PARTICLE SIMULATION OF PLASMA PHENOMENA IN HALL THRUSTERS

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Abstract

Particle simulation codes were developed to investigate low-density plasma phenomena caused by charge separation in a Hall thruster. Electron and ion trajectories were followed in time using a Particle-in-Cell(PIC) method. A Monte Carlo model was coupled to simulate electron collision processes. Using these codes, we examined electrostatic plasma fluctuation moving in the azimuthal direction, which has an effect on the electron motion across the magnetic field. In addition, distributions of plasma properties such as the electron and ion densities, their flux densities, and the space potential were numerically obtained. Compared with experimental results, the calculated distributions in a fluctuant azimuthal field were in better agreement than those without the azimuthal field. This fact indicates that this model is useful to investigate plasma properties in the Hall thruster.

Nomenclature

(SI units unless noted otherwise)

\( A \) : magnetic vector potential
\( A_{\perp} \) : defined by Eq. (9)
\( B \) : magnetic flux density
\( B_{\parallel} \) : magnetic field strength, component of the magnetic flux density
\( E \) : electric field
\( e \) : electronic charge
\( I_a \) : acceleration current
\( I_b \) : ion beam current
\( K_n \) : Knudsen number defined by Eq. (25)
\( k \) : Boltzmann's constant
\( k_f \) : wave number of the plasma fluctuation
\( L \) : acceleration channel length
\( m \) : particle mass
\( m_p \) : propellant mass flow rate
\( n \) : number density
\( q \) : particle charge
\( r \) : distance from the center line of the thruster
\( r_i \) : radius of the inner wall of the acceleration channel
\( r_o \) : radius of the outer wall of the acceleration channel
\( T \) : thrust
\( T_e \) : electron temperature
\( t \) : time
\( V_a \) : acceleration voltage
\( v \) : velocity

\( r^* \) : dimensionless quantity
\( r^* \) : mean quantity

Introduction

Hall thrusters, also known as stationary plasma thrusters, have been flown on over 50 Russian spacecraft[11]. Moreover, it was pronounced that an overall thrust efficiency of 50% has been achieved with the Russian 1.35-kW Stationary Plasma Thruster(SPT-100). Numerical studies have shown that for Earth orbit raising and north/south stationkeeping applications of electric propulsion, the optimum specific impulse is in the range of 1,000 to 2,000 sec. This range coincides
with the range in which the Hall thruster has been optimized to operate. As a result, Hall thrusters have attracted so much interest of spacecraft propulsion specialists as many projects dealing with SPT-100 have been formed in western countries. The first step of the research seems to be evaluating its performance[2].

The thrust efficiency of any Hall thruster can be defined as

$$\eta_T = \frac{\zeta^2}{2nm_eV_a}$$  \hspace{1cm} (1)

This efficiency can be expressed as

$$\eta_T = \eta_a \eta_e \eta_{sp} \xi$$  \hspace{1cm} (2)

where $\eta_a$, $\eta_e$, $\eta_{sp}$, and $\xi$ are acceleration efficiency, beam energy efficiency, propellant utilization[2], and thrust loss factor[2], respectively. As for SPT-100, it has been reported that $\eta_a$ and $\xi$ were lower than the others[2]. Also in Ref. 3, it is stated that $\eta_a$ was lower than $\eta_e$ and $\eta_{sp}$ in regard to Hall thrusters designed at the University of Tokyo in 1990. The acceleration efficiency is defined as

$$\eta_a = \frac{I_a}{I_s}$$  \hspace{1cm} (3)

which mainly depends on the ion loss fraction[3], that is, a fraction of ions produced that are lost to the thruster surfaces such as acceleration channel walls. The thrust loss factor includes beam divergence and profile losses. Thus, it is expected that the thruster performance is influenced not only by the electric field in the acceleration channel but also by that in the downstream region. Accordingly, we have decided to develop a computational code with which we can investigate plasma phenomena in both regions.

In low-density plasma with magnetic fields, charge neutrality can be violated owing to the difference between electrons' and ions' Larmor radii, which results in the electrostatic field. This phenomenon is expected to occur in the downstream region of the Hall thruster. When analyzing such plasma, we should follow ion and electron motion separately. In addition, ions cannot be treated as a fluid in almost all the Hall thrusters since ions are almost freely accelerated by the electric field on the condition of the present mass flow rate.

