DEVELOPMENT OF A STATIONARY PLASMA THRUSTER MATHEMATICAL MODEL ON THE BASE OF EXPERIMENTAL DATA

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ABSTRACT

The mathematical model of a Stationary Plasma Thruster (SPT) as of object of automatic control is necessary at the problem solving of synthesis and research of automatic control system for electrical propulsion engine module. The results of experimental researches of SPT-100 as initial data are used. In result of researches the dependence of the level of average value and amplitude of a discharge current oscillations from the level of an electromagnet current were received at fixed values of the discharge voltage and working substance mass flow rate, and also the spectrums of the discharge current oscillation at various values of an electromagnet current were obtained. The analysis of received experimental data has shown that the static characteristics of the thruster have extreme character and for design of mathematical model of SPT-100 the bell-form function are suggested, the parameters of which were determined on a basis of regression models. The mathematical model of SPT-100 consists of several sub-models. The first component of the model determines the average value of discharge current I_d. The second part of the making mathematical model specifies the amplitude of discharge current oscillations I_p. The third component of the making model reflects the frequent properties of the discharge current oscillations. The diagrams of experimental data fixed in a basis mathematical model and also results of digital simulations of the SPT-100 characteristics are submitted. The represented diagrams show good concurrence of experimental data to data which are received on the basis of offered mathematical model.

1. ANALYSIS OF EXPERIMENTAL DATA FIXED IN A BASIS OF MODEL

During experimental researches of the characteristics and parameters of the SPT-100 search of optimum modes of its work was made which are characterised by a minimum level of average value of the discharge current and the minimum amplitude of the discharge current oscillations at given level of the working substance mass flow rate [Ref. 1,2]. Thus minimisation of power expenses on creation of the thrust is provided, requirements to output power of a discharge power supply are much reduced. Preliminary experiments have shown that determining means for achievement of optimum modes of operation SPT-100 are a choice of size of an electromagnet current and also choice of structure and determination of optimum parameters of the output filter of a discharge power supply. For designing of mathematical model SPT-100 in which specified its peculiarities would be reflected the experimental researches were spent. The purpose of these researches was determination of the dependence between the variables of the thruster which can be considered as input and output. Following variables were considered as input variables of SPT: the level of a discharge voltage V_d, the mass flow rate of working substance through the anode block $\dot{m}$, the level of an electromagnet current, specifying intensity and a configuration of a magnetic field in the accelerating channel SPT - I_m. As output variables of the mathematical model are considered: amplitude of the discharge current oscillations I_p and size of average value of a discharge current I_d. The results of experimental researches of parameters and characteristics of the SPT-100 which appropriate to optimum structure and parameters of the output filter and fixed in a basis of synthesis of the first and second components of mathematical model SPT are represented in Fig. 1 [Ref. 2]. In this figure dependence of amplitude of the discharge current oscillations I_p and average value of the discharge current I_d from the level of an electromagnet current I_m are submitted. The character of the dependence between variables has extreme character which is kept at change of working modes of the thruster (changing of a discharge voltage and the level of the working substance mass flow rate) and at change of structure and parameters of the discharge current power supply output filter.

Alongside with determination of the static characteristics of the thruster SPT-100 the determination of its dynamic characteristics received as spectrums of the discharge current oscillations at various levels of the electromagnet current of an electromagnet were obtained the typical kinds of which are submitted in Fig. 2 [Ref. 3,4]. Received spectrums characterise the variable component of a discharge current $\tilde{I}_d$. The frequencies of registered fluctuations lay in a range from 1.0 up to 400 kHz. As it is visible from the submitted diagrams the basic capacity of fluctuations lays in the range up to 120 kHz, where the basic peaks of a spectrum of oscillations of a discharge current are located. With reduction of a current of an electromagnet from 4.5 A up to 2.6 A the amplitude of peaks sharply decreases (on 20 dB). The character of a spectrum of the oscillations in a range from 120 up to 400 kHz at change of a current of an electromagnet does not practically change.

