Abstract

The relative importance of electromagnetic and electrothermal acceleration mechanisms in pulsed plasma thrusters (PPT) was investigated using theoretical analysis and computer simulations. An analysis based on railgun accelerators quantifies the maximum electromagnetic acceleration expected from plasma accelerators. The impulse calculated from this analysis was found to be significantly smaller than that measured experimentally. Computer simulations of a coaxial accelerator using the MACH2 magnetohydrodynamic (MHD) code showed that there can be a significant electrothermal contribution to the thrust. The magnitude of this contribution depends on the quantity of mass injected, and simulations with reduced propellant mass revealed that the acceleration mode changes at low densities. The MHD simulations provide insight into the complex plasmadynamic processes in pulsed plasma thrusters. This insight should be valuable in the development of improved pulsed plasma thrusters.

Introduction

Pulsed plasma thrusters (PPT) are currently undergoing a strong revival of interest as candidate propulsion systems for small satellites. They are commonly described as electromagnetic plasma thrusters in which a Teflon propellant is ablated into the breech region of a rail accelerator. Recent measurements indicate that the PPT's operate with a specific impulse of approximately 1000 s and a thrust efficiency of approximately 6%. Some attempts at optimization have improved these values to 1430 s and 11.5 %. The specific impulse is similar to that obtained by electrothermal (arcjet) thrusters, but the efficiency is much less. There are a number of reasons why these thrusters do not provide better performance, due both to fundamental limits on the maximum electromagnetic acceleration that can be expected and on the details of the plasma interactions that occur during the acceleration process. There is also some evidence that a significant part of the acceleration results from electrothermal rather than electromagnetic processes.

Electromagnetic Acceleration

A simple conservation analysis has been used to determine the maximum force that can be exerted by a railgun. This analysis neglects certain losses such as eddy current losses in the accelerating electrodes and assumes that all work done by the circuit is exerted to accelerate the moving armature (the propellant gas). The resulting force is,

\[ F(t) = \frac{1}{2} L i^2(t) \]  

(1)
where $F$ is the total force exerted on the propellant, $L'$ is the inductance gradient of the accelerator structure and $i(t)$ is the instantaneous current supplied to the rail structure. It should be noted that this is the maximum total force that can be exerted on the propellant, regardless of the details of the plasma interaction with the electromagnetic field. Although this expression was developed for the railgun, it also gives the correct expression for a simple MHD accelerator. Using this expression for the force, the total impulse that can be generated during a single pulse (the impulse bit) is given by,

$$I_b = \int_0^t F(t) dt = \frac{1}{2} L' \int_0^t i^2(t) dt$$  \hspace{1cm} (2)

where $I_b$ is the total impulse generated over the total pulse time $t$.

The specific impulse and the thrust efficiency associated with this total impulse depend on the velocity with which the propellant is ejected from the accelerator. If we assume the propellant starts at rest and is ejected with an effective velocity $v_{ex}$, the total impulse can also be written,

$$I_b = m v_{ex} = mL' g$$  \hspace{1cm} (3)

where $m$ is the total mass expelled during a single pulse, $L'$ is the specific impulse and $g$ is the sea level acceleration of gravity. The thrust kinetic energy for a single pulse is given by,

$$E_T = \frac{1}{2} m v^2_{ex} = \frac{1}{2} m \left( L' g \right)^2 = \frac{1}{2} I_b \left( I_b g \right)$$  \hspace{1cm} (4)

If the thrust efficiency is defined as the ratio of the thrust energy to the initial stored electrical pulse energy, $E_0$, then the thrust efficiency is given by,

$$\eta_T = \frac{E_T}{E_0}$$  \hspace{1cm} (5)

In order to estimate the expected pulse efficiency, we will assume that the pulse energy is supplied by a simple LRC circuit. The current produced by an underdamped LRC circuit is given approximately by,

$$i(t) = V_0 \sqrt{C/L} e^{-\frac{R}{2L}} \sin \left( \frac{t}{\sqrt{LC}} \right)$$  \hspace{1cm} (6)

where $V_0$ is the initial voltage on the capacitor of capacitance $C$, $L$ is the circuit inductance and $R$ is the total circuit resistance. In the spirit of making a best-case estimate, we will assume that $Rv2L \ll I$ and make the acceleration estimate based only on the first positive cycle of the damped sinusoid, i.e.,

$$\tau = \pi \sqrt{LC}$$  \hspace{1cm} (7)

This pulse form can be approximated in practice by crowbarring the circuit at the first zero crossing of capacitor voltage.