It is empirically known that in a large magnetic field observed electron mobility and diffusion coefficient normal to the magnetic field are larger than values calculated on the assumption of "classical diffusion" due to collisions of electrons with heavy particles. This "anomalous diffusion" is explained by the theory that the scattering of electrons by the walls can play a role analogous to the scattering of electrons by neutral atoms and ions in the acceleration channel[4]. In another theory, it is stated that turbulence in the plasma causes electron "anomalous diffusion". With electron density fluctuation, $\Delta n_e$, electron mobility across the magnetic field is increased, which can be theoretically expressed as

$$\mu_e = \frac{m_e}{eB^2} \left( n_e \langle v \rangle_m + \frac{\pi eB}{4m_e} \left( \frac{\Delta n_e}{n_e} \right)^2 \right)$$  \hspace{1cm} (4)

Applied to a Hall thruster, this theory represents that electrons come to more easily run in the axial direction across the radial magnetic field lines if plasma fluctuation occurs. Therefore, we should examine the plasma fluctuation to correctly estimate plasma properties. This fluctuation cannot be investigated using hydrodynamics, since it occurs when electrons do not have a Maxwellian velocity distribution. Moreover, the effect of scattering on the wall cannot be taken into account in the hydrodynamic model. As a result, the particle model is desirable to treat electrons.

Therefore, the objective of this study is to develop a computational code in which we treat both electrons and ions as particles. Such a code enables us to deal with low-density plasma phenomena caused by charge separation in a Hall thruster. A further objective is to examine the plasma fluctuation which has an effect on the electron motion and finally to numerically calculate plasma properties in a Hall thruster.

**Simulation Model**

Figure 1 is a schematic of a Hall thruster developed at the University of Tokyo in 1992, which is designed so that a hollow cathode can be equipped on the axis. However, a filament cathode was used for its convenience in many experiments. The inner and outer radii of the acceleration channel are 25 mm and 30 mm, respectively. The acceleration channel length is 8 mm. Propellant gas is supplied through the orifice on the anode. The calculated magnetic field configuration is shown in Fig. 2. For effective acceleration of ions by the electrostatic field, the mean free path for the collision between ions and neutral atoms should be larger than the channel length. This condition is satisfied in this thruster unless the propellant mass flow rate exceeds the present actual value(< 5 $A_{eq}$) by a factor of 10. Then, the fluid model is not available to simulate

![Fig. 1 Schematic of a Hall thruster.](image)
charged particles in this Hall thruster. Moreover, we should deal with propellant gas as rarefied gas because of its Knudsen number larger than 0.01.

The block diagram of the code used in this work is shown in Fig. 3. This code includes a charged particle simulation based on the Particle-in-Cell (PIC) and Monte Carlo method. Before this simulation, we should obtain the magnetic field and the neutral atom density distribution. The former is determined only from the thruster design, being calculated using the Finite Element Method (FEM). The latter is obtained using Direct Simulation Monte Carlo (DSMC) method.

**Simulation of Charged Particles**

First we explain physical processes taken into account in this simulation. Ions are produced by ionization collisions between neutral atoms and electrons, moving under the forces of the electric and magnetic fields. Some of them are exhausted as the beam and others are lost because of their recombination with electrons. It occurs on the insulated surfaces such as walls of the acceleration channel and the surface of the front insulator. We assume that trajectories of ions are not altered by collisions. It means that they do not collide with heavy particles since the mean free path for such a collision is probably larger than simulation region. Electrons are emitted from a cathode, causing some kinds of collisions. Here, we assume that emitted electrons have Maxwellian initial velocities. We take into account the following three kinds of collisions: 1) elastic collision with neutral atoms, 2) elastic collision with ions, and 3) inelastic collision with neutral atoms such as ionization collisions and excitation collisions. When an ionization collision occurs, an another low-energy electron is produced. Reaching the insulated surfaces or the anode, electrons are lost. If the surface is isolated, some secondary electrons are emitted from it at the same time.