2. CHOICE OF THE STRUCTURE OF MATHEMATICAL MODEL OF SPT

Mathematical model of SPT-100 is offered for presenting as set of several sub-models, reflecting the various parties of physical processes in SPT. In particular first making sub-model specifying dependence of average level of the discharge current I_d from a discharge voltage V_d, the current of an electromagnet I_m and mass flow rate $\dot{m}$. This model can be submitted as two equations each of which corresponds to a separate site of a range of possible level of a current of an electromagnet:

$$\begin{cases} \bar{I}_d = \bar{I}_{d0}(V_d,\dot{m}) + \dfrac{K_{d1}(V_d,\dot{m})\cdot\alpha_{d1}(V_d,\dot{m})}{(I_m - q_{d1})^2 + \alpha_{d1}(V_d,\dot{m})}, \\ I_m < q_{d2}(V_d,\dot{m}); \\ \bar{I}_d = \bar{I}_{d0}(V_d,\dot{m}) + \dfrac{K_{d1}(V_d,\dot{m})\cdot\alpha_{d1}(V_d,\dot{m})}{(I_m - q_{d1})^2 + \alpha_{d1}(V_d,\dot{m})} + \\ + \dfrac{K_{d2}\cdot\left[I_m - q_{d2}(V_d,\dot{m})\right]^2}{\left[I_m - q_{d2}(V_d,\dot{m})\right]^2 + \alpha_{d2}}, \\ I_m \geq q_{d2}(V_d,\dot{m}) \end{cases} \qquad (1)$$

where $\bar{I}_{d0}(V_d,\dot{m})$ - constant component of a discharge current; $K_{d1}(V_d,\dot{m})$ - peak value of the first bell-form function; K_{d2} - peak value the second of bell-form function; q_{d1} - minimum value of a current of an electromagnet at realisation of experimental researches; $q_{d2}(V_d,\dot{m})$ - the level of a current of an electromagnet, appropriate to a local minimum of average value of a discharge current; $\alpha_{d1}(V_d,\dot{m})$, α_{d2} - parameters, specifying gradient of bell-form functions.

Submitted in (1) the parameters of mathematical model, are the functions of two variables. These functions are received as non-linear regression models on the basis of experimental data. At designing of regression models orthogonal polynomials Chebyshew were used.

The second sub-model of making mathematical model of SPT which reflects dependence of amplitude of the discharge current oscillations I_p from a discharge voltage V_d, current of an electromagnet I_m and mass flow rate $\dot{m}$.

If by analogy with (1) to enter the following designations: $I_{p01}(V_d,\dot{m})$, $I_{p02}(V_d,\dot{m})$ - constant component of the amplitude of the discharge current oscillations; $K_{p1}(V_d,\dot{m})$ - peak value of the first bell-form of function; $K_{p2}(V_d,\dot{m})$ - peak value of the second bell-form function; $q_{p1}(V_d,\dot{m})$ - the value of a current of an electromagnet, appropriate to a minimum of the first bell-form function; $q_{p2}(V_d,\dot{m})$ - the value of a current of an electromagnet, appropriate to a minimum of the second bell-form function; $\alpha_{p1}(V_d,\dot{m})$, $\alpha_{p2}(V_d,\dot{m})$ - parameters, specifying the gradient of the bell-form functions, the mathematical model has the following kind:

$$\begin{cases} I_p = I_{p01}(V_d,\dot{m}) + \\ + \dfrac{K_{p1}(V_d,\dot{m})\cdot\left[I_m - q_{p1}(V_d,\dot{m})\right]^2}{\left[I_m - q_{p1}(V_d,\dot{m})\right]^2 + \alpha_{p1}(V_d,\dot{m})}, \\ I_m \leq q_{p1}(V_d,\dot{m}); \\ I_p = L_1(V_d,\dot{m})\cdot I_m + L_2(V_d,\dot{m}), \\ q_{p1} < I_p < q_{p2}; \\ I_p = I_{p02}(V_d,\dot{m}) + \\ + \dfrac{K_{p2}(V_d,\dot{m})\cdot\left[I_m - q_{p2}(V_d,\dot{m})\right]^2}{\left[I_m - q_{p2}(V_d,\dot{m})\right]^2 + \alpha_{p2}(V_d,\dot{m})}, \\ I_m \geq q_{p2}(V_d,\dot{m}), \end{cases} \qquad (2)$$

The functions of the two variables, included in structure of the equations (2) are regression models and are received similarly to functions, included in the equations (1).

