CALCULATION OF CHARACTERISTICS OF THE ELECTRIC PROPULSION PLUME INTRINSIC RADIATION

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Introduction

Fast development of electric propulsions (EP) allowed to use them on board the modern communication satellites as the attitude control and orbit correction thrusters for many years already.

But EP application for communication satellites equipped by multichannel repeaters imposes high requirements to their electromagnetic compatibility with on-board radio equipment. These requirements are formally stipulated in the standards for the self-radiation level of a noise source within the given frequency range at a distance of 1 m from it (see MIL STD 461B and MIL STD 462 standards, for example).

Application of the standards of this type meets a number of difficulties, because it is necessary to take into account the spatial electrophysical properties for the plasma plume, which may simultaneously be a source of emissions, caused by different kinds of instabilities, and a waveguiding medium (non-uniform plasma waveguide), radiating into the environment.

Thus, EP jointly with plasma plume is not a point, but a spatially distributed source of radiation. The spatial location of the radiation effective center and its directional pattern are characterized by a complicated dependence on frequency.

The knowledge of the above-presented characteristics is of great importance in the tasks of electromagnetic interference (EMI). In the case of EP, these characteristics are still defined experimentally, because currently there no theoretical works, in which a waveguiding task for a plasma plume would present a directional pattern calculation as a result.

Mathematical models, allowing to calculate spatial characteristics for the modern EP self-radiation, would allow to model the EP interaction with on-board radio systems at the stage of technologic designing more quickly and efficiently.

This paper contains results for the study of the processes of generation and emission of inherent electromagnetic radiation, appearing in the plumes of typical plasma thrusters.

Results of [1-5, 8,9] were used as a basis of theoretical approach to the mechanisms of oscillations excitation is plasma accelerators. The task problem consisted of three parts:

- study of the processes of oscillations generation in the accelerator discharge channel;
- study of the processes of plasma plume excitation and the propagation of oscillations inside it as in a waveguiding system;
- study of the processes of electromagnetic radiation generated by a plasma plume with the given electrophysical parameters into the surrounding space.

The first task, connected with the definition of the spectrum of oscillations in the discharge gap of the thruster, requires the knowledge of the exact design of the plasma source and might be solved analytically using the known algorithms or the available experimental data, that is why it is not presented in detail in this paper.

Two other tasks are rather original and their solution is presented in detail in the first and in the second parts. Detailed substantiation of the used approaches and methods is given as well as the main mathematical algorithms, used as a basis in the developed software.

Wave properties of plasma plume.

The propagation of electromagnetic wave of small amplitude is analyzed inside the uniform plasma cylinder, simulating the plume and being the boundary between the plasma and vacuum.

The problem is considered under the following assumptions:

1. Plasma is assumed to be collisionless, i.e. collisions of of all kinds of particles are not taken into account;

2. External magnetic field is absent;

3. F_a function of α particles distribution in velocities and densities diverts from its equilibrium value $f_{0\alpha}$ for a small value f_a, $f_{0\alpha} >> f_a$;

4.Distribution function $f_{0\alpha}$ is supposed to be an equilibrium one.

But in this case, the directed velocity of electrons, exceeding the thermal one, and $V_f \geq V_{T\alpha}$ will be taken into account. The task is solved in kinetic approximation. Kinetical approximation is used when the mean thermal velocity of plasma particles $V_{T\alpha}$ is compatible with the phase velocity of the wave V_f.

The above assumptions take place in a number of practically important cases, for example in the case of ion flow compensation in plasma thrusters of various types. Cylinder coordinate system (r,φ,z) is chosen, the cylinder surface in which is described by the equation $r = R$. This system is considered to be axisymmetric, i.e. $\frac{\partial}{\partial\varphi} = 0$.

The wave is propagating along Z axis and charged particles (plume) are flowing in the same direction. Dependence of variables on Z coordinate and on time is accepted to be of the following form: $\exp i(k_3 z - \omega t)$, where k_3 - longitudinal wave number in plasma (k is the wave number in vacuum).

