NUMERICAL SIMULATION OF PLASMA FLOW DYNAMICS IN SPT

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ABSTRACT

The report is devoted to theoretical investigation of processes in SPT-channel. Two one-dimensional non-stationary models of atoms and ions dynamics are presented. The first model (Hd-model) has pure hydrodynamic character and the second one (Hb-model) is a hybrid type model: The ions ionisation is caused by an electron impact and an equation of electric circuit with given discharge voltage $\mathcal{E}$ are used for determining discharge current $J$. The electron conductivity is assumed to be the known function and depends on a profile of a transverse magnetic field ($B = \text{const}/H^2$).

The investigations were carried out numerically by means of finite-difference methods.

In the framework of Hd-model the practically interesting range of parameters ($m \equiv 2-4 \text{ mg/s, } \mathcal{E} \equiv 200-400 \text{ V, } n \equiv 2 \times 10^{13} \text{ cm}^{-3}$, gas - Xe) was investigated in details. The current-voltage characteristic and efficiency were calculated. The characteristic is almost straight line. The efficiency reaches its maximum value 0.72 at $\mathcal{E} = 350 \text{ v}$.

The first results were obtained on the base of the Hb-model. They had shown a good agreement with integral characteristics of the Hd-model. The oscillograms of these models differ from each another first of all by high frequency component, which is in the Hb-model considerably weaker than in Hd-model.

1. Introduction

In spite of the simplicity of SPT-construction the physical picture of the processes in it is very nontrivial. It is explained mainly by two reasons. The first one is that in the SPT-channel the zone of ionisation and the zone of acceleration are coincided. The second reason is that the transport of electrons on the anode is determined mainly by so-called near-wall conductivity which depends on some rather fine details (Debye layers on the insulators, the structure of their surfaces, the coefficients of the secondary electron emission and so on).

The first models of the SPT-processes had being proposed in 1960 by A.I. Morozov. They were based on the principle of...
The equipotentiality of magnetic force lines [1]. The papers [2], [3] were also very important. The static one-dimensional models were considered in [2] and the two-dimensional ones in [3].

The analytical analysis of linear stability for the models without ionisation was made in [1]. In particular, the criterion of the flow stability with respect to azimuth non-symmetrical perturbations was obtained. This criterion is in the base of all effectively operating SPT. The questions on stability of one-dimensional hydrodynamic models were considered by V.I.Bryzgalov and A.I.Morozov and also by A.G.Sveshnikov [4,5]. Recently the numerical simulation of SPT-flows were carried out by E.Y.Choueiry [6].

2. One-dimensional models of SPT

The complete model of dynamics of electrons, ions and atoms in SPT would be very complicated and bulky. It is reasonable therefore to construct the sequence of relatively simple models, which describe the different aspects of the processes in the SPT and may give together the full picture of these phenomena.

The one-dimensional hydrodynamic model (Hd-model) to be described here is one of such a model. It describes the dynamics of ions and atoms in SPT-channel.

Let $x$ is the longitudinal coordinate in the channel ($x=0$ is the anode, $x=l$ is the exit of the channel). Let $n(x,t)$ is the density of ions (electrons), $v(x,t)$ is the ion velocity, $n_a(x,t)$ is the atom density, $J(t)$ is the electric current.

Then we have the next basic equations of the model.

The continuity equation of ions

$$\frac{\partial n}{\partial t} + \frac{\partial n v}{\partial x} = \beta n n_a$$  \hspace{1cm} (1)

The equation of ion motion

$$\frac{\partial n v}{\partial t} + \frac{\partial}{\partial x} (n v^2) = \frac{e n}{M} E + \beta n n_a v_a$$  \hspace{1cm} (2)

The continuity equation of atoms

$$\frac{\partial n_a}{\partial t} + v_a \frac{\partial n_a}{\partial x} = -\beta n n_a$$  \hspace{1cm} (3)

The equation for electric circuit
\[ L \frac{dI}{dt} + R I + \int_{0}^{l} E \, dx = \varepsilon \]  

