THE MODIFIED BALANCE METHOD OF NEAR-CATHODE PROCESSES CALCULATION CHARACTERISTICS

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Abstract

The modified balance method of calculation such near-cathode processes characteristics as the cathode potential drop $U_c$ and the heat flux $q_o$ which transfers from plasma to cathode surface is discussed. This method which keeps the small expenditure on calculation procedure as in usual balance method takes into account the peculiarities revealed with help of exact calculation method which is founded on the differential equations system solution. It is carry out the comparison of the values $U_c$ and $q_o$ calculated by exact and modified balance methods for arc discharge burning in argon ($P=10^5$ Pa) on the tungsten cathode. It is shown that the calculation results coincide reasonably and the modified balance method may be used for the calculation such characteristics of near-cathode processes as $U_c$ and $q_o$.

Introduction

The arcjet thrusters and MPD-thrusters lifetime is defined in many cases the cathode efficiency which depends from the thermal condition and the kind of the plasma contact with its surface (the spot or the diffuse discharge). In the diffuse discharge case the cathode thermal condition is connected with its geometry, the cooling wise and the plasma parameters in the near-cathode region of arc discharge. Therefore the problem of plasma parameters definition in the arc near-cathode region with diffuse discharge presents the obvious interest.

The physical model of arc near-cathode region.

Mostly the two-layer model of arc near-cathode region [1-3] is used for high pressure discharge ($P \sim 10^5$ Pa). This model as it is shown in fig. 1 includes the collisionalless layer I and the collisional layer II. The collisionalless layer (the space charge layer) contacts with the cathode surface. Usually it is assumed that its thickness $\delta_o$ is smaller than the electron free path length and the interactions between plasma particles within layer I are absent. Moreover it is considered that in the layer I the positive volume charge is concentrated in the main and the great part of the cathode potential drop $U_c$ is realised. The electrical field which exists in the layer I accelerates the emission electrons moving from the cathode surface toward the discharge plasma, and also the plasma ions moving to the cathode surface. But the electric field in this layer brakes

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the plasma electrons which direct to the cathode surface, and only the fast electrons which energy is enough to get over the braking barrier $U_c$ are able to reach the cathode surface. The total current density $j = j_i + j_e + j_{pe}$ and its components (the plasma ion current density $j_i$, the plasma electron current density $j_e$ and the emission electron current density $j_{pe}$) are invariable within the collisionless layer. The emission electrons which are accelerated in layer I have on the boundary between layers I and II the energy $(eU_c + \frac{1}{2}kT_w)$. Here $T_w$ - the cathode surface temperature, $K$ - Boltzmann's constant, $e$ - the electron charge. As for the arc discharge in the arcjet $eU_c \gg kT_w$, it is possible to consider the emission electrons as a monoenergetic stream with energy $eU_c$. This stream brings in the collisional layer the impulse stream $\eta_m v_m e^{\frac{m}{2}}$ and the energy stream $n_e V_e e^{\frac{m}{2}}$, where $n_e$ and $V_e$ - the emission electrons density and their velocity on the boundary between layers I and II, $m_e$ - the electron mass. These impulse and energy streams pass to the plasma particles in the layer II.

The collisional (ionisation) layer thickness $\delta$ is much bigger than the free path length of the plasma particles. Within the layer II the ion current forms, the emission electron stream relaxes and the heavy particles temperature changes from the temperature $T_w$ on the boundary with layer I to the temperature value upon the electron temperature on the boundary with discharge plasma $T_{e\infty}$. Because of the difference between the heavy particles and electron temperatures and also because of the ionisation equilibrium displacement the plasma composition in the layer II differs from the equilibrium composition.

The total current density $j$ also is invariable within the collisional layer. However as it is followed from fig. I the current densities $j_i$ and $j_{pe}$ within the layer II change their values, and the current density $j_e$ changes the value and the sign: on the boundary with layer I the current density $j_{pe}$ is provided with the fast plasma electrons and is directed to the cathode surface, on the boundary with discharge plasma the current density $j_{pe}$ is caused by the motion of all plasma electrons and is directed to the discharge plasma.

The energy losses with the streams of the plasma particles which leave the layer II through the boundaries are compensated by the energy supply with the stream of emission electrons.

Mostly it is used two calculation method for the definition of the near-cathode region characteristics. The first method which we shall call the exact method founds on the differential equations system solution and allows to receive the information about the cathode potential drop $U_c$, the heat flux $q_0$ which passes from the near-cathode plasma to the cathode surface and also the information about the distributions over the collisional layer thickness the temperatures and the concentrations of the particles, the electric field strength $E$, the plasma potential $\phi$ and so on.