Taking these assumptions into account, a system of linearized equations may be written as follows:

$$\frac{d^2\Pi}{dr^2} + \frac{1}{r}\frac{d\Pi}{dr} + k^2\Pi = \frac{j}{i\omega\varepsilon_0} \tag{1}$$

$$\sum_\alpha \frac{\partial f_\alpha}{\partial t} + \bar{V}\frac{\partial f_\alpha}{\partial r} + \frac{e_\alpha}{m_\alpha}\bar{E}\frac{\partial f_{0\alpha}}{\partial V} = 0 \tag{2}$$

$$\bar{j} = e\int_V \bar{V} f_\alpha \partial V \tag{3}$$

where:

$\bar{j} = j(0,0,jz)$ - density of alternating electric current;

$\bar{\Pi} = \Pi(0,0,\Pi_z)$ - Hertz vector, which is connected with electric and magnetic fields by the following relations:

$$\bar{E} = (\Delta + k^2)\bar{\Pi}, \quad \bar{H} = ik\nabla \times \bar{\Pi} \tag{4}$$

System of equations (1-3) should be added by boundary conditions. For electric and magnetic fields the continuous behavior of tangential components at the boundary between plasma and vacuum is such a condition:

$$E_z^{pl}(R) = E_z^{vac}(R), \quad H_\varphi^{pl}(R) = H_\varphi^{vac}(R),$$

while for the distribution function - condition of mirror reflection of particles from the boundary:

$$f_\alpha^{+}(R, V_r \geq 0, V_z) = f_\alpha^{-}(R, V_z \leq 0, V_z), \tag{5}$$

where V_r and V_z - radial and axial velocity components for particles; $f_\alpha = f_\alpha^{+} + f_\alpha^{-}$; $\alpha = e$ - for electrons; $\alpha = i$ - ions.

Deviation of distribution function f_α from the equilibrium function and thus the current j_z might be found by solving the equation (2) jointly with boundary conditions (5), having substituted the electric field components E_z and E_r through Π. Disturbed part of distribution function has the following form:

$$\begin{aligned} f_\alpha = f_\alpha^{+} + f_\alpha^{-} = i\frac{2e}{mV_z}\frac{\partial f_0}{\partial V_z}\int_0^R E_z(\zeta)\cdot K_1(r,\zeta)d\zeta - \\ -\frac{2e}{mV_r}\frac{1}{\gamma}\frac{\partial}{\partial r}\int_0^R E_r(\zeta)\frac{\partial f_0}{\partial V_r}K_2(r,\zeta)d\zeta \end{aligned} \tag{6}$$

where $\gamma = \frac{k_3 V_z - \omega}{V_r}$; $K_1(r,\zeta)$ and $K_2(r,\zeta)$ - integral cores:

$$K_1(r,\zeta) = \frac{1}{\sin\gamma R}\cdot\begin{cases}\cos\gamma_r\cdot\cos\gamma(R-\zeta), & r<\zeta\\ \cos\gamma_\zeta\cdot\cos\gamma(r-R), & r>\zeta\end{cases}$$

Expanding the integral cores $K_1(r,\zeta)$ and $K_2(r,\zeta)$ into Fourier series and substituting the expressions for E_r and E_z through Π into the equation (6), we obtain the values of electric current in the following form:

$$\begin{aligned} j_z = A\sum_{n=0}^{\infty}\frac{\cos\alpha_n r}{(\gamma^2-\alpha_n^2)R}\int_0^R \Pi(\zeta)\cos\alpha_n\zeta d\zeta - \\ -B\sum_{n=0}^{\infty}\frac{\alpha_n\cos\alpha_n r}{(\gamma^2-\alpha_n^2)R}\int_0^R \Pi(\zeta)\sin\alpha_n\zeta d\zeta \end{aligned} \tag{7}$$

where:

$$A = \frac{2e_\alpha^2}{m_\alpha}\int_0^{\infty} V_z dV_z \int_{-\infty}^{\infty}\frac{\gamma}{V_r}\frac{\partial f_0}{\partial V_r}dV_r;$$