The third sub-model reflects frequent properties of discharge current oscillations. For modelling of frequent spectrums of the discharge current oscillations, received experimentally and fixed in a basis of design of SPT model, the model of the forming filter with submission on an input of white noise is used. In result of realisation of experimental researches the spectrums of the discharge current oscillations for various level of a current of an electromagnet and optimum structure and parameters of the discharge power supply output filter were received [Ref. 2, 3]. The typical spectrums of the discharge current oscillations fixed in a basis of design a dynamic sub-model are presented in Fig. 2. Proceeding of a kind of the submitted diagrams, choice of structure of the forming filter was made and its parameters for various sizes of a current of an electromagnet are determined. The values of frequencies of work of this part of model lays in the range 0 - 120 kHz, in which there are the basic resonant frequencies of object and at change of size of a current of an electromagnet I_m the considered range of frequencies remains constant.

The numerical values of transfer factor of forming filters were determined by size I_p, calculated in the equations (2). The typical mathematical model of the forming filter, written down as transfer function, looks as follows:

$$W_{Im=3,2}(s) = \frac{\sum_{i=0}^{n} b_i s^{n-i}}{\sum_{j=0}^{p} c_j s^{p-j}}, n = 6, p = 8, \quad (3)$$

where s - variable of Laplas;

$\xi(s)$ - image by Laplas for white noise, which put on an input of the forming filter;

b_i, c_j - parameters of the forming filter, determined during identification on the basis of experimental data.

3. RESULTS OF DESIGN OF THE MATHEMATICAL MODEL OF SPT

The results of the design of the mathematical model are illustrated by the diagrams, submitted in Fig. 3, 4. In Fig. 3 results of modelling of the static characteristics are submitted

$$I_p = f_1(V_d, I_m, \dot{m});$$

$$I_d = f_2(V_d, I_m, \dot{m}).$$

The received model represents functions of the three variables, which can be used for development and modelling of the automatic control system for control of output variables of SPT which represents as extremal system. The received functions have obviously expressed extremal character and the system of extremal management should ensure maintenance of such values of SPT parameters at which amplitude of the discharge current oscillations and the level of the average value of the discharge current at given size of the mass flow rate will be minimum.

In fig. 4 results of modelling of spectrums of the discharge current oscillations for various values of the discharge current are represented on the basis of offered mathematical model of SPT. The comparison of results of modelling with experimental data shows good concurrence, that confirms adequacy to received mathematical model.

4. CONCLUSIONS

1. Mathematical model of the stationary plasma thruster SPT-100 type received on the basis of experimental static and dynamic characteristics is offered.
2. The received mathematical model reflects the static characteristics of SPT-100 at change of a discharge voltage, mass flow rate, level of the electromagnet current.
3. Mathematical model of SPT-100 enables of modelling of dynamic characteristics of the thruster as time dependence and frequent spectrums in a range of frequencies 0 - 120 kHz.