Secondly we describe about the simulation method. In our code, charged particle simulation is based on the PIC method. In addition, a Monte Carlo method is used to include the effects of collisions of electrons in the PIC code. As output parameters, we can obtain distributions of the electron and ion densities, flux densities, and energies and the space potential.

The PIC method utilizes a large number of test particles to simulate ion and electron flows. Each particle represents a large number of real particles and carries a corresponding electrical charge. Typically, the PIC code has a computational cycle consisting of five steps:

- solving the electric and magnetic field equations on a grid,
- adding new particles,
- interpolating the fields onto the particle positions,
- integrating the equations of motion,
- assigning charge and current densities to the grid.

For explanation, see Ref. 7.

In our particular problem, the simulation presumes the electrostatic approximation, i.e., $E = -\nabla \phi$. The electric field is found self-consistently on each computational cycle, while the magnetic field is given before the simulation. Charged particles move about due to the forces from their own electrostatic field and the applied magnetic field. Accordingly, the governing equations are the Poisson's equation and Newton-Lorentz equations

$$\Delta \phi = -\frac{\varepsilon_0}{\varepsilon_0} (n_i - n_e), \quad (5)$$

$$m \frac{dv}{dt} = q(E + v \times B), \quad (6)$$

where we neglect multi-charged ions.

We consider the cylindrical coordinate system where $r$ is the distance from the thruster axis, along which $z$ extends downstream with its origin on the front insulator surface. Since the magnetic field is uniform in the azimuthal direction in Hall thrusters, it can be written in the form

$$A = (0, A_z, 0), \quad (7)$$

$$B = (B_\theta, 0, B_z) = \left(-\frac{1}{r} \frac{\partial A_z}{\partial z}, 0, \frac{1}{r} \frac{\partial A_z}{\partial r}\right). \quad (8)$$
where
\[ A_R = r A_g. \] (9)

Equation (6) can be rewritten as follows:
\[ m \frac{d \vec{v}_r}{dt} = q (E_r + v_r B_z) + m \frac{v_r^2}{r}, \] (10)
\[ m \frac{d \vec{v}_\theta}{dt} = q (E_\theta - v_r B_z + v_r B_z) - m \frac{v_r v_\theta}{r}, \] (11)
\[ m \frac{d \vec{v}_z}{dt} = q (E_z - v_\theta B_r). \] (12)

Using Eqs. (8) and (11), we find the following equation for the angular momentum:
\[ \frac{d}{dt} (m r \vec{v}_\theta + q A_R) = r q E_\theta. \] (13)

To keep errors less and increase the precision, we use Eq. (13) for the azimuthal motion in place of Eq. (11). For a particle approaching the z-axis, equations of motion in the Cartesian coordinates are used instead of uniformly distributed from those in \( r, \theta, \) and \( z \) directions near the z-axis in order to avoid the singularity problem.

The above governing equations are rewritten in dimensionless units, where time is measured in units of the inverse electron plasma frequency \( \omega_{pe}^{-1} \), lengths in Debye lengths \( \lambda_D \), velocities in electron thermal velocities, space potential in units of a representative electron temperature \( k T_e / e \), magnetic flux density in a representative magnetic field strength \( B_0 \), and number density in a representative electron number density \( n_e \).

We define the Debye length and the plasma frequency by the following equations:
\[ \lambda_D = \frac{e T_e}{\sqrt{n_e \pi^2}}, \] (14)
\[ \omega_{pe} = \frac{n_e e^2}{e_0 m_e}. \] (15)

Rewriting governing equations, Poisson's equation and equations of motion for electrons and ions, we get
\[ \dot{\vec{v}}_r = - (\vec{r} - \vec{r}_s), \] (16)
\[ \dot{\vec{v}}_\theta = - (\vec{r} + \gamma \vec{v}_\theta \vec{B}_z) + \frac{v_\theta^2}{r}, \] (17)
\[ \frac{\dot{\vec{v}}_z}{dt} = \frac{\gamma}{r} \left( \vec{v}_r \vec{v}_z - \vec{v}_\theta \right) - \vec{v}_\theta \vec{B}_z. \] (18)
\[ \frac{\dot{\vec{r}}_\theta}{dt} = \frac{E_z - \gamma A_R}{m_e}, \] (19)
\[ \frac{\dot{\vec{r}}_z}{dt} = \frac{E_z + \gamma v_\theta B_z + \dot{\vec{v}}_\theta}{m_e}, \] (20)
\[ \frac{\dot{\vec{r}}_r}{dt} = \frac{E_r + \gamma v_\theta B_z + \dot{\vec{v}}_\theta}{m_e}, \] (21)
\[ \frac{\dot{\vec{r}}_s}{dt} = \frac{E_s - \gamma v_\theta B_z}{m_s}. \] (22)