$$B = \frac{2e_\alpha^2}{m_\alpha}\int_0^\infty V_z dV_z \int \frac{1}{V_r}\frac{\partial f_0}{\partial V_r} dV_r; \qquad (8)$$

$$\alpha_n = \frac{n\pi}{R}; \quad n = 0,1,2,\ldots..$$

After the substitution of (7) into the equation (1) we obtain a non-uniform differential equation, solution of which causes definite difficulties. Results will be simplified at possible substitution of γ^2 by α_n^2 or in the case when longitudinal velocity is higher than radial one ($V_z > V_r$), or at small n values and large radiuses R.

Assumption of large radius corresponds to a practically semilimited plasma. These assumptions are frequently realized in practice, especially in the case of electron flow propagation through plasma along Z axis. Expanding $(\gamma^2 - \alpha_n^2)^{-1}$ into series and taking first two members of expansion into account, we obtain the following uniform wave equation:

$$\frac{d^2\Pi}{dr^2} + \frac{1}{rL}\frac{d\Pi}{dr} + T^2\Pi = 0 \qquad (9)$$

where:

$$L = 1 + \frac{(k^2 - k_3^2)A}{i\omega\varepsilon_0\gamma^2} + \frac{kB}{\omega\varepsilon_0\gamma^2};$$

$$T^2 = \left[(k^2 - k_3^2) - \frac{k^2 - k_3^2}{i\omega\varepsilon_0\gamma^2}\right]L^{-1}$$

$$\Pi = A_n r^{\frac{L-1}{L}} Z_\nu(TR)\exp(i(k_3 z - \omega t)), \qquad (10)$$

where:

$$\nu = \frac{L-1}{2L} = \frac{1}{2}\sqrt{(1-\frac{1}{L})^2 + 4k\varphi}$$

Z_ν - some cylinder function or a combination of cylinder functions. Using the expressions (10) and (11), let's calculate the components of electric field:

$$E_z = (k^2 + \frac{\partial^2}{\partial z^2})\Pi = (k^2 - k_3^2)\Pi; \quad H_z = 0$$

$$E_\varphi = \frac{\partial^2\Pi}{r\partial\varphi\partial z} = 0; \quad H_\varphi = -\frac{k^2}{i\mu\omega}\frac{\partial\Pi}{\partial r} \qquad (11)$$

$$E_r = \frac{\partial^2\Pi}{\partial r\partial z}; \qquad H_r = \frac{k^2}{i\omega\mu r}\frac{\partial\Pi}{\partial\varphi}$$

Electric field components (11) jointly with boundary conditions (5) define the surface impedances of the plume. Let's make the surface transversal impedance of plasma flow be equal to the impedance of free space. We shall obtain the following equation as a result of this:

$$\frac{T}{k}\frac{H_0(kR)}{H_1(kR)} = \frac{Y_\nu(TR)}{Y_{\nu-1}(TR)}, \qquad (12)$$

where $Y_\nu(TR)$, $H_0(kR)$ - cylinder functions.

Let's study the propagation of high frequency wave with $V_f < V_{Te}$, but $V_f > V_{Ti}$, through the limited plasma, i.e. the phase velocity of the wave is less than the electron thermal velocity, but higher than the ion thermal velocity. The so-called "slow" wave is propagating along the electron flow, moving at V_{Te} velocity. Then, having defined À and B coefficients from expressions (8), we shall find the expressions for T^2 and ν :

$$T^2 = (k^2 - k_3^2)\frac{1 - \sum_\beta\left[\omega_{e\beta}^2(\omega')^{-2} + 3\omega_{e\beta}^2 k_3^2 V_{Te}^2(\omega')^{-4}\right]}{1 + \sum_\beta\left[-\frac{3\omega_{e\beta}^2(k^2 - k_3^2)}{k_3^4 V_{Te}^2} + \frac{\omega_{e\beta}^2 k}{(\omega')^2 k_3}\right]}; \qquad (13)$$

$$\nu = \frac{1}{2} - \left\{\sum_\beta\left[-\frac{3\omega_{e\beta}^2(k^2 - k_3^2)}{k_3^4 V_{Te}^2} + \frac{\omega_{e\beta}^2 k}{(\omega')^2 k_3}\right]\right\}^{-1};$$

where:

index $\beta = p$ refers to plasma, and $\beta = b$ - to the electron flow;