With these assumptions, the current is a single sinusoidal pulse given approximately by,

$$i(t) = V_0 \sqrt{C/L} \sin \left( \frac{t}{\sqrt{LC}} \right)$$  \hspace{1cm} (8)

Using Equations 7 and 8 in Equation 2 we find that the total impulse over the first positive cycle is given by,

$$I_b = \frac{\pi L'}{4} V_0^2 C^{1/2} L^{-1/2}$$  \hspace{1cm} (9)

Noting that the initial stored capacitor energy is,

$$E_0 = \frac{1}{2} C V_0^2$$  \hspace{1cm} (10)

and, using Equation 4, the thrust kinetic energy can be written as,

$$E_T = E_0 \frac{\pi L'}{4} I_b g \sqrt{C/L}$$  \hspace{1cm} (11)

This expression shows that the thrust energy is directly proportional to the inductance gradient, the specific impulse and $\sqrt{C/L}$.

Specific impulse is an important measure of thruster effectiveness and is directly proportional to thrust efficiency. An estimate for the specific impulse that can be generated through electromagnetic acceleration can be obtained using experimental relations for the mass ablated per pulse.

In the typical PPT the mass ablated during a single pulse is directly proportional to the initial stored energy.
In an ideal railgun-type accelerator all the mass ablated during a pulse would be injected before the acceleration began. This would insure that all of the mass exited the accelerator with the exhaust velocity, \( v_{ei} \). With this assumption, and using Equations 3, 9 and 10, the exhaust velocity is given by,

\[
v_{ei}(\tau) = E_0 \frac{\pi L'}{2m} \frac{C}{L}
\]

If the mass is given by Equation 12, then the exhaust velocity is,

\[
v_{ei} = \frac{\pi L'}{2\alpha} \frac{\sqrt{C}}{L}
\]

and the specific impulse is,

\[
I_p = \frac{\pi L'}{2\alpha g} \frac{\sqrt{C}}{L}
\]

The velocity given in Equation 14 is based on only the first positive cycle of the damped sinusoid and, therefore, represents the maximum velocity which the entire ablated mass, \( m \), can achieve. If we base the thrust efficiency on this velocity, then the maximum possible value of thrust efficiency is obtained from Equations 5, 11 and 15,

\[
\eta_r = \frac{(\pi L')^2}{8\alpha} \frac{C}{L}
\]

From Equations 15 and 16 we find that the thruster performance depends on three parameters: the inductance gradient, \( L' \), the ablation parameter, \( \alpha \), and the ratio of capacitance to circuit inductance, \( C/L \). It is clear that restricting the analysis to the first half-cycle provides the most optimistic estimate, since subsequent cycles will have smaller values of current and acceleration.

The value of inductance gradient for the present PPT configurations is approximately \( 2 \times 10^7 \). Recent measurements using a baseline PPT gave a value of \( 1.325 \times 10^8 \) kg/J for the ablation coefficient of Teflon using a 30 \( \mu \)F capacitor with a circuit inductance of 0.093 \( \mu \)H [8]. Using these values we estimate a specific impulse of 435 s and a thrust efficiency of 1.2%. Although these values are based on a best-case model, they are significantly less than the measured performance of the thruster (specific impulse of 960 s and thrust efficiency of 5.9%). This strongly suggests that a significant part of the thrust was derived from electrothermal processes.

### Plasmadynamic Effects in Electromagnetic Accelerators

In current versions of the PPT, the propellant mass is ablated during the accelerating pulse and only a portion of that mass undergoes electromagnetic acceleration, which results in poor propellant utilization. In the analysis presented above, it was assumed that the magnetic field swept up all the propellant gas and accelerated it to the exhaust velocity without losses. This corresponds to a one-dimensional "snowplow" model, commonly used to analyze electromagnetic acceleration of perfectly conducting plasma. There are two factors that limit the validity of this model: 1) resistive diffusion of the magnetic field through the plasma and 2) the Hall effect. Both of these mechanisms allow penetration of the magnetic field into the plasma and can result in a significant fraction of the propellant being left behind the accelerating magnetic field.