where \( \gamma \) is the ratio of electron cyclotron frequency to plasma frequency defined by the equation
\[ \gamma = \frac{\omega_{ce}}{\omega_{pe}}. \] (23)
\[ \omega_{ce} = \frac{e B_0}{m_e}. \] (24)

From above dimensionless equations, it is expected that the frequency ratio \( \gamma \) has a significant effect on the plasma behavior.

We apply the Monte Carlo technique to simulation of electron collisions. The method for selecting which collision process occurs has been described in Ref. 8. Once an elastic collision occurs, an another random number is chosen to determine the deflection angle based on the differential elastic scattering cross section. The energy of the electron is assumed to remain constant. In case of ionization collision, the produced electron's energy is less than a few electron volts when the injected electron's energy is less than a few times of the ionization energy. Thus, we chose a random number uniformly distributed from 0 to 3 eV to determine the produced electron's energy. With such a low energy, the direction of the initial velocity can be regarded as uniformly distributed. As for injected electrons, the new velocity is determined with reference to the differential ionization collision cross section. When a secondary electron is emitted from the insulated surface, we also use random numbers to determine their velocities dependent on the surface temperature.

**Simulation of Neutral Atoms**

The neutral atom density distribution is required for the estimation of the mean free path for electron-atom collision. In this study, we use the DSMC method applicable to rarefied gas.

Propellant gas is supplied into the acceleration channel through the orifice on the anode. As for initial conditions, we assume that the initial velocity is the speed of sound at the temperature of the anode and insulated surfaces. Neutral atoms collide with each other, changing the direction of their motion. On the thruster surfaces, diffused reflection occurs. Unless the propellant is almost fully ionized, we take into account neither generation on the thruster surface, that is the recombination of ions and electrons, nor extinction caused by ionization collisions.

**Results and Discussions**

First we examine plasma fluctuation which may influence the electron motion across the magnetic field and secondly calculate distributions of plasma properties in the \( r-z \) plane.

When simulating neutral atoms, we use the region as shown in Fig. 4 with reference to the thruster in Fig. 2. In the following simulations, we chose that \( W_s = 20 \) mm and \( W_r = 50 \) mm. Figure 5 is a calculated neutral atom density distribution on the condition that the inner and outer radii of the channel, \( r_1 \) and \( r_2 \), are 19 mm and 25 mm, the channel length \( L = 8 \) mm, the inner and outer radii of the orifice are 21 mm and 23 mm, the thruster surface temperature is 300 K, and the Knudsen
plasma fluctuation, charged particles drift in the axial direction. To investigate this phenomenon, we performed the particle simulation only in the acceleration channel using the coordinates shown in Fig. 6. Because of the restriction of the computer memory capacity, we assumed a uniform axial electric field given by \( E_z = V_z / L \). In other words, we obtain the space potential distribution in the \( r-\theta \) plane.

Examples of the results are shown in Figs. 7-9. Here, we chose \( \gamma = 0.2 \) and \( V_z = 100 \text{ V} \). The mass ratio smaller than that of the actual propellant, \( m_1/m_2 = 1840 \), is chosen so that the computation time may be relatively short. Fig. 7 shows azimuthal distributions of the space potential at \( r=22 \text{ mm} \), that is the center of the acceleration channel. Two curves correspond to the distributions at different times. This figure indicates that the space potential changes with the position and with time and that the frequency of the fluctuation is about \( 1/200 \) of the plasma frequency. Distributions of the azimuthal electric field and electron density at one time are shown in Figs. 8 and 9.

