THE ELECTRON DYNAMICS IN SPT-CHANNEL
AND THE PROBLEM OF ANOMALOUS EROSION

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ABSTRACT

The report is devoted to numerical simulation of some problems on electron dynamics in SPT-channel. The results of the simulation give the possibility to see some special and typical properties of electron trajectories, their dependence on initial parameters of electrons and on a character of their interaction with the walls of the channel. As a result of averaging of large number of the trajectories some mean values determined by electron dynamics were obtained. It is proposed the new numerical model of anomalous erosion of insulators. In this model the anomaly is connected with the singularities of electron dynamics in SPT-channel.

1. Introduction

The theoretical [1,2] and experimental [3] investigations of SPT-processes had shown that electron dynamics in SPT is very complicated. It is known, in particular, that there exist three groups of electrons: 1) "slow" electrons, which oscillate along magnetic force lines and do not reach the insulators because of near-walls Debye layers; 2) "fast" electrons, which, on the contrary, freely go through these layers and elastically (but not mirror-like) reflect; and 3) "intermediate" electron, which reach the walls and reflect inelastically. Now there are not direct measurements of the Debye layers but exist some indirect data, showing that a value of potential jump $U_d$ is not homogeneous and depend on time. The energy distribution function of reflected electrons is known (see, for example [4]). The experimental investigation [5] had shown that near the external insulator there exist strong high-frequency oscillations, which may be a cause for additional electron scattering. There exist also electron collisions with neutral atoms and ions. It is rather difficult to take into account all the fac-

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tors mentioned above in numerical simulation. Therefore, the main attention was directed to the dynamics of fast electrons. Calculating trajectories of intermediate electrons, we will also neglect by variability of the Debye layers. In the model of anomalous erosion the value $U_d$ will be known function of coordinates. The distribution of electric–magnetic field is considered to be a given one and close to the field of "ATON".

2. The statement of the problem

It is shown in [1], that motion of electrons in SPT is not drift-like and the transverse adiabatic invariant does not remain constant. Therefore to calculate the electron trajectories, we must use exact equations of electron motion.

There are two basic assumptions – the electromagnetic field has axial symmetry and does not depend on time. Further we shall use the cylindrical coordinates $(r, \phi, z)$.

Let the electrical field $E$ and the magnetic field $H$ are given in the following way $E = (E_r, 0, E_z)$, $H = (H_r, 0, H_z)$. It is conveniently to introduce the electrical potential $\Phi$ and the magnetic flux function $\Psi$ by the next formulas

$$ E_z = -\frac{\partial \Phi}{\partial z}, \quad E_r = -\frac{\partial \Phi}{\partial r}, \quad H_z = \frac{\partial \Psi}{r \partial r}, \quad H_r = -\frac{\partial \Psi}{r \partial z} \quad (1) $$

Then the exact equations of electron motion can be written as follows

$$ m \ddot{z} = -\frac{\partial U}{\partial z}, \quad m \ddot{r} = -\frac{\partial U}{\partial r} \quad (2) $$

and at $t=0$ $z = z_0$, $r = r_0$, $v_z = v_{z0}$, $v_r = v_{r0}$, $v_\phi = v_{\phi0}$.

Here $U(r,z) = -e\Phi + 1/2mr^2(D + e/c \Psi)$ – is the effective potential, $D = mr^2v_\phi^2 - e/c \Psi$ – is the generalized integral of angular momentum.

Let an electron moves in a region $0 \leq z \leq z_1$, $r_1 \leq r \leq r_2$. The boundaries $z = 0$, $z = z_1$ correspond to the anode and to the exit of the channel, the values $r_1$ and $r_2$ are the internal and external radiuses of the insulators.

Next we define the electric and magnetic fields in this region by the following way

$$ \Psi = r_1 (h_o(z_1 - z) + \frac{H_0 - h_0}{3 z_1^2} (z_1^3 - z^3)) \quad (3) $$

$$ \Phi = \kappa \Psi \quad (4) $$
The relation (4) is in a good agreement with experimental data. The values $H_o, h_o, \kappa$ are the parameters of the model.

