APPLICATION OF ADAPTIVE NUMERICAL SCHEMES FOR MPD THRUSTER SIMULATION

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Abstract
A numerical simulation program has been developed to investigate coaxial self-field magnetoplasmadynamic thrusters of arbitrary geometry including thermal and electrical behavior of the electrodes. An explicit Finite Volume scheme and Runge-Kutta time-stepping are used to solve the mass, momentum and energy conservation equations in the flow field for an Argon plasma in ionization equilibrium. The electron temperature and the magnetic field are determined by solving the stationary electron energy equation and Maxwell’s equations together with Ohm’s law. In the electrodes, the current density and temperature distribution are obtained by solving Maxwell’s equations and the heat conduction equation considering Ohmic heating. The elliptical equations are solved using the Finite Element Method. Planar unstructured triangular meshes are used for each computational domain assuming that all variables are constant in azimuthal direction and appropriate boundary conditions are introduced at the interfaces. Adaptive mesh refinement has been implemented to enhance the spatial resolution depending on relative gradients of the plasma parameters. Numerical results are discussed for the cylindrical ZT3 thruster and the nozzle type hot anode thruster HAT which have been developed and operated at IRS.

Nomenclature

- Richardson constant
- Magnetic field
- Specific heat at constant volume
- Electron charge
- Electric field
- Mesh node distance
- Planck’s constant
- Electric current density
- Boltzmann’s constant
- Equilibrium constant
- Particle mass
- Number density
- Pressure
- Heat flux vector
- Collision cross section
- Temperature
- Cathode fall voltage
- Internal partition function
- Mean flow velocity
- Thermal relaxation coefficient
- Permittivity of free space
- Thermal conductivity
- Viscosity
- Permeability of free space
- Collision frequency
- Mass density
- Electrical conductivity
- Viscous stress tensor
- Work function
- Ionization potential
- Stream function
- Control volume

Subscripts
- electron
- electron-electronic (internal)
- heavy particle
- Ar
- Ar
- Ar
- i

1 Introduction
Steady state self-field MPD thrusters can deliver high specific impulse at a high thrust level which will be of major importance for future space missions. To take advantage of magnetic acceleration and to achieve high specific impulse, it is desirable to operate MPD thrusters at
high currents. Limitations due to thermal problems and plasma instabilities have still to be overcome to achieve high efficiency and thrust. Coaxial devices with several nozzle shapes and also cylindrical types without a nozzle have been built and operated in a vacuum chamber to study their performance and to obtain experimental data [1, 2]. In parallel, numerical simulations of these thrusters for several propellants have been conducted to gain a better understanding of the physical mechanisms which limit the performance and lifetime [3].

This paper is devoted to the application of new numerical approaches, i.e. finite volume TVD schemes on unstructured triangular meshes and adaptive mesh refinement for MPD thruster simulation. It has been motivated by the need to resolve strong gradients in the plasma parameters with high accuracy to predict the occurrence of plasma instabilities and to allow the computation of arbitrarily shaped coaxial accelerators without difficulty. The physical model is based on our previous work on MPD thruster simulation assuming axisymmetric flow and thermal non-equilibrium with slight improvements [4]. The electrodes are included in the computation to determine the electrode temperature and current distribution along the cathode.

The numerical results reflect the occurrence of steady shocks in the flow field which are induced by the nozzle geometry depending on the local transport properties of the plasma and are observed in experiments. The parameter ranges for which steady shocks can be observed are predicted by the numerical calculation.

In sections 2 and 3 the underlying equations are described in detail. The mesh generation procedure and numerical aspects are outlined in section 4 followed by a discussion of some numerical results and comparison with experimental data as well as some conclusions in section 5 and 6.