5. REFERENCE

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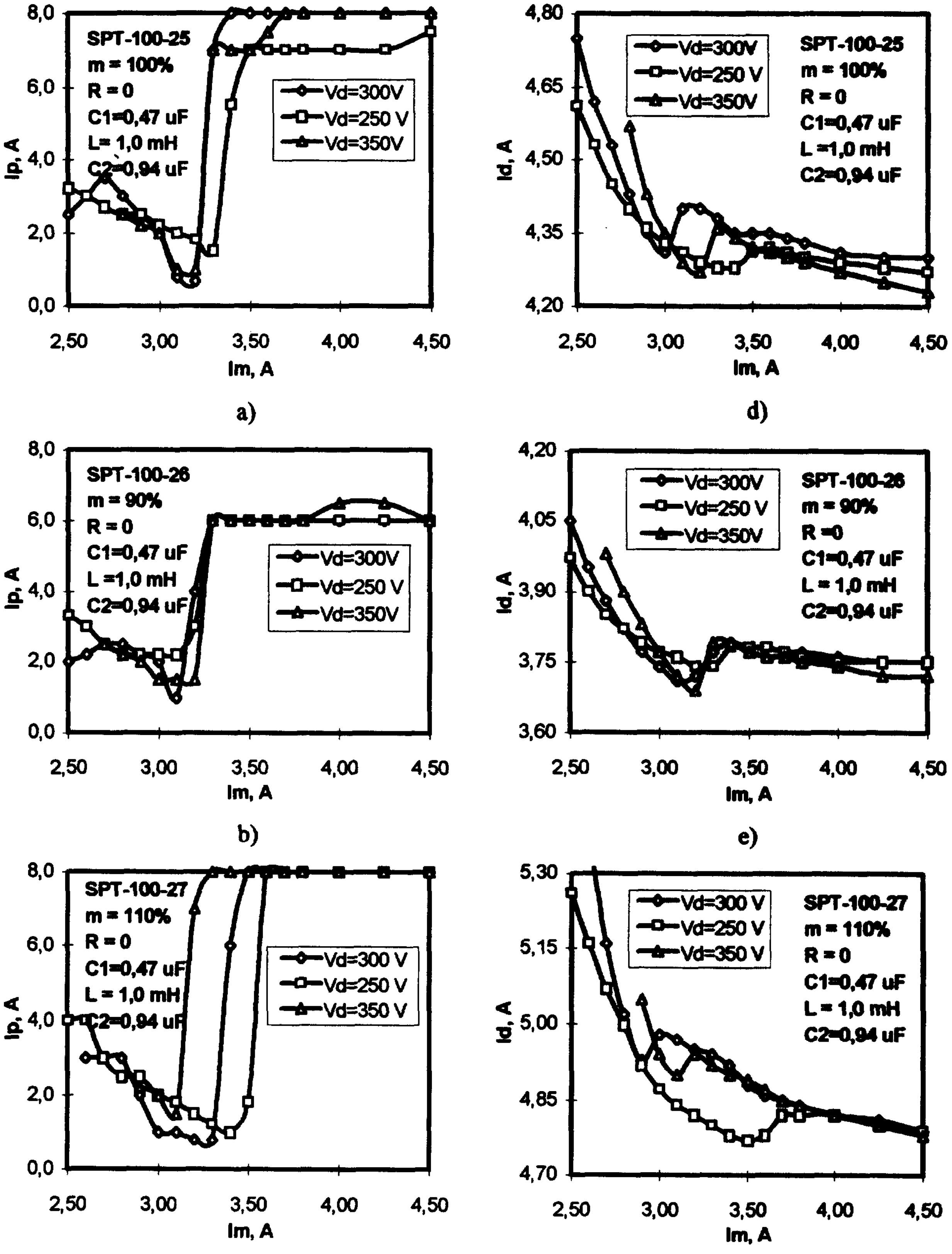

a) d)

b) e)

c) f)

Figure 1. The dependence of amplitude oscillations I_p and average value of the discharge current I_d from the electromagnet current I_m.