$\omega_{e\beta} = \sqrt{\varepsilon_0 e^2 n_\beta m^{-1}}$ - Lengmuir electron frequency for plasma and flow;

$\omega' = \omega$ at summing up for $\beta = p$;

$\omega' = \omega - k_3 u$ at summing up for $\beta = b$;

u - electron flow velocity along Z axis;

V_{Te} - thermal velocity of electrons.

Within the broad range of plasma parameters variation the value of ν is close to 1/2. If $\nu = 1/2$ is accepted and left and right parts of equation (12) are simplified, we obtain:

$$\frac{1}{k}\frac{H_1(kR)}{H_0(kR)} = -\frac{1}{T}ctgTR \qquad (14)$$

Let's substitute Henkel functions by McDonald functions. Such substitution is valid, because dispersion equations are obtained for the case when impedance boundary conditions are used. Finally, the dispersion equation has the following form:

$$\frac{1}{kR}\frac{K_0(kR)}{K_1(kR)} = \frac{1}{T(k_3)R}tgT(k_3)R \qquad (15)$$

Thus, for this case the dispersion equation (15) is obtained, which defines the entire spectrum of plasma waves.

Calculation algorithm for a plasma plume directional pattern

Calculation of directional pattern for a plasma plume is based on a numerical solution of dispersion equation, defining a number of axial wave numbers for a slow surface of the wave. For each wave number the directional pattern is calculated using a model of equivalent dielectric cylinder rod with radiating end.

Dispersion equation, solved relative to an axial wave number k_3, has the following form:

$$\frac{1}{kR}\frac{K_0(kR)}{K_1(kR)} = \frac{1}{T(k_3)R}tg\left[T(k_3)R\right] \qquad (16)$$

or $y = f(k_3) = 0$,

where:

$$y = f(k_3) = \frac{1}{T(k_3)R}tg\left[T(k_3)R\right] - \frac{1}{kR}\frac{K_0(kR)}{K_1(kR)}. \qquad (17)$$

Here $k = \omega/c < k_3$ - wave number in free space, $\omega = 2\pi f$ - angular frequency of the wave, $c = 3\cdot10^8$ m/s - light velocity in vacuum, R - cylinder radius.

$$T^2(k_3) = (k^2 - k_3^2)\frac{1 - F_1(k_3,p) - F_1(k_3,b)}{1 + F_2(k_3,p) - F_2(k_3,b)}, \qquad (18)$$

$$F_1(k_3,\beta) = \frac{\omega_{e\beta}^2}{(\omega'_\beta)^2} + \frac{3\omega_{e\beta}^2 k_3^2 \vartheta_{Te}^2}{(\omega'_\beta)^4}, \qquad (19)$$

$$F_2(k_3,\beta) = \frac{\omega_{e\beta}^2 k}{(\omega'_\beta)^2 k_3} - \frac{3\omega_{e\beta}^2 (k^2 - k_3^2)}{k_3^4 \vartheta_{Te}^2}, \qquad (20)$$

$$\beta = \begin{cases} \text{p for plasma,} \\ \text{b for electron flow,} \end{cases}$$

$\omega_{e\beta} = \sqrt{\dfrac{n_\beta e^2}{\varepsilon_0 m}}$ - plasma electron frequency,

n_β - electron density,

e - absolute value of the electron charge,

m - electron mass,

ε_0 - electric constant,

$$\omega_\beta' = \begin{cases} \omega \text{ for } \beta = p \\ \omega - k_3 u \text{ for } \beta = b, \end{cases}$$

u - electron flow velocity,

V_{Te} - thermal velocity of electrons,

$K_0(\cdot), K_1(\cdot)$ - McDonnald's function.