2 Basic Assumptions

Since the devices are rotationally symmetric, cylindrical coordinates are used. The φ dependence of all variables is neglected assuming axisymmetric flow and pure azimuthal magnetic field. There is no azimuthal velocity in MPD thrusters. Argon is considered as propellant which has been used in most of the experimental investigations of MPD thrusters at our institute. It is assumed that the plasma is in ionization equilibrium, i.e. the concentrations of the species are governed by the law of mass action and can be calculated from total mass density ρ and electron temperature $T_e$. This is a good approximation since the reaction rates for ionization reactions are quite high in the case of Argon [5]. A two temperature model with collisional heat transfer between electrons and heavy particles is used to account for thermal nonequilibrium. The electron velocity is

$$\vec{v}_e = \vec{v} - \frac{\vec{J}}{n_e e}$$

with current density $\vec{J}$ and electron particle density $n_e$ neglecting the ion current. In addition quasineutrality is assumed in the bulk plasma. The electrode sheath region is separately treated in connection with the boundary conditions at the electrodes. The flow velocity $\vec{v}$ is considered to be the velocity of the heavy species neglecting ion slip and the electron contribution to the total momentum. Since the pressure correction due to electrostatic micro fields is negligibly small, ideal gas law is used, i.e.

$$p = n_e k T_e + n_h k T_h$$

The stationary Maxwell's equations are used neglecting displacement current since a steady state solution is sought.

The model is valid in the interior of the thruster up to the nozzle exit where the particles can be assumed to be nearly Maxwellian.

3 Governing Equations

The plasma flow is governed by the mass, momentum and energy conservation equations for the species together with Maxwell's equations and expressions for the transport coefficients [6]. Under the abovementioned assumptions, the species mass and momentum conservation equations can be added to yield

$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial}{\partial t} \rho \vec{v} + \nabla \cdot (\rho \vec{v} \otimes \vec{v} + p) = \nabla \cdot \vec{F} + [\vec{J} \times \vec{B}]$$

neglecting products of diffusion velocities. $F$ is the viscosity tensor of the plasma and $p$ the total pressure. Only two momentum equations are obtained after transformation to cylindrical coordinates since $v_\phi = 0$. They have the form

$$\frac{\partial}{\partial t} \rho v_r + \text{div}(\vec{f}_r) = Q_r - j_z B_\phi$$

$$\frac{\partial}{\partial t} \rho v_z + \text{div}(\vec{f}_z) = Q_z + j_z B_\phi$$

with the flux vectors

$$\vec{f}_r = \begin{pmatrix} \rho v_r^2 + \frac{p}{\rho} + \mu (\frac{\partial}{\partial r} v_r - \frac{2}{3} \frac{\partial}{\partial r} v_r) \\ 0 \\ \rho v_r v_z + \mu (\frac{\partial}{\partial r} v_z + \frac{\partial}{\partial r} v_z - \frac{2}{3} \frac{\partial}{\partial r} v_r) \end{pmatrix}$$

$$\vec{f}_z = \begin{pmatrix} \rho v_z v_r + \mu (\frac{\partial}{\partial z} v_r + \frac{\partial}{\partial z} v_z - \frac{2}{3} \frac{\partial}{\partial z} v_r) \\ 0 \\ \rho v_z^2 + p + \mu (\frac{\partial}{\partial z} v_z - \frac{2}{3} \frac{\partial}{\partial z} v_z) \end{pmatrix}$$
and source terms
\[
Q_r = \frac{p}{r} + \frac{\mu v_r}{r^2} + \frac{2}{3} \frac{v_r \frac{\partial \mu}{\partial r} - \frac{\partial \mu}{\partial z} \frac{\partial v_z}{\partial r} + \frac{\partial \mu}{\partial r} \frac{\partial v_z}{\partial z}}{r},
\]
\[
Q_z = \frac{2}{3} \left( \frac{v_r}{r} \frac{\partial \mu}{\partial z} - \frac{\partial \mu}{\partial r} \frac{\partial v_r}{\partial z} + \frac{\partial \mu}{\partial z} \frac{\partial v_r}{\partial r} \right).
\]
\(\mu\) is the coefficient of viscosity which depends on the local plasma state.