Moving in the fields (3),(4) the electron may reach the side walls of the channel ($r = r_1$ or $r = r_2$). We suppose that in such cases it reflects inside the channel. The result of the reflection depends on the group which it belongs to. For the fast electrons it is assumed that their energy does not change ($\varepsilon = m v^2/2 = \text{const}$) and all possible directions of motion have equal probabilities (diffusion reflection). Namely, let $\xi$ and $\eta$ are random variables and they have homogeneous distributions on the intervals $[0,1]$ and $[0,2\pi]$ correspondingly. Then after collision the electron velocity has the next value:

$$
\begin{align*}
v_r &= \bar{r} (2\varepsilon/m)^{1/2} (1 - \xi) \\
v_z &= (2\varepsilon/m)^{1/2} \sqrt{\xi(2-\xi)} \cos(\eta) \\
v_\theta &= (2\varepsilon/m)^{1/2} \sqrt{\xi(2-\xi)} \sin(\eta)
\end{align*}
$$

Having determined the electron velocity by (5) we may to find its further motion. Certainly the new pair of random parameters $\xi, \eta$ is used for each next collision. The electron trajectory "finishes" then $z = 0$ (the electron reached the anode) or $z = z_1$ (the electron went out the channel).

In the following we use dimensionless units of measurements: $r_1$ is the unit of length; $H_o$ is the unit of magnetic field; $t_o = \omega^{-1}$ is the unit of time ($\omega = eH_o/mc$); $v_o = r_1/t_o$ is the unit of velocity, $E_o = \kappa r_1 H_o$ is the unit of electrical field.

There are two important dimensionless parameters in the problem. It is $\alpha = \kappa c/\omega_o$ the parameter that characterizes the relation between the electric and magnetic fields, and it is $\gamma = h_o/H_o$ that determines the spatial distribution of the fields.

Equations (2) are solved numerically by means of explicit difference scheme of second order approximation [6].

On the whole the electron trajectory depends of course on the initial electron parameters and on a sequence of the random parameters $\xi, \eta$ determining results of each collision with the
walls. However, it is natural to think that if a number calculated electron trajectories will be high enough, some average characteristics will not depend on singularities of separate trajectories.

3. Numerical results

The geometry of the channel is fixed by $z_1 = 3$, $r_1 = 1$, $r_2 = 2$. Let $\gamma = 0.1$. If $H_0 \approx 100$ Oe, $E_0 \approx 100$ v/cm, then $\alpha \approx 0.03$.

It is known [7], that collisions with the walls of insulators are a cause of so called near-wall conductivity. This effect was detected experimentally [3] by spatial oscillations of $z$ - component of electron current. In connection with this effect we undertook to calculate a radial distribution of the current. The trajectories of $N=5000$ electrons were calculated and statistically treated in the sense of the Monte-Carlo method. The radial distributions of longitudinal electron current for cross-sections $z = 0$, $z = 0.8$, $z = 1.6$, $z = 2.0$ are shown on the fig.1. For fast electrons the oscillations are not very strong. This results of calculations are in good agreement with experimental data. On fig.2 one can see the projections of typical trajectory of fast electrons on the planes $(r,z)$ and $(r,\theta)$.

Now we shall consider the singularities of trajectories of intermediate electrons. It is assumed that $U_d = 0$ but the energy of electron may change after collisions. The probability that the electron with the energy $E_i$ will have after collision the energy $E_f$ is given by the next formula

$$ W(E_f) = \frac{2}{\nu} (1 - \frac{E_f}{(\nu E_i)}) \quad E_f < \nu E_i $$

$$ = 0 \quad E_f > \nu E_i $$

(6)

where $\nu \approx 1$ is the parameter of the function $W(\cdot)$.

On fig.3 the projections of typical trajectory of intermediate electrons are presented ($\alpha = 0.01$). The comparison of trajectories of fast and intermediate electrons shows that in the last case an electron losing as a rule the larger part of its energy "adheres" to the external wall of the channel. The collisions with the internal wall are much more infrequent than in the case of the fast electrons.

It is not difficult to take into account the slow electrons. In the case $U_d \neq 0$, and if $\varepsilon < U_d$ we have an
elastic reflection \(-\nu_r \rightarrow -\nu_r, \nu_z \rightarrow \nu_z, \nu_\theta \rightarrow \nu_\theta\). The slow electron may move during long time along some magnetic force line, reflecting either from the internal wall or the external one.