One of the requirements for a good railgun-type pulsed plasma accelerator is that there must be a relatively thin current sheet between the rails. This occurs when there is a strong axial gradient in the magnetic field. The strong axial gradient forms when the magnetic field interacts with the conductive plasma and the current is confined to the distance that the field diffuses into the plasma. In a high conductivity plasma this produces a thin current sheet, but if the plasma conductivity is small, the magnetic field can diffuse through the plasma, leaving weakly ionized material behind. In space thrusters the accelerator channel will vent to low conductivity space-plasma conditions between pulses, and the injected plasma must retain a plasma density sufficient to prevent diffusion of the magnetic field.

The Hall effect becomes important when the plasma electrons become magnetized. This occurs at low plasma density when the Hall parameter, the ratio of electron cyclotron frequency to electron collision frequency, becomes of the order unity or greater,
\[ \Omega_H = \frac{eB}{m_e v_{coll}} \geq 1 \]  \tag{17}

where \( e \) is the electron charge, \( m_e \) is the electron mass, \( B \) is the magnetic induction, and \( v_{coll} \) is the electron collision frequency. When this occurs, an electric field component appears perpendicular to both the current and magnetic field. This Hall electric field can propagate the magnetic field through the plasma as a magnetic shock\(^{13}\). This effect also uncouples the magnetic field from the plasma and allows it to penetrate, leaving only weakly accelerated propellant plasma behind.

**MACH2 Simulations for Coaxial Pulsed Plasma Thrusters**

Some earlier simulations of a PPT have been performed by Mikellides and Turchi\(^{12}\) using the MHD code, MACH2. They developed a detailed model for the ablation of Teflon propellant in a two-dimensional rail accelerator. We have also used the MACH2 code to explore and illustrate some of the plasmadynamic effects that occur during the electromagnetic acceleration of the plasma propellant. In the simulations reported here the propellant ablation was not modeled. A cylindrical accelerator, rather than the rail accelerator, was chosen for these simulations to avoid the three-dimensional effects of rail accelerators that tend to reduce their performance and are difficult to simulate in two dimensions\(^{13}\).

The MACH2 code was used for computer simulations of a coaxial electromagnetic thruster to illustrate some of the plasmadynamic effects discussed in the previous section. A coaxial geometry was chosen to eliminate the important three-dimensional effects inherent in the rectangular geometry of the PPT. MACH2 is an MHD code originally developed by Mission Research Corporation for Phillips Laboratory to study pulsed plasma devices\(^{14}\). A new Hall effect algorithm was developed at the University of Tennessee Space Institute (UTSI) and implemented in MACH2 for simulation of plasma opening switches\(^{15}\). MACH2 simulations were used to investigate the behavior of a coaxial pulsed plasma accelerator in which the propellant is placed into the accelerator channel before the acceleration pulse begins. This allows the propellant mass to be determined independently of the initial stored energy. The computational domain used for the simulations is shown in Figure 1. The accelerator has a cathode diameter of 1 cm and an anode diameter of 3 cm with an overall length of 5.5 cm. The propellant is initially uniformly distributed in the upstream 5 mm of the accelerator channel. This geometry results in an inductance gradient, \( L' \), of \( 2.2 \times 10^{-3} \), similar to the PPT.

![Figure 1. Computational grid used for the closed base MACH2 coaxial thruster simulations.](image)

The current pulse is generated by a simple LRC circuit with the external inductance, \( L = 93 \, \text{nH} \), the external resistance, \( R = 0 \), and the capacitance \( C = 30 \, \mu\text{F} \) charged to an initial voltage of 182 V to provide an initial stored energy of 50 J. The maximum current occurs at approximately 2.5 \( \mu\text{s} \). These values were chosen to compare with experiments reported in Reference 8.