Since the total momentum of the electrons can be neglected, the electron momentum equation reduces to
\[
\vec{E} = \vec{\nabla} \times \vec{B} + \frac{1}{n_e e} \left( \vec{j} \times \vec{B} \right) - \frac{1}{n_e e} \nabla p_e
\]
which is Ohm’s law including the Hall term. Under steady state conditions, Maxwell’s equations reduce to
\[
\text{rot} \vec{E} = 0, \quad \text{rot} \vec{B} = \mu_0 \vec{j}.
\]
Combined with Eq. (11) they determine the current density and magnetic field in the plasma for a given flow and plasma state. In cylindrical coordinates we obtain the elliptical differential equation
\[
0 = \frac{\partial^2 B_\phi}{\partial r^2} + \frac{\partial^2 B_\phi}{\partial z^2} - \frac{\partial B_\phi}{\partial r} \left( \frac{1}{r} + \frac{1}{2} \frac{\partial \sigma}{\partial r} + \mu_0 \sigma v_r - \sigma B_\phi \frac{\partial \beta}{\partial z} \right) - \frac{\partial B_\phi}{\partial z} \left( \frac{1}{\sigma} \frac{\partial \sigma}{\partial r} + \mu_0 \sigma v_z + \sigma B_\phi \frac{\partial \beta}{\partial r} - \frac{2 \sigma B_\phi \partial \beta}{\partial z} \right) - D_\phi \left( \mu_0 \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{1}{r^2} + \frac{1}{\sigma} \frac{\partial \sigma}{\partial r} - \frac{B_\phi \partial \beta}{\partial z} \right)
\]
with the Hall factor \(\beta = \frac{1}{n_e e}\). The current density is given by Eq. (12) as
\[
j = \frac{1}{\mu_0 r} \begin{pmatrix}
-\frac{\partial \Psi}{\partial z} \\
0 \\
\frac{\partial \Psi}{\partial r}
\end{pmatrix}
\]
with the stream function \(\Psi = r B_\phi\).

Designating with \(e_h\) the sum of translational and kinetic energy of the heavy particles
\[
e_h = \frac{3}{2} n_h k T_h + \frac{1}{2} \rho v^2,
\]
the heavy particle energy equation becomes
\[
\frac{\partial}{\partial t} e_h + \nabla \cdot \left( (e_h + p) \vec{v} \right) = p_e \nabla \cdot \vec{v} + \nabla \cdot \left( \lambda_h \nabla T_h \right)
\]
\[
+ \nabla \cdot \left( \vec{j} \cdot \vec{r} \right) + \nabla \cdot \left( \vec{j} \times \vec{B} \right) + \sum_{i=0}^{N} n_{e_i} n_i \alpha_{ei} (T_e - T_h),
\]
with the electron pressure \(p_e\), the thermal relaxation factor \(\alpha_{ei}\) and the particle densities \(n_i\) of the species \(Ar^{+}\).

If we write \(\vec{a} = \vec{v} \cdot \vec{r}\) in cylindrical coordinates, we get
\[
a_r = \frac{4 v_r}{3} \frac{\partial v_r}{\partial r} + \frac{v_z}{3} \frac{\partial v_z}{\partial z} - \frac{2 v_r}{3} \frac{\partial v_z}{\partial z} - \frac{2 v_r^2}{r},
\]
\[
a_z = \frac{4 v_r}{3} \frac{\partial v_z}{\partial r} + \frac{v_z}{3} \frac{\partial v_z}{\partial z} - \frac{2 v_r}{3} \frac{\partial v_r}{\partial r} - \frac{2 v_r v_z}{r}.
\]

The internal energy \(e_{int}(\rho, T_e)\) which comprises the ionization and excitation energy of the species in ionization equilibrium is added to the electron thermal energy \(e_e = \frac{1}{2} n_e k T_e\). The sum is designated with \(e_t\) and obeys the electron-electronic energy equation in stationary form
\[
\nabla \cdot \left( (e_t + p) \vec{v} \right) + \frac{5}{2} \frac{k}{e} \nabla T_e + \frac{1}{n_e e} \frac{1}{\sigma} \nabla p_e = \sum_{i=0}^{N} n_e n_i \alpha_{ei} (T_e - T_h) + \nabla \cdot \left( \lambda_e \nabla T_e \right) + \frac{\dot{j}^2}{\sigma},
\]
\(\lambda_e\) is the sum of electron and reaction thermal conductivity and \(\sigma\) the electrical conductivity. If both energy equations are added, the total energy conservation equation of the plasma
\[
\frac{\partial}{\partial t} e + \nabla \cdot ((e + p) \vec{v}) = \nabla \cdot (\lambda_h \nabla T_h + \lambda_e \nabla T_e)
\]
\[
+ \nabla \cdot (\vec{j} \cdot \vec{r} + \vec{j} \cdot \vec{E}) + \nabla \cdot \left( \frac{5}{2} \frac{k}{e} T_e \vec{j} \right)
\]
is obtained. The total electrical power which is transferred per cubic meter to the plasma
\[
\vec{j} \cdot \vec{E} = \vec{v} \cdot \left( \vec{j} \times \vec{B} \right) + \frac{\dot{j}^2}{\sigma} - \frac{1}{n_e e} \frac{1}{\sigma} \nabla p_e
\]
results from Eq. (11) by multiplication with \(\vec{j}\).