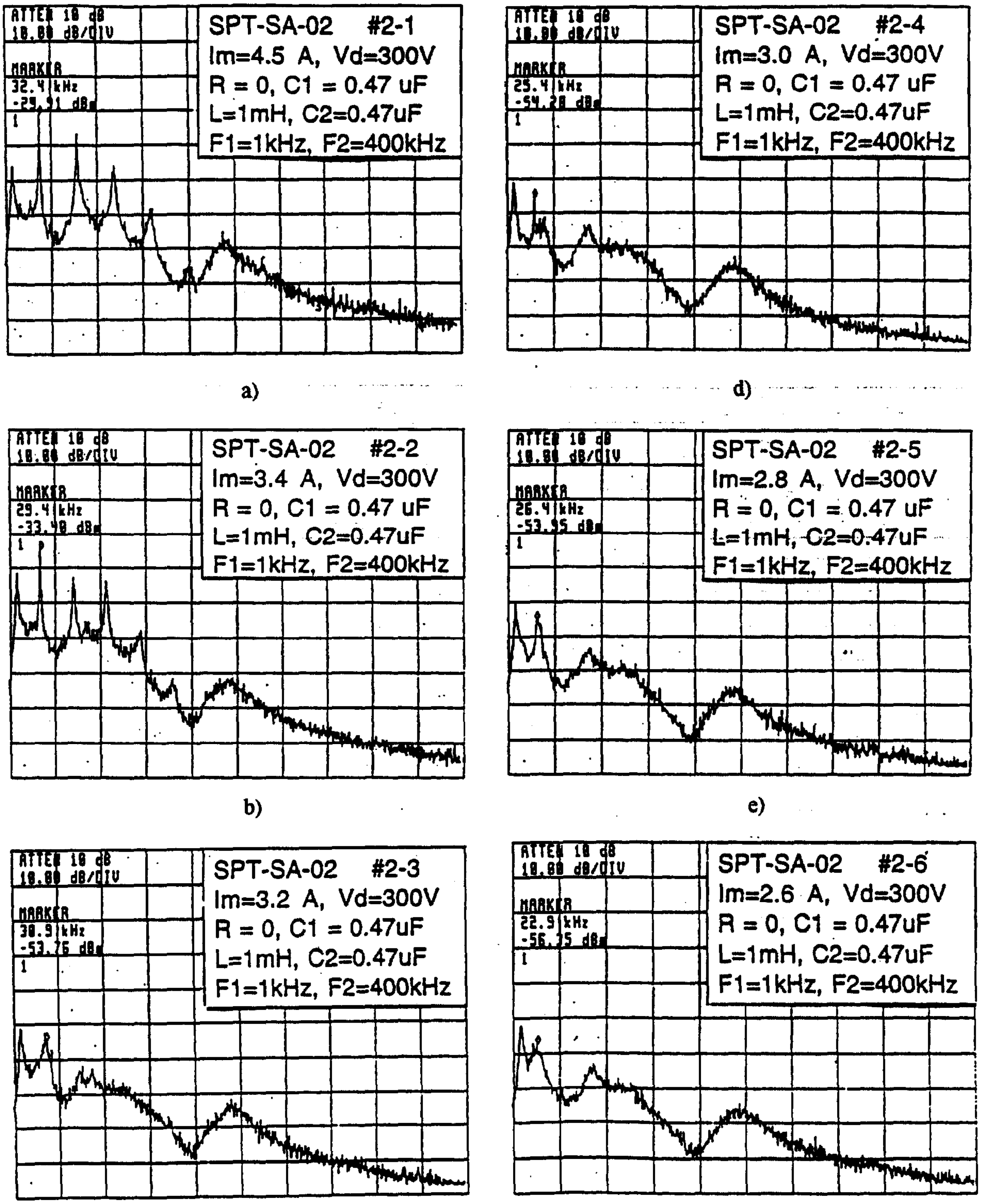

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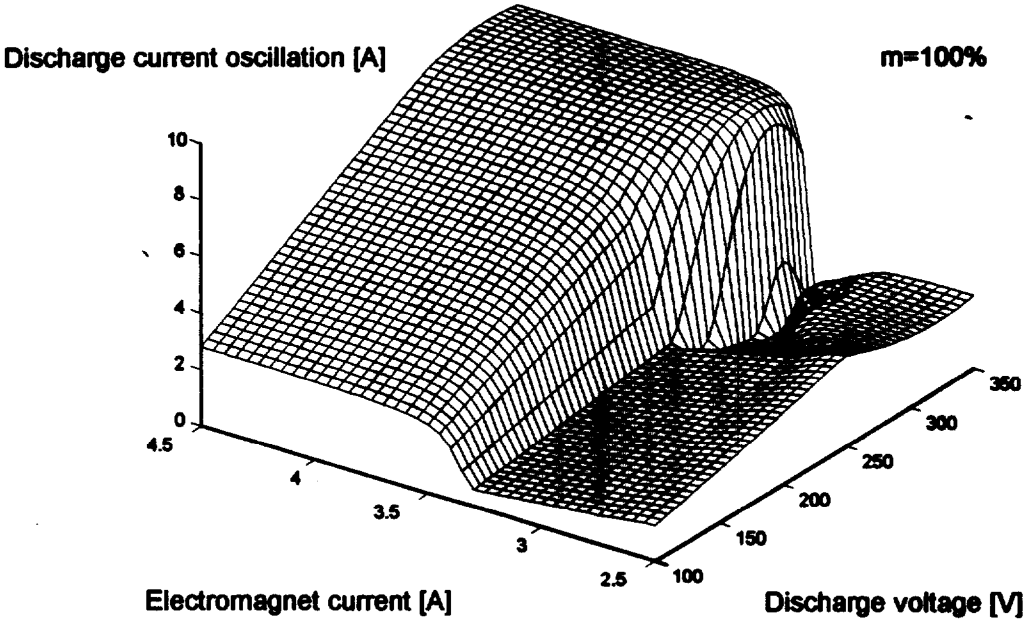