Approximation of the following form is used when McDonnald's functions $K_0(x), K_1(x)$ are calculated:

a) at $0 < x \le 2$

$$K_0(x) = -\ln(x/2)I_0(x) - 0{,}57721566 + 0{,}42278420(x/2)^2 + 0{,}23069756(x/2)^4 + 0{,}03488590(x/2)^6 + 0{,}00262698(x/2)^8 + 0{,}00010750(x/2)^{10} + 0{,}00000740(x/2)^{12}, \qquad (21)$$

where $I_0(x)$ - modified Bessel function of the first kind of zero order:

$$I_0(x) = 1 + 3{,}5156229(x/3{,}75)^2 + 3{,}0899424(x/3{,}75)^4 + 1{,}2067492(x/3{,}75)^6 + 0{,}2659732(x/3{,}75)^8 + 0{,}0360768(x/3{,}75)^{10} + 0{,}0045813(x/3{,}75)^{12}, \qquad (22)$$

$$K_1(x) = [x\ln(x/2)I_1(x) + 1 + 0{,}15443144(x/2)^2 - 0{,}67278579(x/2)^4 - 0{,}18156897(x/2)^6 - 0{,}01919402(x/2)^8 - 0{,}00110404(x/2)^{10} - 0{,}00004686(x/2)^{12}]\cdot x^{-1}, \qquad (23)$$

where $I_1(x)$ - modified Bessel function of the first kind of the first order:

$$I_1(x) = [\ \frac{1}{2} + 0{,}87980594(x/3{,}75)^2 + +0{,}51498869(x/3{,}75)^4 + 0{,}15084934(x/3{,}75)^6 + +0{,}02658733(x/3{,}75)^8 + 0{,}00301532(x/3{,}75)^{10} + +0{,}00032411(x/3{,}75)^{12}\,]\cdot x\,; \qquad (24)$$

b) at $2 \le x < \infty$

$$K_0(x) = [\ 1{,}25331414 - 0{,}07832358(2/x) + + 0{,}02189568(2/x)^2 + 0{,},01062446(2/x)^3 + + 0{,}00587872(2/x)^4 + 0{,}00251540(2/x)^5 + + 0{,}00053208(2/x)^6\,]\cdot x^{-1/2}e^{-x}; \qquad (25)$$

$$K_1(x) = [1{,}25331414 + 0{,}23498619(2/x) + + 0{,}03655620(2/x)^2 + 0{,}01504268(2/x)^3 + + 0{,}00587872(2/x)^4 + 0{,}00251540(2/x)^5 + + 0{,}00053208(2/x)^6]x^{-1/2}e^{-x}; \qquad (26)$$

Transcendental equation is solved by two steps:

1) roots are found with a step, allowing to obtain all transitions through zero in the argument values area of interest; this task is solved by a graphical method on the basis of sequential visual analysis, made by the user;

2) calculation of roots at a given accuracy by a method of division into halves.

After that, directional pattern for the plasma plume radiation is calculated and visualized for each of the obtained values of axial wave number k_ς, being the roots of dispersion equation:

$$F_E(\theta) = \left|\cos\theta \frac{\sin b(\theta)}{b(\theta)} \frac{2J_1(d(\theta))}{d(\theta)}\right| \qquad (27)$$

in the E-plane;

$$F_H(\theta) = \left|\frac{\sin b(\theta)}{b(\theta)} \frac{2J_1(d(\theta))}{d(\theta)}\right| \qquad (28)$$

in the H-plane;

where: $b(\theta) = \frac{kL}{2}(\gamma - \cos\theta)$, (29)