4. Qualitative model of the anomalous erosion

The anomalous erosion of insulators was revealed for the first time in OKB Fakel [8] in the process of prolonged (> 1000 hours) resource tests. For this type of erosion there are three characteristic singularities: 1) The appearance of periodical structure with space scale \(\approx 1\) mm; 2) The orientation of these nonhomogeneities (especially on the initial stage) along the flow axes but not transverse it as in the case of the classical ion bombardment; 3) The difference between the characters of erosion on the external and internal walls. It was suggested in [8] that this type of erosion is stipulated mainly by electrons. The energetic electrons hitting an insulator may tear completely or partially the ties of "atoms" with the basic mass of the insulator. Then due to either evaporation or ion bombardment the weakly bound "atoms" go out from their places. The process of formation of the erosion structure may be presented by the following way. On the surface of the "new" insulator there are always small hollows, protuberances and also granulars of different solidity. These nonhomogeneities are destroyed differently and may form random "grooves" with orientation mainly from the cathode to the anode. The effect of electron "adhesion" to the external walls will promote this process. The formation of the "grooves" structure influences on the value \(U_d\). The simple arguments show that \(U_d\) will be larger on the side walls of the grooves and smaller on their bottoms and tops.

The next is devoted to the testing and to the refinement of this exclusively qualitative model.

5. Numerical model of the anomalous erosion

We shall consider only the processes near the external insulator. Let the inner surface of this insulator has at \(t=0\) the form \(r = R_o(\theta)\), where \(R_o(\theta)\) is a smooth periodical function with a period \(\Theta_m\). Let also for \(0 \leq \theta \leq \Theta_m\) we have the next analytical expression
\[ R_0(\theta) = A_0^0 + A_1^0 \cos(\omega \theta) + A_2^0 \cos(2\omega \theta) + A_3^0 \cos(3\omega \theta) + \]
\[ B_1^0 \sin(\omega \theta) + B_2^0 \sin(2\omega \theta) + B_3^0 \sin(3\omega \theta) \]

where \( \omega = 2\pi/\Phi_m \)

Thus, \( R_0(\theta) \) is the shape of the "new" external insulator.

The numerical model of the erosion gives an algorithm for a transition from the function \( R_0(\theta) \) to the function \( R_1(\theta) \) that presents a shape of the insulator at the "next" time \( t_1 \).

Here we suggest that the analytical expression (7) is valid always, but the coefficients in (7) may change.

Now we give a brief description of the algorithm.

1) The "length" of the profile \( R_0(\theta) \) is calculated

\[ l_0 = \int_{0}^{\Phi_m} \sqrt{R_0^2 + (R'_0)^2} \, d\theta \]

2) The profile is divided into \( N_1 \) (\( N_1 \approx 50 \)) equal intervals.

3) Being given the integer number \( N_p \) (\( N_p \approx 1000 \)) we calculate in the region \( z_{mn} \leq z \leq z_{mx}, r_{mn} \leq r \leq r_{mx}, 0 \leq \theta \leq \Phi_m \) the trajectories of \( N_p \) electrons. The number of passings through the Debye layer for each of the intervals is counted. Let it will be the numbers \( I_1,...,I_{N_1} \). Then the numbers \( \beta_1,...,\beta_{N_1} \) are defined as follows

\[ \beta_i = I_i / (\sum_k I_k) \]

4) Next we find the coordinates of the points which divide the intervals into two equal parts \(-r_1, \theta_1, \ldots, r_{N_1}, \theta_{N_1} \) and move these points along the external normal vector on the distance \( \sigma \beta_i \). Thus we obtain the points \((r_1, \theta_1), \ldots, (r_{N_1}, \theta_{N_1})\).

5) Using these points by means of method of least squares we construct the \( R_1(\theta) \) of the form (7) i.e. we find the coefficients \( A_0^1, A_1^1, \ldots, B_3^1 \).

6) Further we set \( R_0(\theta) = R_1(\theta) \) and repeat the points 1) - 5).

The functions \( R_0(\theta), R_1(\theta), \ldots, R_N(\theta) \) give the "time"-evolution of the external insulator. \( N_1, N_p, \sigma \) are the parameters of the model.

On the fig.4 for \( \alpha = 0.005, \gamma = 0.1, \nu = 1.0, U_d = 0.02, \sigma = 0.001, N_1 = 40, N_p = 500 \) there be given the insulator profiles at different moments of the "time".
References


[2]. Morozov A.I., Savelyev V.V., Numerical simulation of electron dynamics in SPT-channel, 2-nd German-Russian Conf. on Electric propulsion engines. Russia, Moscow, 1993, p. 45.