The whole system is therefore reduced under the simplifying assumptions to six equations with six unknowns, namely \(\rho, v_r, v_z, T_h, T_e, B_\phi\) which has to be solved simultaneously.

In the cathode, the temperature \(T_c\) and the current density \(\dot{j}\) are determined by solving
\[
\lambda_c \left( \frac{\partial^2 T_c}{\partial r^2} + \frac{\partial^2 T_c}{\partial z^2} \right) + \frac{\partial T_c}{\partial r} \left( \frac{\lambda_c}{r} + \frac{\partial \lambda_c}{\partial r} \right) + \frac{\partial T_c}{\partial z} \frac{\partial \lambda_c}{\partial z} = -\frac{\dot{j}^2}{\sigma},
\]
\[
\sigma \left( \frac{\partial^2 \Psi}{\partial r^2} + \frac{\partial^2 \Psi}{\partial z^2} \right) - \frac{\partial \Psi}{\partial r} \left( \frac{\sigma_c}{r} + \frac{\partial \sigma_c}{\partial r} \right) - \frac{\partial \Psi}{\partial z} \frac{\partial \sigma_c}{\partial z} = 0
\]
with the thermal conductivity \( \lambda_e(T_e) \) and the electrical conductivity \( \sigma_e(T_e) \) for which the approximations given in [7] have been used.

### 3.1 Ionization Equilibrium

The reaction rates for ionization reactions must take into account all possible reactions from ground states and all possible excited states of the Argon atoms and ions under quasi steady state conditions [8]. This leads to rates which are several orders of magnitude higher than those considering only ionization from ground state. Therefore, equilibrium ionization may be assumed at least in the discharge region. The electron particle density is obtained by finding the single positive root of

\[
n_{e}^{N+1} + \sum_{\nu=1}^{N} n_{e}^{N-\nu}(n_e - \nu n_e) \prod_{\mu=1}^{\nu} K_{\mu} = 0 \tag{24}
\]

with the equilibrium constant

\[
K_{\mu} = \frac{n_{i-1} n_{e}}{n_{i+1}} = \frac{z_{i-1}}{z_{i+1}} \left( \frac{2\pi m_e k T_e} {\hbar^3} \right)^{3/2} \exp \left\{ \frac{e\chi_{i}}{k T_e} \right\}. \tag{25}
\]

\( \chi_{i} \) is the ionization potential and \( z_{i} \) is the electronic partition function of \( \text{Ar}^{+} \). \( N \) is the number of ionization stages considered. The particle densities are then given by

\[
n_{0} = \frac{n_{h} n_{e}^{N}} {n_{e}^{N} + \sum_{\nu=1}^{N} n_{e}^{N-\nu} \prod_{\mu=1}^{\nu} K_{\mu}}, \tag{26}
\]

\[
n_{i} = \frac{K_{i}} {n_{e} n_{i-1}}. \tag{27}
\]