(4)

The right hand side of (1), (3) describe the ionisation by electron impact. We assumed here that the electron temperature \( T_e \) is sufficiently high and therefore \( \beta = \text{const} \). In the eq. (3) \( v_a = \text{const} \) is the velocity of atoms. In eq. (4) \( L, R, \varepsilon \) are the inductance, the resistance and the discharge voltage of the circuit. The dynamics of electrons in the Hg-model is determined by the Ohm law in the following way

\[ E = \frac{1}{\sigma(x)} \left( J - e n v \right) \]  

(5)

Where \( \sigma(x) \) is the plasma conductivity. It is assumed that \( \sigma(x) = \sigma_0 (H_0 / H(x))^2 \), \( \sigma_0 = \text{const} \), \( H(x) \) is the profile of the transverse magnetic field, \( H_0 = H(l) \).

For equations (1)–(4) it is investigated the next initial-boundary problem: at \( t = 0 \) let there be given \( n(0,x), v(0,x), n_a(0,x), J(0) \); at \( x = 0 \) given \( n(0,t) = n_0, n_a(0,t) = n_{ao}, v(t,0) = v_o \).

Further we use the new dimensionless units of measurements. Let \( l \) is the unit of length, \( n_{ao} \) is the unit of density, \( E_0 = \varepsilon / l \) is the unit of electrical field, \( v_o \) is the unit of velocity, \( t_0 = l / v_o \) is the unit of time, \( I_0 = \varepsilon / R_{ch} \) is the unit of electric current \( (R_{ch} = l / \sigma_o \sim \text{"resistance" of the channel}) \), \( H_o \) is the unit of magnetic field. The next dimensionless parameters are the main parameters of the problem:

\[ \mu = e \varepsilon / M v_o^2, \quad \nu = \beta n_a l / v_o, \quad \eta = e n_a v_o R_{ch} / \varepsilon \]

\[ \lambda = L v_o / l R_{ch}, \quad r = R / R_{ch}, \quad h_o = H(0) / H_o \]

The profile of the external magnetic field has the following form

\[ H(x) = h_o + (1 - h_o) x^2 \]  

(7)

3. Parameters of the problem

Here we evaluate the basic parameters of the problem. Let \( M = 3 \, \text{mg/s (gas Xe)} \). If \( v_a = 2 \times 10^4 \, \text{cm/s}, \) then \( n_{ao} = 3 \times 10^{13} \, \text{cm}^{-3} \).

The ion energy near the anode is about \( 3 \, \text{ev}, \) so we have \( v_o = 2 \times 10^5 \, \text{cm/s}. \) If \( l = 3 \, \text{cm}, \) then \( t_o = 15 \, \text{mcs}. \) The evaluation of \( \beta \) gives \( \beta = 5 \times 10^{-8} \, \text{cm}^3 / \text{s}. \) It is more difficult to obtain the evaluation of parameter \( R_{ch} \approx 8 \times 10^4 \, \text{Om cm}^2. \) Thus we have the
following expressions for the dimensionless parameters

\[ \mu = \varepsilon / \delta \quad \eta = 7 \times 10^4 / \varepsilon \quad \nu = 20 \]
\[ \lambda = 10^{-2} \quad r = 3 \times 10^{-3} \quad \eta_0 = 0.1 \]

We have also \( n_o / n_{ao} = 10^{-2} \), \( v_{ao} / v_o = 0.1 \). If \( \varepsilon = 300 \nu \) then \( \mu = 50, \eta = 233 \).

4. Steady-state Hd-model

The steady-state solutions of the Hd-model may be obtained by two different ways. The first one is the solving of non-stationary equations. By this way one can find only stable solutions. The second one is the solving steady-state equations. If \( f = n \nu, p = n \nu^2 \) then these equations can be written as follows (\( H(x) = 1 \)):

\[ \frac{df}{dx} = \frac{v}{v_{ao}} \frac{j^2 (M - f)}{p} \]
\[ \frac{dp}{dx} = \frac{j^2}{p} (\mu f + \nu M - (\mu \eta + \nu) f) \]
\[ (r + 1) J = \eta \int_0^\infty j \, dx = 1 \]

Here \( M = n_o + v_{ao} \), \( f(0) = p(0) = n_o \).