Neglecting the axial magnetic field, we can define the average electron drift velocity by the equation:

\[
\lambda_n = \frac{\lambda_n}{W_n} \quad \text{(25)}
\]

where \( \lambda_n \) is the mean free path for atom-atom collision at the orifice on the anode. The above Knudsen number corresponds to the operating condition that argon is used as the propellant and \( m = 2 \text{ Aeq} \). However, the calculation results with various Knudsen numbers from 0.1 to 1.0 demonstrate no remarkable difference. The distribution mainly depends on the thruster design rather than the Knudsen number.

In the following simulations of changed particles, we choose \( r_1 = 19 \text{ mm} \), \( r_2 = 25 \text{ mm} \), \( L=8 \text{ mm} \), the secondary electron emission yield \( \delta = 0.6 \text{[2]} \), and we use the neutral atom density distribution shown in Fig. 5.

Plasma Fluctuation

Magnetic field lines are roughly radial in the acceleration channel of the Hall thruster as shown in Fig. 2. Once azimuthal electric field being induced by

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Fig. 4 Region for the neutral atom simulation.

number \( K_n=0.26 \). Here, \( K_n \) is defined by

\[
K_n = \frac{\lambda_n}{W_n} \quad \text{(25)}
\]

Fig. 6 Region for plasma fluctuation simulation.

Fig. 5 Neutral atom density distribution \( (K_n = 0.26) \).

(a) In the whole region. (b) In the acceleration channel.

Fig. 7 Space potential distributions in the azimuthal direction. (a) At one time. (b) At \( 110/\omega_{pe} \text{ sec after (a)} \).
estimated that $E_y$ ranges from 400 V/m to 700 V/m. As for ions, use of the ion density distribution in place of the electron density distribution leads to the other imaginary electric field that is about 200 V/m. This result suggests that an influence on electrons is bigger than that on ions. With reference to “Bohm diffusion theory”, axial electron motion in a Hall thruster can be approximately written by

$$\bar{n}_e \bar{v}_e = \bar{I}_e = \frac{kT_e}{16eB_r} \frac{\delta n_e}{\delta z} = \frac{n_i}{16B_r} E_z$$

(31)

Considering that electron temperature is one or two orders in magnitude smaller than the acceleration voltage, the average axial velocity of electrons can be roughly given by

$$\bar{v}_z = \frac{V_e}{16B_r L}$$

(32)

After calculating from Eqs. (26) and (32), we obtained $-700 / B_r = -400 / B_r$ and $\bar{v}_z = -780 / B_r$. Thus, we can consider that electron drift velocity observed in this simulation is close to the velocity expected from the “Bohm diffusion theory”.

**Distributions of Plasma Properties in the r-z Plane**

Before simulation in the downstream region, we measured the ion beam density distribution of the thruster shown in Fig. 1 in such a way that a small ion collector with the voltage of -70 V was swept at 18 centimeter’s distance from the thruster. The obtained distribution is shown in Fig. 10, which is practically independent of operating parameters. Using this figure, the half width of the ion beam is estimated to be 20 degrees and the thrust loss factor $\xi$ to be about 0.76. This result supports the fact ions actually spread outward, indicating the importance of investigation of the beam divergence.

To obtain distributions of plasma properties in r-z plane, we performed the simulation in the same region.

![Graph](image_url)

Fig. 10 Measured distribution of ion beam density of the thruster shown in Fig. 1.
as shown in Fig. 4. We solved Poisson's equation Eq. (5) for the space potential as a function of \( r \) and \( z \), adopting the condition,
\[
\phi = \text{const. at anode,} \quad (E)_r = 0 \quad \text{at } r=0.
\]
where \((E)_r\) represents the normal line component which is directed outward in an electric field. On the insulated surfaces, we adopted the condition that
\[
(E)_n = -\frac{\sigma}{\epsilon_0}.
\]

For simplicity, we used the same condition also on the plasma boundaries. A cathode is equipped at \((z, r)=(8 \text{ mm, 22 mm})\), that is near the acceleration channel exit.

Figure 11 is an example of calculated distributions of electron density and space potential in the acceleration channel. The electron density has its peak near the center of the channel and equipotential lines are curved. These tendencies agree well with experimental distributions\(^{[6]}\) measured on the condition of the same \( \gamma \) and \( K \), as those used in the simulation. These dimensionless values correspond to the following operating parameters: \( m = 2 \text{ Aeq.}, I=1.5 \text{ A, and the maximum magnetic field strength is 0.1 T.} \) Form this simulation, the ion loss fraction was estimated to be 44% in this case, which is almost equal to the experimental value\(^{[6]}\). As a result, it has become clear that this code is useful to investigate plasma properties in the acceleration channel.