The second method which is used for the estimation of $U_c$ and $q_0$ values proceeds from the integral balance correlations. This method we shall call the balance method. Let us dwell on these methods essence in detail.

### The exact calculation method.

As the collisional layer thickness $\delta$ in the diffuse discharge is much smaller than the radial dimension of the discharge, the equations system for the layer II is written in the onedimensional approach. Besides it is suggested that the emission electrons current density may be written with the help of Richardson-Deshman's equation with the Shottky correction.

The equations system which is taken into account the processes of relaxation of emission electrons stream, the ion current forming, the ionisation equilibrium displacement and so on was suggested in [3]

$$
\frac{5}{2}Kn_e UdTe + \frac{5}{2}Kn_e V_e dTe = \frac{d}{dx}\left(\lambda_e dTe + \frac{5}{2}kTe\right) - \frac{5}{2}kTe \cdot \frac{d^2Te}{dx^2} + \frac{d(eU_e + \frac{1}{2}kT_e)}{dx} \; .
$$

(1)
\[
\frac{5}{2} \frac{d}{dx} \left[ \frac{n_v (U+V_v) + n_i (U+V_i)}{n_e} \right] = \frac{d}{dx} \left( \lambda \frac{d T}{dx} \right) + W_e + j_i E, \tag{2}
\]

\[
W_e = 3 \frac{n_e}{n_i} \frac{m_e}{M_i} K_{ee} \left( V_{ee} + V_{ei} \right) (T_e - T), \tag{3}
\]

\[
-e_n E - \frac{d P_e}{d x} = \frac{m_e n_e V_{ee}}{K_{ee}} (V_e - V_a) - \frac{m_e n_e V_{ei}}{K_{ee}} (V_e - V_i) = 0, \tag{4}
\]

\[
-e_i E - \frac{d P_i}{d x} = \frac{m_i n_i V_{ei}}{K_{ei}} (V_e - V_i) - \frac{n_i V_{ia}}{2} (V_i - V_a) = 0, \tag{5}
\]

\[
m_e n_e V_e + M_i n_i V_i + M_a n_a V_a = 0, \tag{6}
\]

\[
n_e = n_i; (n_e \ll n_i), \tag{7}
\]

\[
P = K \left( n_{Te} + n_i T + n_a T \right) = \text{const}, \tag{8}
\]

\[
P = K n_e T_e, \tag{9}
\]

\[
P_i = K n_i T, \tag{10}
\]

\[
\dot{n}_e = \beta(T_e) n_e n_n - \alpha(T_n) n_e^3, \tag{11}
\]

\[
\dot{n}_e = \frac{1}{e} \frac{d j_i}{d x} = \frac{d (n_i V_i)}{d x}, \tag{12}
\]

\[
\dot{j}_e = -e_n V_e, \tag{13}
\]

\[
\dot{j}_i = e_n V_i, \tag{14}
\]

\[
\dot{j}_e = -e_n V_e^m, \tag{15}
\]

\[
\dot{j}_i = \dot{j}_e + \dot{j} + \dot{j} = \text{const}, \tag{16}
\]

\[
\dot{j}_e^m = j_e^m \left[ \exp \left[ - \int \left( n_a <Q_{ea} \right) + n_i <Q_{ei} \right] + \frac{1}{2} n_e <Q_{ee} > \right] dx, \tag{17}
\]

\[
\int \dot{j}_e^m = m_e n_e + M_i n_i + M_a n_a, \tag{18}
\]

\[
\int \frac{d (P_e u^i)}{d x} = \int \left( m_e n_e^m \cdot V_e^{em^2} \right), \tag{19}
\]

\[
E = - \frac{d \phi}{d x}, \tag{20}
\]

\[
\frac{d \phi}{d x} = \frac{1}{2} \left( \frac{3}{2} K T_e \right) n_e <Q_{ee} >, \tag{21}
\]

\[
\frac{d V_e^{em}}{d x} = \frac{1}{2} \left( V_e^{em} - V_e \right) n_e <Q_{ee} > + \left( V_e^{em} - V_i \right) n_i <Q_{ei} >, \tag{22}
\]