### 3.2 Transport Coefficients

Based on the collision frequencies

\[
\nu_{ee} = \sqrt{2n_{e} Q_{ee}} \sqrt{\frac{8k T_e} {\pi m_{e}}}, \tag{28}
\]

\[
\nu_{ei} = n_{i} Q_{ei} \sqrt{\frac{8k T_e} {\pi m_{e}}}, \tag{29}
\]

\[
\nu_{ie} = n_{e} Q_{ei} \sqrt{\frac{8k T_e} {\pi m_{h}}} 2\sqrt{\frac{m_{e}} {m_{h}}}, \tag{30}
\]

\[
\nu_{ij} = \sqrt{2n_{j} Q_{ij}} \sqrt{\frac{8k T_h} {\pi m_{h}}}, \tag{31}
\]

the transport coefficients are computed as

\[
\alpha_{ei} = 3 \nu_{ei} \frac{n_{e} k} {m_{e} n_{i}}, \tag{33}
\]

\[
\lambda_{e} = 1.45 \cdot \frac{15}{4} \frac{n_{e} k^2 T_e} {m_{e}(\nu_{ee} + \sum_{i=0}^{N} \nu_{ei})}, \tag{34}
\]

\[
\mu = 1.2 \cdot \frac{3k T_h} {2} \sum_{i=0}^{N} \frac{n_{i}} {\nu_{ie} + \sum_{j=0}^{N} \nu_{ij}}, \tag{35}
\]

\[
\lambda_{h} = 0.8 \cdot \frac{15 k T_e} {4 m_{h}}, \tag{36}
\]

\[
\lambda_{\text{react.}} = \frac{5}{2} c_{v} \mu. \tag{37}
\]

In the reaction heat conductivity, \( c_{v} \) is the specific heat at constant volume with respect to \( T_e \) which is obtained by numerical derivation of \( e_{\text{int.}}(\rho, T_e) \). Constant factors have been introduced in Eq. (32) to (37) to adapt this simple theory to more exact calculations [9, 10].

### 3.3 Boundary Conditions

The hyperbolic character of the fluid dynamic equations demands three characteristic variables to be prescribed at subsonic inlet boundaries and one at subsonic outflow boundaries. If the flow is supersonic at the outflow boundary, no information is needed at all. However, a full set of boundary values has to be prescribed due to the parabolic character of the viscous part.

At the inflow boundary, variables are controlled in such a way that a defined total mass flow \( \bar{n} \) at a defined temperature results. The heavy particle temperature is given at all boundaries except the outflow boundary, where \( \bar{n} \cdot \nabla T_h = 0 \) is assumed. For the electron temperature \( \bar{n} \cdot \nabla T_e = 0 \) is assumed at all boundaries. The velocity vector is set to zero at the solid walls and its normal derivative is set to zero at the outflow boundary. The pressure is given at the outflow boundary and controlled at the inlet boundary to obtain a defined total mass flow. The value of \( rB_{0} \) is constant along all boundaries but the electrodes.

#### Electrodes

Since the sheath potential may vary along the cathode, the cathode boundary is not an equipotential. Instead, the current is distributed in a different way depending on the thermionic emission properties of the cathode.

Due to the sheath potential the current of electrons reaching the cathode from the sheath edge is very small in general. The local current density consists therefore of the ion current density \( j_{i} \) and the electron current density \( j_{e} \) caused by thermionic emission. The ion current...
is approximately equal to the ion saturation current, i.e.
\[ j_i = \frac{1}{2} e n_e \sqrt{\frac{k T_e}{m_h}}. \]  (38)

The ions which reach the cathode surface recombine and the released ionization energy minus the work function heats the cathode. The electron current density due to thermionic emission is given by the Richardson formula
\[ j_e = A_R T_e^2 \exp\left(\frac{-e \Phi}{k T_e}\right), \]  (39)

with the work function \( e \Phi = 2.6 \text{eV} \), the cathode temperature \( T_e \) and the Richardson constant \( A_R = 10^4 A/m^2K^2 \) \[11\]. It is several times larger than the ion current under the considered working conditions.

Therefore, the normal component of the current density on the cathode surface is known for a given cathode temperature distribution which yields the boundary condition for \( B_\perp \). The global cathode fall voltage \( U_c \) can be calculated from
\[ U_c I = \int_{\text{cathode}} \frac{5 k}{2} e^{-j} T_e + j_i (x_i + U_c) \, dx \]  (40)

assuming that the fall region input power heats the electrons up to \( T_e \) and is consumed by ionization to maintain the ion current. Besides the heat flux from recombining ions, the cathode is cooled by thermionic emission and by radiation according to Stefan Boltzmann's law yielding the boundary condition for the heat conduction equation of the cathode.