Two different plasma models were used for these simulations: The first used a plasma that consisted of doubly ionized carbon, \( C^{++} \), traditionally used to model plasma opening switches operated with Teflon plasmas. The second used thermodynamic properties for equilibrium Teflon plasma calculated at UTSI. In both cases a modified Spitzer conductivity model was used for electrical conductivity.

In order to illustrate the electrothermal effects two types of simulation were run. In the first the coaxial thruster had a closed base as shown in Figure 1. In the second an additional computational zone was added upstream but was left open so propellant was free to move backward in the thruster. The results of a simulation for the closed-base thruster in which the total mass was equal to 73 \( \mu\text{g} \), similar to that measured in Reference 8, is shown in Figure 2.
Figure 2. MACH2 Simulation of impulse as a function of time for the closed base coaxial thruster with an initial mass of 73 μg. The curve labeled Total is the total impulse, the curve labeled Pressure is the integral of fluid pressure over the closed base of the thruster, the curve labeled JxB is the volume integral of the electromagnetic force on the fluid, and \( \frac{1}{2} L' I^2 \) is the theoretical maximum electromagnetic impulse given by Equation 2.

In this simulation the initial plasma in the zone next to the base was Teflon with a density of 2.32 × 10^{-2} kg/m^3. The rest of the computational domain was filled with Teflon at a density of 3.2 × 10^{-7} kg/m^3 and an electron density of 3.19 × 10^{19} m^{-3}.

In this simulation the thruster functioned essentially as an MHD accelerator, and the arc remained nearly stationary near the base of the thruster. This is shown in Figure 3.

The total impulse on the thruster consists of the sum of the electromagnetic reaction, equal to the integral of the JxB forces, and the integral of the base pressure over the base area and time, since there is no divergence in the thruster walls. At 1 μs the base pressure is approximately 4 atmospheres and the temperature is approximately 2.7 eV. As shown in Figure 2, the pressure pulse is essentially complete by 2 μs. This is followed by additional electromagnetic acceleration as indicated by the maximum pressure region moving downstream from 2 to 4 μs. Some indication of the “snowplow” effect can be seen in the 3 μs plot where the current peak has started to move downstream. The impulse is essentially complete at 4 μs as seen from Figure 2. For this example the total impulse is 932 μN-s and the electrothermal contribution, indicated by the base pressure, to the total impulse is approximately 79 percent. The specific impulse is 1305 s and the thrust efficiency is 11.9 percent.

Another simulation was run for the same conditions except that the base was left open so that propellant could flow backwards out of the base, eliminating the pressure contribution to the impulse. The result is shown in Figure 4.

In the open base simulation the total impulse is essentially all electromagnetic. The total impulse is only 255 μN-s compared with 932 μN-s for the closed base case. In both cases the electromagnetic contribution to the impulse is well predicted by the simple expression in Equation 2.

Equations 14 and 16 indicate that specific impulse and thrust efficiency depend inversely on the ablation coefficient, α. This parameter controls the amount of propellant material ablated during the pulse. If this could be reduced, performance would be expected to improve. A simulation was run to investigate the effect of using a significantly smaller quantity of propellant. The simulation for the closed base thruster was the same as before except that the total mass was reduced from 73 μg to 7.3 μg. The result
of this simulation is shown in Figure 5. At 3 μs the total impulse is 148 μN·s, significantly smaller than either of the high-density cases. The explanation can be found in the detailed plasmodynamics of the acceleration process. Figure 6 shows the plasma density, cathode current density and current streamlines at times of 1.0, 1.5, 2.0 and 2.5 μs.