Here $T_e$ and $T$ - the electron and heavy particles temperatures; $\lambda_e$ and $\lambda$ - the electron and heavy particles heat conductivities; $n_e$, $n_i$, $n_a$ - the electron, ion and atom concentration; $n_e^{em}$ - the emission electrons density; $V_e$, $V_i$, $V_a$ - the directed velocities of the electron, ion and atom motion; $V_e^{em}$ - the emission electron velocity; $\phi = \frac{m_e V_e^{em}}{2}$ - the kinetic energy of emission electrons; $M_i$ and $M_a$ - the atom and ion masses; $\varphi$ - the plasma electric potential; $\beta$ - the ionisation potential; $U$ - the directed velocity of motion of plasma as whole in the layer II because the transfer of the emission electrons impulse stream to the plasma particles: $\dot{n}_e$ - the net electron (ion) generation rate; $V_{ea}$, $V_{ei}$, $V_{ia}$ - the frequencies of electron - atom, electron - ion and ion - atom elastic collisions; $<Q_{ea}>, <Q_{ei}>, <Q_{ia}>$ - the average cross sections of impulse transfer by elastic collisions of particles; $K_{ea}$, $K_{ei}$, $K_{ia}$ - kinetic coefficients; $P_e$, $P_i$ - the partial pressures of electrons and ions.
The influence of the collisionalless layer on the system of equations (1)-(22) is taken into account in the boundary conditions: as the cathode potential drop it is understood the potential drop in the layer I - \( U_c \). Besides because of the small thickness of the collisionalless layer \( \delta \), the parameters on the boundary between the layers I and II and the cathode surface are identified.

The boundary conditions for the system (1)-(22) are written the next manner:

a) on the boundary between the layers I and II (\( x = 0 \))

\[
T = T_w; \quad \varphi = U_c; \quad U = 0; \quad \varepsilon = e U_c; \quad j_i = j_{iw};
\]

\[
\left( \frac{dT}{dx} \right)_{x=0} = \frac{dT}{dx} \left|_{x=0} \right.; \quad \lambda_e \frac{dTe}{dx} \left|_{x=0} \right. = -j_{pe} \left|_{x=0} \right. \cdot U_c;
\]

\[
j_{pe} \left|_{x=0} \right. = j_{ach} \cdot \exp \left( -\frac{eU_c}{KTe_w} \right); \quad j_{em} = AT_w^2 \exp \left( -\frac{e\varphi_{ef}}{KTe_w} \right); \quad (23)
\]

b) on the boundary with the discharge plasma (\( x = \delta \))

\[
j_{e} = 0; \quad \left( \frac{dT}{dx} \right)_{x=\infty} = \left. \frac{dT}{dx} \right|_{x=\infty} = C_t; \quad \left. \frac{dn_e}{dx} \right|_{x=\infty} = C_e \quad (24)
\]

Here \( A \) - the Richardson's constant, \( \varphi_{ef} \) - the effective material work function. Subscriptions " \( \infty \) " and " \( \omega \) " - mean the conditions on the boundary between the layers I and II and on the boundary with the discharge plasma accordingly.

The value of the heat flux \( q_o \) which is passed into the cathode body is defined from the energy balance on the cathode surface

\[
j_{em} = \left( \frac{5}{2} \frac{KTe_w}{e} + \varphi_{ef} \right) + j_{iw} (U_c + \lambda_i - \varphi_{ef}) + \lambda \frac{dTe}{dx} = q_o + \varepsilon_w \xi T_w^4 + j_{em} \left( \frac{5}{2} \frac{KTe_w}{e} + \varphi_{ef} \right), \quad (25)
\]

where \( \varepsilon_w \) - the blackness degree of the cathode surface. \( \xi \) - the Stephan - Boltzmann's constant.

As it is shown in [3] the solution of the equations system (1)-(22) is the enough labour-consuming procedure and requires the large expenditure of the calculation time. Therefore the set of the received solutions is limited: it was calculated the near-cathode processes characteristics for the arc discharge burning in argon (\( P = 10^5 \) Pa) on the tungsten cathode for the difference values of \( T_{ew} \) when the values \( T_w \) or \( q_o \) remain invariable.

The balance calculation method.

The balance calculation method which is used usually is estimational and is not taken into account all completeness of the collisional layer. In this method it is assumed that the electron and heavy particles temperatures have the constant value in layer II. Besides it is suggested that the energy flux \( j_{em} \) carried in the layer II by the emission electrons is spent on the compensation of energy losses with the ion current \( j_i \left( \lambda_i + \frac{5}{2} \frac{KTe_w}{e} \right) \) , with the fast plasma electrons current \( \dot{j}_{pe} \left( U_c + \frac{5}{2} \frac{KTe_w}{e} \right) \) and with total current on the boundary with the discharge plasma \( j \left( \frac{5}{2} \frac{KTe_w}{e} \right) \).