L - cylinder length,

$\gamma = k_3/k > 1$ - deceleration coefficient,

$$d(\theta) = |kR\sin\theta|, \qquad (30)$$

$J_1(\cdot)$ - Bessel function of the first kind; it is calculated using the following formulae:

a) at $0 \le x \le 3$:

$$J_1(x) = [1/2 - 0{,}56249985(x/3)^2 + +0{,}21093573(x/3)^4 - 0{,}03954289(x/3)^6 + +0{,}00443319(x/3)^8 - 0{,}00031761(x/3)^{10} + +0{,}00001109(x/3)^{12}]\cdot x \qquad (31)$$

b) at $3 \le x \le \infty$:

$$J_1(x) = f_1\cos\theta_1/\sqrt{x}\,, \qquad (32)$$

$$f_1 = 0{,}79788456 + 0{,}00000156(3/x) + + 0{,}01659667(3/x)^2 + 0{,}00017105(3/x)^3 - - 0{,}00249511(3/x)^4 + 0{,}00113653(3/x)^5 - - 0{,}00020033(3/x)^6; \qquad (33)$$

$$\theta_1 = x - 2{,}35619449 + 0{,}12499612(3/x) + + 0{,}00005650(3/x)^2 + 0{,}00637879(3/x)^3 + + 0{,}00074348(3/x)^4 + 0{,}00079824(3/x)^5 - - 0{,}00029166(3/x)^6. \qquad (34)$$

Directional patterns (27), (28) are normalized in such a way that $F_{E,H}(0) = 1$ for a case of ideal axial radiation, corresponding to $\gamma = 1$.

The following parameters are specified as initial data:

- n_p - plasma electron density, m^{-3};
- n_b - electron flow density, m^{-3};
- u - electron flow velocity, m/s;
- V_{Te} - thermal velocity of electrons, m/s;
- f - wave frequency, Hz;
- R - plasma cylinder radius, m;
- L - plasma cylinder length, m;
- $\theta_i, \theta_e, \Delta\theta$ - initial and final values and step for the argument of directional pattern, degree;
- names of directories, containing the output files;
- $k_{3i}, k_{3e}, \Delta k$ - initial and final values and step of the argument for the function (2), 1/m;

- $\mathcal{E}$ - absolute accuracy for calculating the roots of dispersion equation, 1/m.

While making calculations the initial data is checked for the fulfillment of conditions, specified for the used procedure applicability: $k_3 > k$, $L < 12\pi/k$.

The following data is the output:

- $y = f(k_3)$ function;
- roots of the dispersion equation;
- 2-D arrays of directional pattern in E- and H-planes.

TEST EXAMPLE FOR THE CALCULATION OF PLASMA PLUME EMISSION PATTERN

Test calculations for different parameter values were made using the developed software. Results of these calculations are presented in Fig. 1-4. Test values of the software input data are presented below:

- plasma electron density, m^{-3} .... 10^{16}
- electron flow density, m^{-3} .... 0.5 10^{16}
- electron flow velocity, m/s .... 0.4 10^5
- thermal velocity of electrons, m/s .... 6 10^5
- wave frequency, Hz .... 5 10^7
- plasma cylinder radius, m 0.5, 1
- plasma cylinder length, m 1.5, 3
- initial and final values for the argument of directional pattern, degree .... -180,180

Plots of Fig. 1 and 2 correspond to the plume radius of 0.5 m and its length of 1.5 m and are made in E and H orthogonal planes, correspondingly, while plots of Fig. 3 and 4 correspond to the plume