![Figure 4. MACH2 Simulation of impulse as a function of time for the open base coaxial thruster with an initial mass of 73 μg. The curve labeled Total is the total impulse, the curve labeled JxB is the volume integral of the electromagnetic force on the fluid, and \( \frac{1}{2} L' \cdot F^2 \) is the theoretical maximum electromagnetic impulse given by Equation 2.](image)

For this case the thruster is operating predominately in the railgun mode. This can be seen from the motion of the current density distribution down the accelerator channel with time. In the coaxial accelerator geometry both the current density and the magnetic field vary inversely with radius and this causes the JxB force to be significantly larger near the inner (cathode) wall. The plasma undergoes more acceleration near the cathode and the current and the arc moves toward the exit faster than on the anode wall. At approximately 2.0 μs the plasma near the cathode has thinned, and the arc accelerates rapidly to the muzzle leaving a large portion of the plasma near the outer (anode) wall behind with little acceleration. This results in a small fraction of the mass being accelerated to high velocities and the remainder accelerated to much lower velocities. The breaks in the JxB and total impulse curves in Figure 5 near 2.8 μs are a result of a portion of the current density extending beyond the computational domain.

![Figure 5. MACH2 Simulation of impulse as a function of time for the closed base coaxial thruster with an initial mass of 7.3 μg. The curve labeled Total is the total impulse, the curve labeled Pressure is the integral of fluid pressure over the closed base of the thruster, the curve labeled JxB is the volume integral of the electromagnetic force on the fluid, and \( \frac{1}{2} L' \cdot F^2 \) is the theoretical maximum electromagnetic impulse given by Equation 2.](image)

Discussion

Analysis based on railgun concepts and MACH2 computer simulations indicate that both electromagnetic and electrothermal processes contribute to the impulse in pulsed plasma thrusters.
The relative importance of these two mechanisms depends on the specific thruster parameters, especially the total mass injected per pulse. For the amount of mass used by current versions of the PPT it appears that the electrothermal component is at least comparable to the electromagnetic component.

The MACH2 simulations utilized a coaxial geometry having a similar value of the inductance gradient and the same circuit characteristics and mass as the benchmark PPT experiments. At a stored energy input of 50 J the MACH2 simulation predicted an impulse bit of 932 \( \mu \text{N-s} \) and the experiment measured an impulse bit of 623 \( \mu \text{N-s} \). Both of these values exceed the maximum expected electromagnetic impulse of approximately 255 \( \mu \text{N-s} \) and indicate the importance of electrothermal contributions to the total impulse. The larger impulse bit predicted by the MACH2 simulation is probably a result of the closed coaxial geometry which does not permit leakage of plasma out the sides as is possible with the rail structure.

The analysis predicts that both the specific impulse and the thrust efficiency of an electromagnetic accelerator are inversely proportional to the quantity of mass used (Equations 15 and 16). MACH2 simulations using a factor of ten smaller mass predicted a larger value of specific impulse, but also revealed a different mode of operation than found in the simulations with larger mass. With larger mass the accelerator operated like an MHD accelerator with the arc remaining near the base of the accelerator. There was little inducance change as a result of arc motion. With the smaller mass the arc accelerated down the accelerator pushing propellant in front of it. However, the mass expulsion was incomplete because of the variation of the \( \mathbf{J} \times \mathbf{B} \) force with radius and the increase in specific impulse was significantly less than a factor of ten. This effect is important in low density coaxial accelerators, but can be compensated by an initial propellant distribution which places more mass near the cathode.

MACH2 simulations were run using two different plasma models: a perfect gas of doubly ionized carbon and equilibrium Teflon. Only the results using Teflon were presented here. With carbon the temperatures are significantly higher, but the performance is similar to Teflon. It should be noted that the electromagnetic portion of the impulse is relatively independent of the propellant used.

Some MACH2 calculations were run with the Hall effect turned off. For the high-density cases there is little effect. In the low-density cases the arc accelerates somewhat faster and the arc exits the muzzle sooner. The low-density simulation shown in Figures 5 and 6 were run with the Hall effect disabled. The other simulations had the Hall effect included.

These simulations and the earlier simulations of Mikellides and Turchi using MACH2 show the value of a general purpose MHD code for the simulation and understanding of the complex plasmadynamics in pulsed plasma thrusters. These simulations can serve as a valuable tool to guide the development of improved thruster configurations. Mikellides and Turchi developed a more detailed ablation model to determine the quantity of propellant removed during the pulse. At the present time the accuracy of the code simulations is limited by the available data for thermodynamic and transport properties of plastic plasma propellants. The determination of thermal diffusion and radiation transfer to an ablating surface from a nonequilibrium plasma is a particularly difficult problem.

References