The solution of the problem (9)-(11) may not exist. For the simplicity we consider the case \( \nu = 0 \). The solution is very simple

\[ v = (1 + 2\mu \left( \frac{1}{1 + \nu} \right) x)^{1/2}, \quad n = 1 / \nu, \quad J = \frac{1 + \eta}{1 + r} \]

If \( r \eta > 1, \eta \to \infty \) then \( x \to x_* = \frac{1 + r}{2\mu(r \eta - 1)} \). If \( x_* < 1 \), then the solution does not exist. The similar situation takes place also in the general case, then \( H(x) \neq 0 \) and \( p \neq 0 \). Under the conditions of SPT then \( M = 2 - 4 \text{ mg/s}, \varepsilon = 150 - 800 \nu \) the solutions of the steady-state problem exist. In fig.1 there are profiles of \( n, v, n_a \) at \( M = 3 \text{ mg/s}, \varepsilon = 300 \nu \).

5. Non-stationary Hd-model.

The numerical simulation of the eqs. (1)-(5) had given the next general results: 1) There exist steady-state stable solutions; 2) There exist steady-state unstable solutions; 3) There exist periodical solutions; 4) There exist non-stationary non-periodical solutions. This sequence of transitions realizes if we shall increase the value of the parameter \( \eta \). Under conditions of SPT (8) all steady-state solutions are unstable. How-
ever, the calculations had shown that time-averaged values have spatial distributions, which are very similar to the steady-state ones.

The volt-ampere characteristic and oscillograms of the discharge current $I(t)$ and the voltage $U(t) = \int E \, dx$ are very important characteristics of plasma flow. We introduce also the efficiency $\text{Eff}$ of the system in the following way

$$
\text{Eff} = \frac{\langle n \, v^2 \rangle_{x=1}}{2n \, a_0 \, v} \frac{1}{\langle J \rangle} = \frac{1}{2} \frac{\eta}{V_0} \frac{\langle n \, v^2 \rangle_{x=1}}{\langle J \rangle}
$$

(13)

The brackets $<...>$ means the time-averaging.

In fig.2 it is presented the volt-ampere characteristic ($<J>$ as a function of $\varepsilon$) and the efficiency $\text{Eff}(\varepsilon)$ (dotted lines give the results of steady-state model). $\text{Eff}(\varepsilon)$ has its maximum value 0.72 for $\varepsilon = 350 \, \text{V}$. In fig.3 there being given the oscillograms of $J(t)$, $U(t)$ and $(nv^2)_{x=1}$ for $\varepsilon = 300 \, \text{V}$. The basic period of oscillations $\tau_0 \cong 100 \, \text{mcs}$. The $U(t)$ oscillogram shows an existence of high frequency oscillations with typical period $\tau_h \cong 2 \, \text{mcs}$. The basic period increases then $\varepsilon$ decreases: at $\varepsilon = 400 \, \text{V} - \tau_0 \cong 75 \, \text{mcs}$, at $\varepsilon = 200 \, \text{V} - \tau_0 \cong 170 \, \text{mcs}$.

6. Stability steady-state solutions in Hd-model

The investigation of a stability of steady-state solutions of the problem (9)–(11) is a very difficult task even in linear approximation. Here we consider only one simple case then $H(x) = 1$, $\nu = 0$ and $r\eta = 1$. In this case (see (12)) the steady-state solution is $\nu = 1$, $n = 1$. In linear approximation we may obtain the next linear equations for small perturbations $n, \nu, j$

$$
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (n + \psi) = 0
$$

$$
\frac{\partial \nu}{\partial t} + \frac{\partial \nu}{\partial x} = \mu \left( j - \eta (n + \psi) \right)
$$

$$
\lambda \frac{\partial j}{\partial t} + (r + 1) j - \eta \int (n + \psi) \, dx = 0
$$

(14)

with boundary conditions $n(t,0) = \psi(t,0) = 0$.