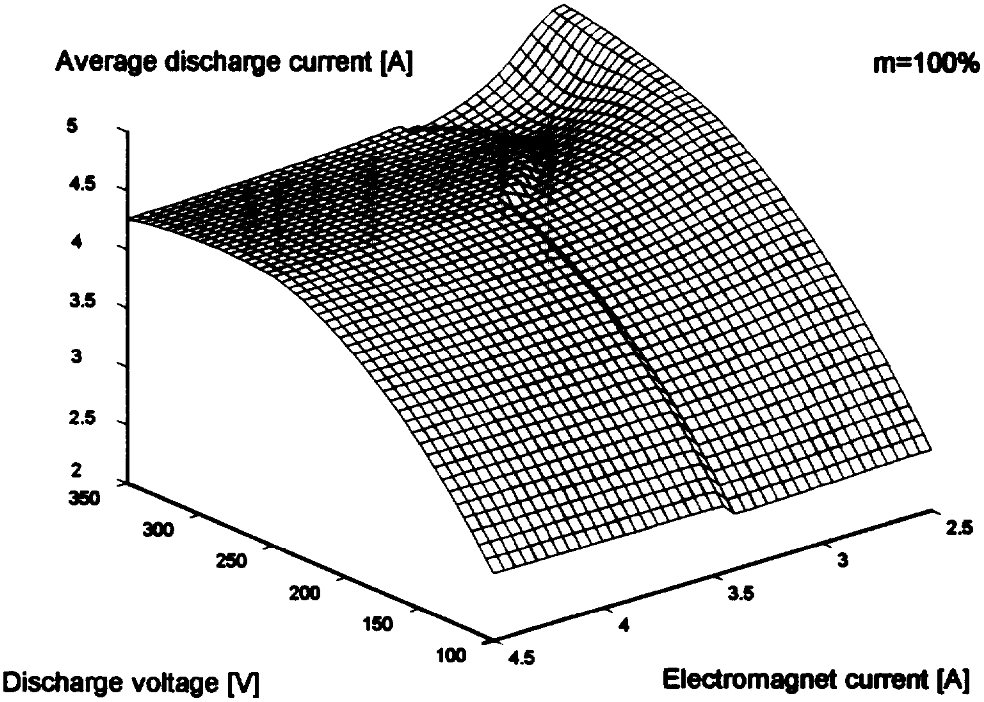
Figure 2. Spectrums of the discharge current oscillations.