At the anode, the boundary is considered to be an equipotential, i.e. \( E_{\|} = 0 \) on the surface.

### 4 Numerical Solution

#### 4.1 Mesh Generation

For each computational domain the boundary is described by a set of boundary segments. Each segment has a physical type which determines the boundary conditions and may have an arbitrary shape \( r = (r(s), z(s)), s \in [0, 1] \). The mesh generation process is controlled by a function \( g(r, z) \) prescribing the distance of neighboring nodes at each position and proceeds in three steps.

1. Generation of boundary nodes according to \( g(r, z) \)
2. Generation of inner nodes according to \( g(r, z) \)
3. Optimization by means of changing edges and moving nodes

Boundary nodes are generated and moved along their boundary segment retaining the exact boundary shape. At common boundaries, the node positions are copied from the neighboring mesh to simplify the exchange of boundary values. For the first mesh, \( g \) is defined as a linear function along the boundary segments and is extrapolated in the computational domain.

To enhance the spatial resolution in the vicinity of strong gradients, the mesh generation process has been controlled in the MPD thruster calculations using
\[ g(r, z) = \frac{C}{|\nabla T_h|} \]  (41)

based on a coarse mesh solution. The resulting flow field mesh for the Hot Anode Thruster computation is shown in Fig. 1.

**Figure 1:** Refined computational mesh for Hot Anode Thruster computation (7819 nodes)

#### 4.2 Fluid Dynamic Equations

As can be seen from Eq. (3), (4) and (20) all terms but the electromagnetic source terms are in flux formulation. This suggests to solve the fluid dynamic equations using a Finite Volume discretization which guarantees conservation of mass, momentum and energy during the iteration process. Although source terms due to conversion to cylindrical coordinates and splitting of the energy equation in a heavy particle and electron-electronic part lead to small conservation errors, the hyperbolic character of the whole system is retained.

Dual cell control volumes \( \Omega_i \) are used containing node \( i \) and connecting the centers of mass of the neighboring triangles around node \( i \) (see Fig. 2). The conserved quantities are attached to the mesh nodes as well as \( T_e \) and \( B_\perp \) and are considered to be mean values of the quantities in \( \Omega_i \). An explicit upwind scheme with Runge-Kutta time-stepping has been implemented based on the Osher
approximate Riemann solver considering the total pressure $p = p_h + p_c$. Linear reconstruction of the primitive variables is used and the gradients are limited as proposed by Barth [12] to obtain a total variation diminishing scheme of second order accuracy. The viscous fluxes are computed on the FEM mesh using linear basis functions. The global time-step is restricted through the CFL criterion and the local values of viscosity, heat conductivity and specific heat.

4.3 Elliptical Equations

The Finite Element Method has been used for the discretization of the elliptical differential equations. Basis functions $N_i(r, z)$ are defined in the nodes of the triangular mesh which are piecewise linear in the triangles containing node $i$ and zero elsewhere. Galerkin's Method is used to obtain a system of linear equations from the differential equation using Gaussian quadrature. For this purpose, the control volume mean values of the conserved quantities in $\Omega_i$ enter the FEM discretization as node values $U_i$. The linear system is solved iteratively by simultaneous over-relaxation coupled with the fluid dynamic equations. Since the reciprocal behavior of $B_\varphi$ near the cathode root is poorly approximated by linear functions, the variable $\Psi = r B_\varphi$ is used instead. The differential equation is reformulated in $\Psi$ and is solved in regions far from the centerline instead of Eq. (13). Additionally for the computation of $j^2/\sigma$ and $\mathbf{J} \times \mathbf{B}$, switching between linear approximation of $B_\varphi$ or $\Psi$ between the nodes is used to enhance the accuracy.

4.4 Implementation

An integrated tool has been developed for the simulation of MPD thrusters including mesh generation, FEM and FVM solver and visualization. The whole system has been implemented in C++ using separate classes for each component. Since the equations are strongly coupled by means of source terms and transport coefficients, they are solved iteratively starting from initial conditions. At each time step, the transport coefficients are updated and the elliptical PDE's (electron energy equation, discharge equation and electrodes) are iterated. The computation is continued until the residuum is decreased by at least four order of magnitudes.

