mathbf{Q},
\]

\[
\frac{\partial \mathbf{H}}{\partial t} = \text{rot}[(\mathbf{v} \times \mathbf{H}) - \text{rot}(\mathbf{v} \mathbf{j})], \quad \mathbf{j} = \text{rot} \mathbf{H},
\]

\[
\Pi_{\alpha\beta} = \rho \mathbf{v}_\alpha \mathbf{v}_\beta + \rho \delta_{\alpha\beta} - \frac{1}{2} (\mathbf{H}_\alpha \mathbf{H}_\beta - \mathbf{H}^2 \delta_{\alpha\beta}),
\]

\[
\kappa = \kappa_e + \kappa_i, \quad \mathbf{v} = 1/R_e, \quad \mathbf{p} = (\gamma - 1) \rho \mathbf{e} = \beta \rho \mathbf{T}/2.
\]

Here \(\Pi_{\alpha\beta}\) is the momentum flux density tensor, \(\kappa_e\) and \(\kappa_i\) are the thermal conductivities across the field of the electronic and ionic components of the plasma, which in the single-fluid model in question differ only with respect to the particle masses, \(R_e\) is the magnetic Reinoolds number which is proportional to the electrical conductivity.

Equations (1) are written in dimensionless variables. As the units of measurement we chose the characteristic values of physical quantities given at the channel inlet. The dimensionless parameter \(\beta\) is the ratio of such gas and magnetic pressures.

The dimensional transport coefficients can be expressed in a known way [10] in terms of the unknown functions. The quantity \(\mathbf{Q}\) is introduced to simulate radiation losses of the optically thin plasma in the form [11,12]. It also includes the radiation of the impurities contained in the plasma on the basis of [13].

In solving the system (1) the boundary conditions are as follows. At the channel inlet (section \(AB\)): \(\rho = 1, \mathbf{T} = 1, \mathbf{H} = (r_A + r_B)/2r, \mathbf{V}_r = 0\). The four boundary conditions correspond to subsonic inflow of the plasma into the channel, i.e., to the condition \(V < \sqrt{\gamma \beta T/2 + H^2/\rho}\) (the plasma velocity is less than the fast magnetic sound speed). The electrodes are assumed to be impermeable, thermally insulated and equipotential. At the channel outlet (section \(CD\)) it is assumed that the flow is supersonic, and the heat fluxes across the field in the \(z\) direction and the radial component of the drift currents are fairly small: \(\delta T/\delta z = 0, \delta H/\delta z = 0\).

For the purpose of a detailed investigation of the compression flows,
it is convenient to divide the channel into two parts (Fig.1): the curvi-
linear accelerating domain ABFE and the domain OEFCD, which is located
beyond the cut OE of the inner electrode and includes the compression
region. The calculations for the latter were separately carried out with
allowance for the mentioned above dissipative processes which are unim-
portant in the smooth flow domain ABFE. The flow in the section EF is
assumed to be supersonic, and in this section the boundary conditions ne-
cessary for the autonomous calculation of the domain OEFCD are taken from
preliminary calculations for the domain ABFE [15]. On the channel axis
the obvious symmetry conditions are imposed. Since current work is fo-
cused on the compression flows, only the domain OEFCD is considered below

Numerical Results

The solution of the system of equations (1) in the domain OEFCD in-
volves the partitioning of these equations with respect to both the phy-
sical processes and the space variables r and z. The method is based on
finite-difference algorithms: Harten's TVD-scheme [13] for solving the
hyperbolic part of the system and an implicit scheme, implemented by
means of the longitudinal-transverse sweep method, for taking into
account the finite conductivity and heat conduction.

The calculation [15] of the domain ABFE, the source of the boundary
conditions in the section EF, was carried out using the flux-corrected
algorithm SHASTA [16].

Dimensional constants participating in the formulation of the problem
are taken from [3].

According to the formulation of the problem an ideal MHD-flow, i.e.,
nonviscous, non-heat-conducting, infinitely conductive and radiationless
plasma flow are completely determined by the parameter $\beta$. The calcula-
tions for such flows have shown [17] that beyond the cut of the inner
electrode a conical shock wave is formed, and there is a region of eleva-
ted values of the plasma density and temperature, the so-called com-
pression region, beyond the shock wave in the neighbourhood of the z axis.

Figures 2, 3a, and 3b show the velocity vectors, the equidistant elec-
tric current lines ($H_r = \text{const}$) and isotherms, respectively, for the value
$\beta = 0.1$. The fragment marked by the rectangle in Fig.2a is on a large
scale illustrated in Fig.2b, where the solid lines are the boundaries
between subsonic and supersonic flows.

Figure 2a shows that beyond the cut of the inner electrode the super-
sonic plasma flow converges to the channel axis and is then reflected
from it. It is seen from Fig.2b that in the vicinity of the cut a recircu-
lation flow is formed. Figures 2 and 3 clearly demonstrate the front of
the forming shock wave. It is also seen that the shock wave is less
strong as a distance from the axis increases.