The equations system in the balance method which is used usually has the next form [4]

\[
\dot{j} = \dot{j}_{em} + \dot{j}_{i} + \dot{j}_{pe}, \quad (26) \quad \dot{j}_{em} = AT_w^2 \cdot \exp \left( -\frac{e\varphi_{ef}}{KTe_w} \right), \quad (27)
\]

\[
\varphi_{ef} = \varphi \omega - \Delta \varphi, \quad (28) \quad \Delta \varphi = 3.75 \cdot 10^{-4} E \omega, \quad (29)
\]
\[ E_w = 5700 (A_i)^{1/4} U_c^{1/4} i^{1/2}, \]
\[ j_e^e = j_o e^* \exp \left( -\frac{e u_e}{k T_w} \right) = \frac{e n_i e^*}{4} \exp \left( -\frac{e u_e}{k T_w} \right), \]
\[ j^e = j^e ch = \frac{e n_i e^*}{4}, \]
\[ P = K (n_e T_e + n_i T + n_o T_o), \]
\[ n_e = n_i, \quad (34) \]
\[ \beta (T_e) n_e n_o = \sigma (T_e) n_e^8 \]

where \( A_i \) - the atom weight, \( \bar{U}_e = \sqrt{\frac{8 k T_e}{\pi m_e}} \), \( \bar{U}_i = \sqrt{\frac{8 k T_i}{\pi m_i}} \), \( \varphi_w \) - the cathode material work function.

The energy balance equations on the cathode surface and for the collisional layer as a whole are solved in common with the equations (26)-(35)

\[ Q_o + E_w + T_e^4 + j^e \left( \frac{5}{2} \frac{k T_w}{e} + \varphi_e \right) = j^i \left( U_o + X_i - \varphi_e \right) + j^e \left( \frac{5}{2} \frac{k T_w}{e} \right), \]
\[ j^e \left( U_e + \frac{5}{2} \frac{k T_w}{e} \right) = j^i \left( X_i + \frac{5}{2} \frac{k T_w}{e} \right) + j^e \left( U_e + \frac{5}{2} \frac{k T_w}{e} \right) + \dot{j} \left( \frac{5}{2} \frac{k T_w}{e} \right), \]

The equations system (26)-(37) was solved for such conditions as a system (1)-(22). The comparison of the calculation results which was obtained the exact and balance methods in [3] showed that the cathode potential drop values \( U_c \) calculated with the help of the balance method exceed on 1-2 V the values of the cathode potential drop which was defined by the exact method. The difference of the values of the heat flux \( Q_o \) which is passed into the cathode body is more essential. The values \( Q_o \) which was defined by the balance method can twice as much than the values \( Q_o \) calculated by the exact method.

Since the balance method possesses essential lesser labour-consuming than the exact method, it is imagined expedient the modification of this method by which it is necessary to remain small labour-consuming of calculation procedure and to rise the precision of the values \( U_c \) and \( Q_o \) determination.

**The modified balance method.**

The considerable difference for the values \( Q_o \) calculated by the different methods which was peculiarities set in the collisional layer of near-cathode region of the arc discharge which is not taken account by the balance method calculations. For the arc discharge burning in argon (P~10^1 Pa) on the tungsten cathode these peculiarities may be written in the following way:
1) the electron temperature near the cathode surface \( T_{e}\) exceeds the electron temperature on the discharge plasma boundary \( T_{e}^{\infty} \) approximately by 2000 K;
2) the heavy particles temperature \( T \) near the cathode surface is equal to the cathode surface temperature \( T_{w} \) and on the discharge plasma boundary \( T_{e} \approx T_{e}^{\infty} \);
3) the heat flux \( \lambda \frac{dT}{dx} \) which is determined by the heavy particles heat conductivity changes very weakly and is equal approximately \( 4 \times 10^4 \text{ W/cm}^2 \).
4) the current density on the cathode surface $j_{\text{w}}$ is defined by the diffusion in the near-cathode layer and may be written as $j_{\text{w}} = B(T_{\text{w}}) \cdot j_{\text{w}}$, where $B(T_{\text{w}})$ - the coefficient which depends on temperature $T_{\text{w}}$. 

5) the near-cathode plasma composition is non-equilibrium, the net electron generation rate $\dot{n}_e$ values by the temperatures $T_{\text{w}}$ and $T_{\text{e}}$ make up about 0.95 from value $\dot{n}_e^{\text{max}}$ which is the maximum value by the corresponding temperatures $T$ and $T_e$.

6) under the emission electrons action the plasma motion as a whole in the collisional layer is arised towards the discharge plasma and through the boundary with the discharge plasma the energy flux $U = \frac{1}{2} k T_{\text{p}} (n_e^2 + n_i^2 + n_n^2)$ goes out.