5 Results

Numerical calculations have been carried out for several thrusters developed at IRS under a range of working conditions. Simulation input parameters are the total current $I$, the inlet mass flow $\dot{m}$ and the geometry of the device. The pressure behind the outflow boundary has been set to 0.1 Pa in all calculations to avoid recirculation which leads to numerical problems. The temperature of the inflowing gas, cathode root and neutral walls (water-cooled) have been set to 300 K and 500 K respectively. For a fully converged solution about $10^8$ timesteps are needed depending on initial conditions and mesh resolution.

In general, the predicted total voltages are in the correct range with deviations up to 15% except for the cylindrical thruster at low currents. The comparison of thrust yields deviations up to 30% again with exception of the cylindrical device. This may be explained by uncertainties in the thrust measurements and the fact that some computations have not yet reached full convergence in the sense that the mass in and outflow are exactly equal which leads to errors in the calculation of total thrust. In the following sections, two cases are discussed in detail.

5.1 Cylindrical Thruster

The ZT3 thruster has been operated at currents up to 15\,000\,A and at mass flow rates of 1 to 3 g/s [2, 13]. In Fig. 3 to 6 results are shown for $I = 10\,000\,A$ and $\dot{m} = 2 g/s$. The cathode has been shifted back by 2.5 cm to reduce the heat load on the first anode segment. The calculated total voltage agrees well with the measured value of 20 V. However, the measured thrust of 7 N is much lower than the calculated thrust (12.1 N) and even lower than the magnetic thrust alone.

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Figure 2: Triangular mesh and dual mesh control volumes

Figure 3: Current lines (500 A between lines)
In Fig. 3 the current streamlines are shown. Compared with solutions obtained assuming equipotential electrode boundary condition, the current is shifted somewhat upstream which is still enforced by the Hall effect leading to a reduced magnetic thrust. As can be seen in Fig. 4 the cathode temperature is nearly constant at the tip although the maximum ohmic heating power density in the cathode is reached 2.5 cm upstream on the surface. This is due to the large heat flux from the plasma in the tip region. Comparing the heavy particle and electron temperature in Fig. 5 and Fig. 6 it is obvious that thermal equilibrium is only reached near the cathode tip. The electron temperature is strongly coupled to the degree of ionization of the plasma due to the equilibrium assumption and decreases slowly in the expansion region compared with the heavy particle temperature.

A complicated flow structure results from the combination of magnetic forces, ohmic heating and expansion above the cathode tip as shown in Fig. 7 leading to a maximum flow velocity slightly apart from the axis.

5.2 Hot Anode Thruster

The HAT thruster has recently been operated with up to 5000 A total current in a vacuum chamber and emission spectroscopic measurements have been carried out. Numerical results are shown in Fig. 8 to 9 for I=2000 A and m=0.8 g/s. A steady shock appears in the solution which is induced by the nozzle shape of the device reducing the thrust considerably. The shock is clearly seen in the experiments, however it reaches the axis somewhat too far downstream in the calculation. Further, a contact discontinuity is observed in the calculation separating the low density region in the upper part of the computational domain from the plume.

The computed voltage of 41.2 V and thrust of 4.9 N are in the correct range, the first measured values were 40.2 V and 3.7 N respectively.
the anode leading to low particle densities. This is due to the deceleration of the flow near the wall.

6 Conclusions

Numerical concepts such as unstructured mesh TVD schemes and mesh adaption have been applied to investigate the detailed flow structure and acceleration mechanisms of MPD thrusters. Arbitrarily shaped devices can be simulated with the developed program enabling the study of geometrical influence on thruster performance.

Calculations have been performed for a number of MPD devices developed at IRS in Stuttgart. Parameter ranges for which steady shocks can be observed are well predicted by the computations. The computed thrust and power consumption are in the correct range compared with the experimental data. However, deviations up to 70% have been obtained for the cylindrical MPD thruster. This may be due to problems with the thrust balance or may be an indication that viscosity is higher than calculated by our model. Further comparisons of the numerical results with spectroscopic measurements are presented in [13].

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