When the accelerated flow converges, the plasma density increases at
the beginning adiabatically and then at the shock front, in front of
which the flow is braked. Therefore, the adiabatic compression is mainly
connected with converting of magnetic energy into internal energy. It is
important that the latter is small (its quantity is proportional to $\beta$),
and so even small (relatively total energy quantity) addition to internal
energy increases it, and hence the plasma density, appreciably. However,
due to the shock wave the plasma is greater heated than compressed in the
compression region that shapes a plasma cord stretched along z axis.

It should be emphasized that the compression region essentially differs from such supercompressed plasma formation as plasma focus: the former is formed in case of steady-state flows in a channel and is connected with plasma transport, the latter results from only once compression of fixed plasma mass.

In Fig.3a one can see the thickening of electric current lines at the shock front because of that magnetic field is frozen in plasma. This means a strong electric current flows along the front. Beyond it current loops are formed. By the way, the quantity of the drift currents (i.e., the quantity of the currents which are drifted from the domain involved) is appreciable and, in particular, is approximately 10% of the value of total discharge current. Thus, the plasma stream flowing out the accelerator channel is, by A.I.Morozov definition, the "four-factor", i.e., carries along mass, momentum, energy, and magnetic field.

It should be noted that when the value $\beta$ decreases (for instance, owing to an increase in the discharge current) the acceleration of plasma is stronger. As a result the flow converging to the axis is rarer, i.e., the plasma density decreases in front of the shock front. At the same time the shock wave strength and therefore the fractional density jump increases. Hence there is the optimal value $\beta$ when the plasma density in the compression region is maximum. The calculations show this value is close to unity.

Theoretical analysis of the limiting case $\beta = 0$, which corresponds to infinitely large values of the discharge current, within the framework a model of narrow flowtubes [8] shows that at $\beta \to 0$ the plasma density tends to the certain finite distribution beyond the shock front, but the temperature unlimitedly increases such that asymptotically it is inversely proportional to $\beta$. The calculations for small values of $\beta$ confirm this feature.

The calculations of steady plasma compression flows with allowance for the dissipative processes modeled in accordance with [10] have shown that finite conductivity and heat conduction do not have much effect [18]. This is because on the range of plasma parameters in question the electron component is magnetized which, in the last analysis, leads to a decrease in the thermal conductivity coefficient with increase in temperature across the azimuthal magnetic field. In this case the thermal conductivity coefficient along the field is higher, as a result of which most of the conductive heat transfer takes place along the field. The unimportance of the electrical conductivity of the plasma at high temperatures follows from its weak dependence on the magnetic field and proportionality to $T^{3/2}$ with rather large coefficient. Allowing for the self-radiation of the plasma and the radiation of the impurities it contains showed that the former is negligibly small, while the latter becomes significant when the total impurity ion concentration is approximately 2% or more of the basic ion concentration. In this case the plasma density in the compression region increases as the plasma cools.

In the calculations with isotropic heat conduction the variation of the plasma parameters in the compression region is more significant: both the temperature and the density fall. Figures 4a and 4b show the same distributions as Fig.3a and Fig.3b, respectively, but for the isotropic case in the absence of impurities when $Q = 0$. The electric current lines (Fig.4a) now no longer express the shock front so clearly (a transition
zone taking the form of a shock front "smeared out" by dissipative processes, to be exact). And here the closed current loop beyond the front in Fig.3a is closed on the outer electrode together with the other current lines. From the isotherm distributions (Fig.4b) it is clear that the shock wave is formed closer to the inner electrode than in the anisotropic case.

In addition, as compared with the anisotropic case the impurity radiation losses have a more appreciable effect on the plasma characteristics in the compression region. In particular, 1% impurities is sufficient for the values of the plasma density in the compression region obtained in the ideal flow calculations to be even slightly exceeded. This is because the high thermal conductivity leads to a marked fall in temperature which on the temperature range considered produces a sharp increase in radiation intensity. By the way, as a rule, the maximum of the impurity radiation losses corresponds to the maximum of the plasma density and not the temperature. In general, the plasma parameters in the compression region are mainly determined by the relation between the heat fluxes and the radiation from the impurity ions, as well as by their role in the energy balance of the plasma flow.

The numerical results are in good agreement with the available experimental data. In particular, this regards a location of the compression region and evaluated values of the plasma density in it [4], and a distribution of the energy flux density in the plasma flows obtained by calorimeter measurements [3,19] as well.

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References

Fig. 1. Scheme of a coaxial plasma accelerator and computational domain.

Fig. 2. Velocity vectors in MHD-flow.

Fig. 3. Electric current lines (a) and isotherms (b) for MHD-flow.

Fig. 4. Electric current lines (a) and isotherms (b) for the isotropic case.