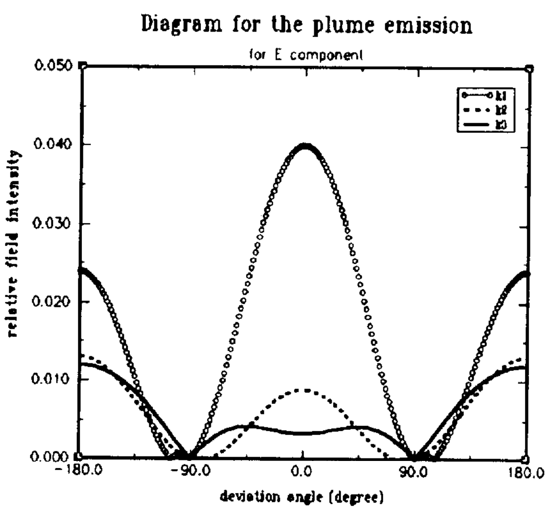

Fig.1.a

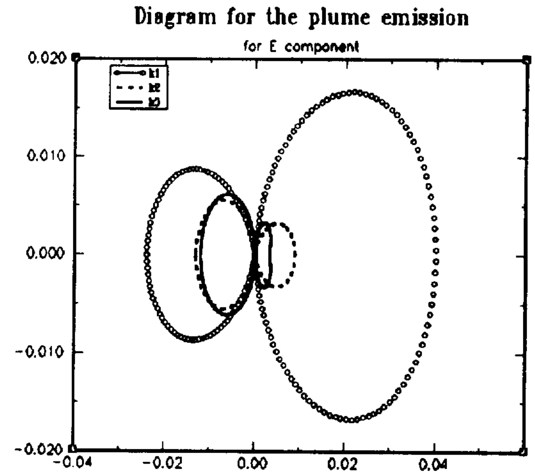

Fig.1.b

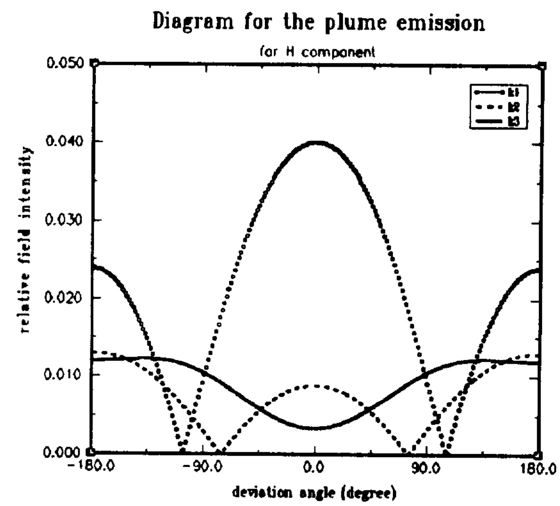

Fig.2.a

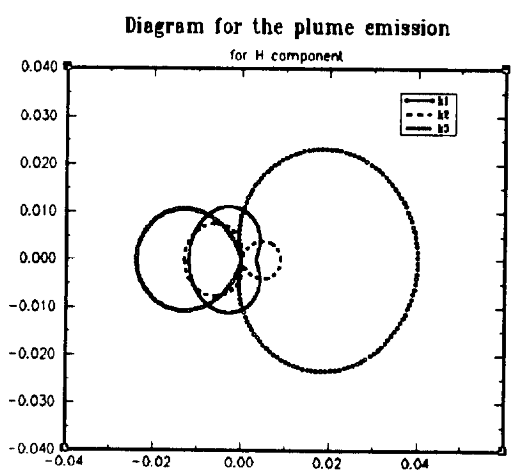

Fig.2.b

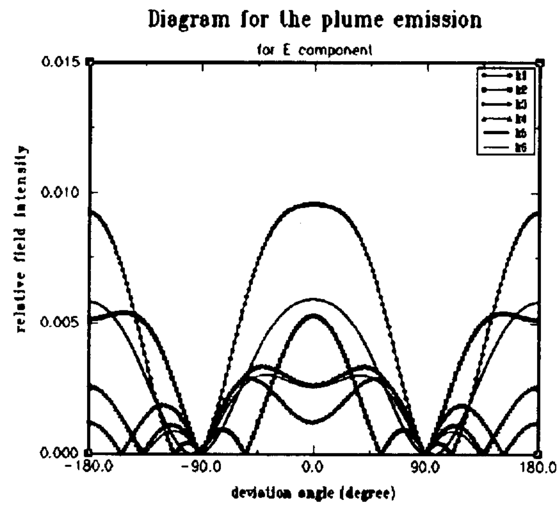

Fig.3.a

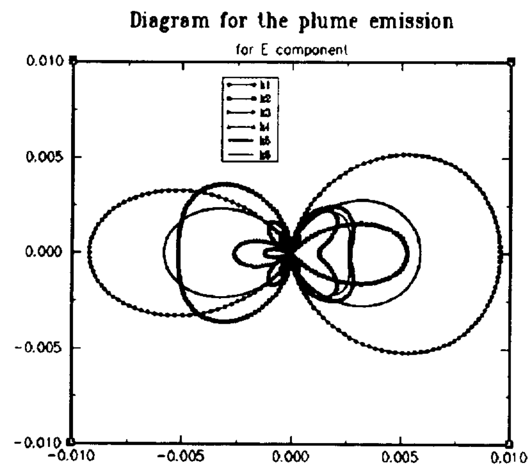

Fig.3.b

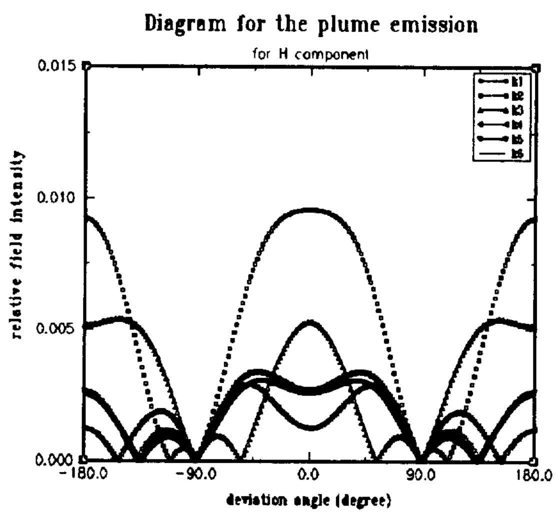

Fig.4.a

of I m in radius and 3 m in length. Upper plots of figures I-4 are made in rectangular coordinate system and bottom ones - in the polar coordinate system.

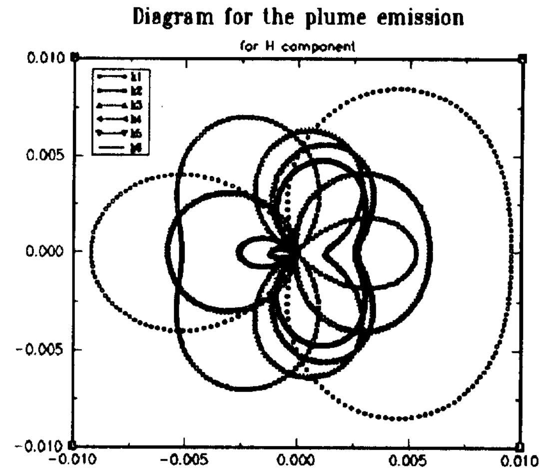

Fig.4.b

Analysis of calculation results shows that at the variation of plume geometric parameters (radius and length) the number of dispersion equation roots changes from 3 to 6 at corresponding variation of spatial directivity of emission.

As to the normalization of the emission diagrams presented, all of them are normalized to the maximum radiation intensity of an equivalent traveling wave antenna, the directivity characteristics of which are defined by the formulae (27) and (28) for the following case: $\gamma = k_3/k=1$ and axial direction of radiation.

Thus, the presented diagrams characterize the loss in emission of plasma plume relative to the emission of corresponding equivalent antenna.

CONCLUSION

Mathematical model describing the emissive properties of the plume is developed. It is based on the plume presentation in the form of a set of traveling wave antennae for each mode of oscillations.

Algorithms are developed for simulating the processes of the inherent electromagnetic emission of plasma plume. Cylinder model of plasma plume is used as a basis. New solution of dispersion equation, defining the set of modes of oscillations in the plasma plume of given structure, is obtained.

Software package is developed for the calculation of characteristics of plasma plume radiation into the surrounding space within the broad range of wave lengths for plasma blobs of given structure.

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