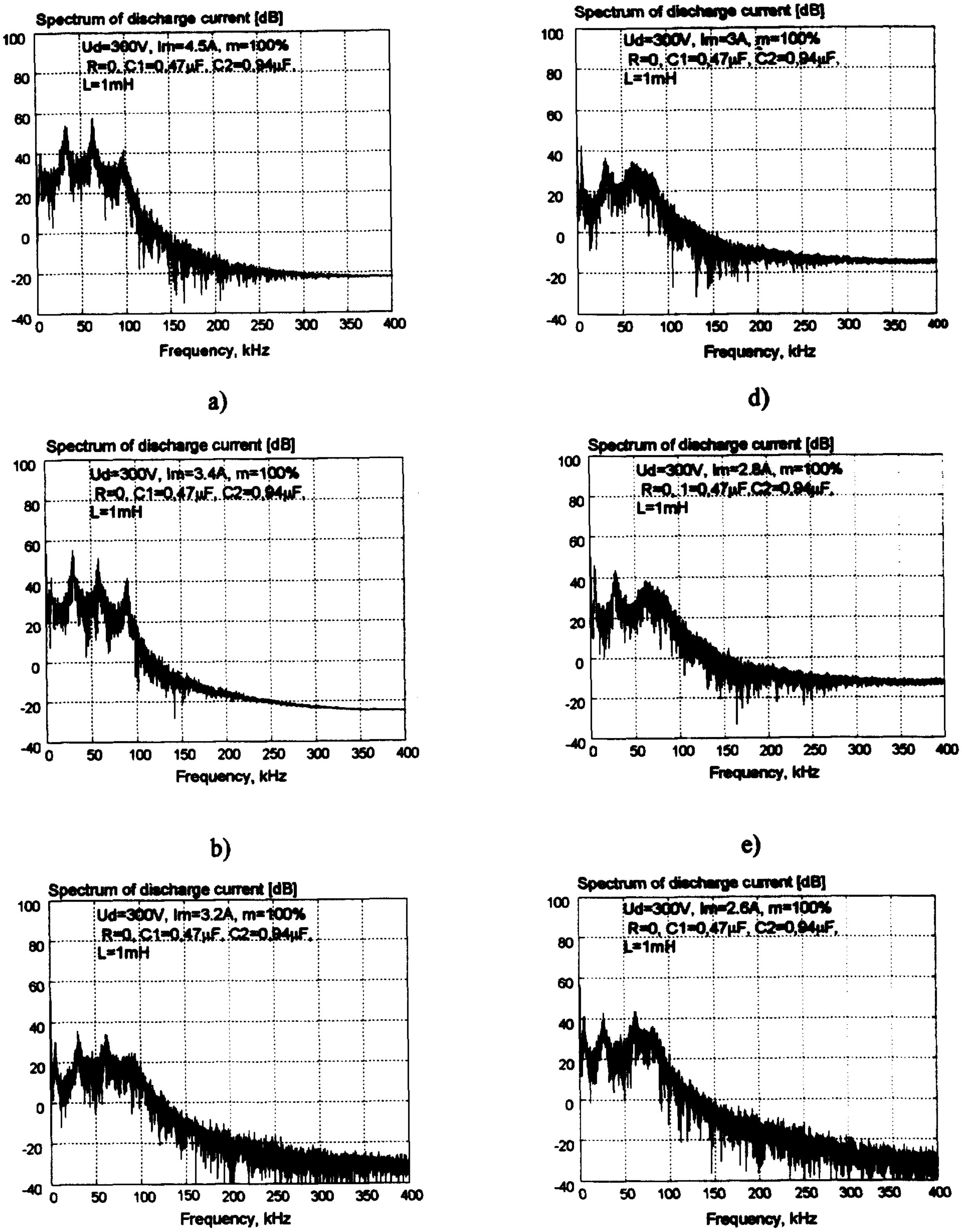


a)



b)

Figure 3. Results of the modeling amplitude of oscillations (a) and average value (b) of the SPT-100 discharge current



a) d) b) e) c) f)

Figure 4. Spectrums of the discharge current oscillations.