With account of these peculiarities the equations system for the modified balance method has the following form:

$$
j = j_{\text{em}} + j_{\text{w}} + j_{\text{ew}}, \quad (38)
$$

$$
j_{\text{em}} = AT_e^2 \exp\left( - \frac{e \phi_{\text{ef}}}{kT_{\text{w}}} \right), \quad (39)
$$

$$
\phi_{\text{ef}} = \phi_w - \Delta \phi, \quad (40) \quad \Delta \phi = 3.75 \cdot 10^{-4} \cdot E_{\text{w}}, \quad (41)
$$

$$
E_w = 5700 \left( \frac{A_i}{A_e} \right) \cdot U_{\text{c}}^{-1/2} \cdot \frac{d}{d_e}, \quad (42)
$$

$$
\dot{j}_{\text{ew}} = \dot{j}_{\text{ch}} \cdot \exp\left( - \frac{e U_c}{kT_{\text{w}}} \right) = \frac{en}{4} \exp\left( - \frac{e U_c}{kT_{\text{w}}} \right), \quad (43)
$$

$$
\dot{j}_{\text{w}} = B(T_{\text{w}}) \cdot \dot{j}_{\text{w}} = \frac{en}{4} \cdot B(T_{\text{w}}), \quad (44)
$$

$$
P = k(n_e T_e + n_i T_i + n_n T_n)_w = k(n_e T_e + n_i T_i + n_n T_n)_\infty, \quad (45)
$$

$$
T_\infty = T_{\text{e}}, \quad (46) \quad n_e = n_i, \quad (47)
$$

$$
T_{\text{e}} = T_{\text{w}} - 2000, \quad (48) \quad \dot{n}_e = \beta n_e n_a - d n_e^3, \quad (49)
$$

$$
\dot{n}_e = 0.95 \cdot \dot{n}_e^{\text{max}}, \quad (50) \quad U = \sqrt{\frac{2 e U_c}{m_e}} \cdot \sqrt{\frac{m_{n_e} n_{e}^{\text{em}}}{(m_e + m_i + m_n + M_n)}}, \quad (51)
$$

The energy balance equations in the modified balance method may be written as

$$
q + E_w \delta T_w + j_{\text{em}} \left( \frac{5}{2} \frac{kT_{\text{w}}}{e} + \phi_{\text{ef}} \right) = \lambda \frac{dT}{dx} + \dot{j}_{\text{w}} \left( U_c + \phi - \phi_{\text{ef}} + \frac{5}{2} \frac{kT_{\text{w}}}{e} \right) +
$$

$$
+ \dot{j}_{\text{ew}} \left( \frac{5}{2} \frac{kT_{\text{w}}}{e} + \phi_{\text{ef}} \right), \quad (52)
$$

$$
\dot{j}_{\text{em}} \left( U_c + \frac{5}{2} \frac{kT_{\text{w}}}{e} \right) = \lambda \frac{dT}{dx} + \dot{j}_{\text{ew}} \left( U_c + \frac{5}{2} \frac{kT_{\text{w}}}{e} \right) + \dot{j}_{\text{w}} \left( \phi - \phi_{\text{ef}} + \frac{5}{2} \frac{kT_{\text{w}}}{e} \right) +
$$

$$
+ \dot{j} \left( \frac{5}{2} \frac{kT_{\text{w}}}{e} + U \frac{5}{2} kT_{\text{w}} \right) \left( n_e + n_i + n_n \right), \quad (53)
$$

The results of equations system (38)-(53) calculations for the arc discharge burning in argon ($P=10^5$ Pa) on the tungsten cathode are presented on fig 2 and fig 3.
By the calculations the diapason of change of the cathode surface temperature $T_w$ maked up from 3500 K to 4000 K, of the near cathode electron temperature $T_{e_w}$ maked up from 15000 K to 23000 K. On fig.2 and fig.3 the dependencies of the cathode potential drop $U_c$ and the heat flux $q_o$ from the total current density $j$ are presented.

Here it is shown the influence on the values $U_c$ and $q_o$ such parameters as the temperature $T_w$ and $T_{e_w}$. On fig.2 and fig.3 for the comparison the values $U_c$ and $q_o$ which were calculated by the exact method are shown. The sufficiently correspondence of the results which were received by the exact and modified balance methods allows to apply this modified balance method for the calculation of the argon near-cathode region characteristics.

The application of the modified balance method for other $P$ : $T$ : $T_e$ diapasons in argon or the arc discharge burning in other gases requires the additional calculations of the exact method.

References