On the basis of the result of plasma fluctuation simulation described above, we made an approximation of the azimuthal electric field by the equation
\[
E_\theta (\theta) = E_{\phi 1} \cos(k_\theta \beta - \omega \beta t) \quad (36)
\]
This equation is used so that the effect of the azimuthal fluctuation can be included in the simulation where the space potential is calculated as a function of \( r \) and \( z \). Simulation results on the condition of various acceleration voltages and frequency ratios showed that
\[
k_\theta = 2, 3, 4, \quad 6 \times 10^3 \text{ V/m} = E_{\phi 1} \approx 8 \times 10^3 \text{ V/m}, \quad 200/\omega_{rf} \leq 2\pi/\omega_f \leq 300/\omega_{pe},
\]

![Fig. 11 Calculated distributions in the acceleration channel (\( \gamma = 0.3 \))](#)

(a) Electron density. (b) Space potential (The contour interval is 5 V.).

Then, we chose \( k_\theta = 3, \quad E_{\phi 1} = 6 \times 10^3 \text{ V/m}, \quad \omega_f = 2\omega_{pe}/300 \) as representative values. We also simulated plasma without azimuthal electric field to make clear the effect of the electric field fluctuation. Figure 12 shows the distributions of the electron density, ion density, space potential, and ion flux vector with \( E_\theta \) given by Eq. (36), while Fig. 13 is those without \( E_\theta \). As shown in Fig. 12, the level of the ion density is slightly higher than that of the electron density although electron and ion density distributions appear similar. The difference between electron density and ion density produces large electric fields both in the acceleration channel and in the downstream region of the thruster. Resulting radial potential gradient makes ions flow spreading outward, increasing beam divergent angle. Comparing Fig. 12(c) with Fig. 13(b), we find that equipotential lines are more dense when the azimuthal field is not induced. This fact means that the azimuthal electric field caused by the plasma fluctuation increases the electron mobility perpendicular to magnetic

![Fig. 12 Calculated distributions of plasma properties with \( E_\theta (\theta = 0.2, \text{In the above figures, a small circle denotes a filament cathode.}) \)](##)

(a) Electron density. (b) Ion density. (c) Space potential (The contour interval is 10 V). (d) Ion flux vector.
field lines, decreasing the electric field and the acceleration voltage. Thus, we can regard this phenomenon as "anomalous diffusion".

To discuss the validity of this simulation, we measured plasma density with Langmuir probes in the downstream region of the thruster shown in Fig. 1. Figure 14 is the obtained plasma density distribution when a filament cathode is equipped near the exit of the acceleration channel and \( \gamma = 0.2 \) as same as in Figs. 12 and 13. As shown in Fig. 14, plasma density has its peak in the acceleration channel and on the thruster axis. Comparing Fig. 12(b) with Fig. 13(a), we find that more ions can approach the \( z \)-axis and the density distribution is more similar to Fig. 14 in the azimuthal field. It is because an increase in the electron mobility reduces charge separation resulting in a decrease in the potential on the thruster axis.

From these results, it is suggested that we were able to simulate the fluctuation which has a significant effect on the plasma properties in a Hall thruster. Considering that the simulation result with the fluctuation shows better agreement, it is concluded that such a code, by which an anomalous diffusion can be simulated, is necessary to investigate plasma phenomena and further to estimate thruster performance of Hall thrusters.

**Conclusions**

Particle simulation codes based on the PIC method coupled with Monte Carlo method were developed to investigate low-density plasma phenomena in a Hall thruster. This code enables us to simulate electrostatic plasma fluctuation moving in the azimuthal direction and further to obtain distributions of plasma properties such as electron and ion densities, their flux densities and the space potential. The fluctuating azimuthal electric fields being taken into account, electrons come to move easily run across the magnetic field lines and simulation results show better agreement with measured distributions than those without azimuthal fields. From these results, it is concluded that this method is very useful to numerically investigate plasma properties in Hall thrusters.

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**References**