We may obtain more simple problem suggesting that the current $\mathcal{J}(t) = \text{const}$ and $j = 0$. In this case we have

$$
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (n + \psi) = 0
$$

(15)
We seek the solutions of (15) in the form \( \exp(\omega t - ikx) \) with \( \text{Im} k = 0 \) and obtain

\[
\omega = k + i/2 \left( 1 \pm \sqrt{1 - 4ik} \right)
\]  

There exist an increasing perturbation and the increment grows to infinity then \( k \to \infty \). So the system of equations (15) are not evolutionary.

7. Hybrid model

For more complete and detail description of the processes in SPT we must use the kinetic equation of ions dynamics and take into account an electron temperature \( T(t,x) \). This model we shall call as the hybrid model (Hb-model).

Let \( f(t,x,v) \) is the ion distribution function. We have now

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + e \frac{e}{M} E \frac{\partial f}{\partial v} = \beta(T) n n_a \delta(v - v_a)
\]

The dynamics of \( T \) is described by the next equation of hydro-dynamical type

\[
\frac{3}{2} n \left( \frac{\partial T}{\partial t} + v_e \frac{\partial T}{\partial x} \right) + n T \frac{\partial v_e}{\partial x} = \frac{\partial}{\partial x}(\kappa \frac{\partial T}{\partial x}) +
\]

\[
+ j_e E - \alpha \beta(T) n n_a
\]

In (17),(18) \( n = \int f dv \) --the density of ions (electrons); \( j_1 = e n v_1 \) --the ions current; \( v_1 = 1/n \int v f dv \) --the ions velocity; \( j_e = j - j_1 \) --the electron current; \( v_e = -j_e /e n \) --the electron velocity.

The equations (3)–(5), (17),(18) are the basic equations of the Hb-model. The boundary conditions: \( x=0 - n_a = n_0 \), \( \partial T/\partial x = 0 \) and \( f = f_b(v) \) for \( v > 0 \); \( x = l - T = T_0 \). Let

\[
f_b(v) = 0.5 \pi \frac{n_0}{v_0^2} v \exp(-\frac{\pi}{4} (v/v_0)^2).
\]

\( n_0 = \int f_b(v) dv \), \( v_0 = 1/n_0 \int v f_b dv \) --the parameters of \( f_b(v) \).

The coefficient of ionisation \( \beta(T) \) will be used in the following way

\[
\beta(T) = \begin{cases} 
\beta_0(T/T_0 - 1), & T > T_* \\
0, & T < T_*
\end{cases}
\]

where \( T_* \approx 3 - 5 \text{ ev} \), \( \beta = 2.2 \times 10^{-8} \text{ cm}^3/\text{s} \).

The energetic price of ion \( \alpha \approx 30 - 50 \text{ ev} \), the coefficient of
heat-conductivity $\kappa = \kappa_0 T / H(x)$

The first results obtained in the frame of Hb-model had shown the good agreement with the results of the Hd-model in the case $\beta = \text{const}$. For example the current-voltage characteristics are practically identical. In the cases then both models have stable steady-state solution there exist not only qualitative but quantitative correspondence of the results.

Now we present some results of Hb-model in the case $\beta \neq \text{const}$. In the fig. 4 it is given the profiles of $n(x)$, $v(x)$, $n_a(x)$ ($M = 3 \text{ mg/s}, \varepsilon = 300 \text{ v}, T_0 = 20 \text{ ev}, \alpha = 40 \text{ ev}$). In fig. 5 we can see the oscillograms of $I(t)$ and $U(t)$. The basic period of oscillation $\tau_0 \approx 30 \text{ mcs}$. These results allow to make the conclusion: Hb-model on the whole gives more stable regimes of the flows than Hd-model.

References

\[ \text{fig. 4} \]