Trajectory Analysis of Electric Propulsion System with Thrust Misalignment

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Abstract

Trajectory analysis with thrust misalignment has been conducted using the interplanetary trajectory optimization code developed for electric propulsion system study. Electric propulsion missions such as interplanetary exploration mission and Earth-orbit application require low-thrust trajectory optimization in order to fully exert electric propulsion performance, however, low-thrust trajectory optimization is so difficult and time-consuming that parametric study of electric propulsion system is impossible on an usual computation environment. Therefore trajectory analyses of electric propulsion system so far tend to be simplified and deal with ideal conditions. The study in this paper is based on the work conducted by the authors to develop an efficient mission analysis code for electric propulsion researchers, where the code uses two numerical optimization techniques combined. The DCNLP, which is robust in convergence, however, requires huge computation memory and time. The SCGRA, which is superior in computation cost, however, tends to lose convergence when a poor initial estimate is given. In the combined method, the DCNLP is used to calculate a rough estimate and the SCGRA follows to obtain a final estimate with more accuracy. In the first half of the paper the combined method is used to solve interplanetary trajectory optimization problem for an Earth-Nereus transfer in order to show its effectiveness, and in the latter part of the paper the thrust misalignment problem is formulated and the calculation example of the Earth-Nereus case is shown.

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Nomenclature

c_3 = characteristic energy
F = thrust
f = thrust direction vector, $|f| = 1$
g = gravitational acceleration
I = functional
Isp = specific impulse
m = vehicle mass
m_{pl} = payload mass
P = power
r = radius
t = time
T_f = flight time
x = position vector
v = velocity vector
α = specific mass
ΔT = thruster operating time
ϵ = tankage fraction
η = propulsive efficiency
θ = thrust misalignment angle, defined as the angle from the optimal control direction
μ = gravitational constant

Subscripts

a = arrival
d = departure
limit = upper limit
plan = planned trajectory

Introduction

Electric propulsion is expected to offer substantial benefits for planetary exploration missions and Earth orbit applications by reduced spacecraft mass and/or increased payload capacity. As the interest in electric propulsion has been focused on applications to space activities, several electric thrusters have been developed and used for attitude control and north-south station keeping of satellites. Although the number of electric propulsion missions is still limited today, those missions validated electric propulsion system and demonstrated its advantages over chemical propulsion system. Electric propulsion system up to now has been used for auxiliary propulsion functions, however, due to the development of both high-power solar and nuclear space power systems, interest in using electric propulsion system for primary propulsion functions is increasing. With the growing interest, mission analysis and optimization of an electric propulsion system is becoming more important than ever in order to fully utilize electric propulsion performance and to help design electric thrusters more appropriate for use.

Since the performance of an electric propulsion system is greatly influenced by the choice of the trajectory that a vehicle follows, optimization of a low-thrust trajectory is required. Low-thrust trajectory optimization study ranges from analytic solutions[1,2] which obtained for near-Earth missions using the infinitesimally small thrust assumption, to numerical ones for interplanetary missions. In interplanetary missions where electric propulsion exerts superior performance, however, thrust produced by electric thrusters is not negligible to the sun's gravitational force, this assumption is not properly used. Instead, difficult and time-consuming numerical low-thrust trajectory optimization is required. In addition, electric thruster and power plant models should be incorporated to the low thrust trajectory optimization. Although there are many electric propulsion mission studies, their electric thruster models are too simplified to provide useful information for electric propulsion researchers. Hence, the following low-thrust trajectory optimization code is necessary: 1) being robust and reliable, 2) running in shorter time with less amount of memory usage, 3) incorporating realistic electric propulsion models.

Following these circumstances, the University of Tokyo has started to develop a mission analysis code which can be widely used in the area of electric propulsion research. The study so far successfully developed the trajectory optimization code which can deal with interplanetary missions incorporating electric thruster characteristics using the "combined method.[3,4,5] However, these analyses assumed ideal conditions and no perturbation around the trajectory was included. This study is an enhancement to the code by including thrust misalignment estimation.

There may be several causes of thrust misalignment such as thrust misalignment against the center of gravity of the vehicle or navigation error. This thrust misalignment deviates the trajectory from the planned one and the arrival position and velocity change, which must be corrected by chemical thrusters since not enough time is left for electric propulsion to accelerate the vehicle and recover its error. Therefore the estimation of the error is necessary in doing electric propulsion system analysis and planning electric propulsion mission. This study introduces the calculation technique estimating such errors.

The paper consists of two parts. In the first half of the paper, the low thrust trajectory optimization problem for electric propulsion system is formulated and the practical way to solve optimization problems by the "combined method" is shown. In the latter half of the paper, the velocity correction estimation problem to the thrust misalignment is formulated and the example of its analysis is shown.

Trajectory Optimization

Trajectory and vehicle mass model

The trajectory and vehicle mass model given here is for an Earth departure interplanetary transfer. For a practical simplicity, the following assumptions are made; (a) the sun is considered as the only one gravity source, (b) the trajectory of a vehicle is divided into thrusting and coasting phases since the optimized solution is known to have thrusting and coasting phases.

Since the position and velocity of the vehicle in the coasting phase can be calculated analytically, the calculation in the thrusting phase is only necessary. The following equations must be satisfied in the thrusting phase,

$$\ddot{x} = \frac{F}{m} - \frac{\mu}{|x|^3}x \qquad (1)$$

$$\dot{m} = -\frac{|F|}{g\mathrm{Isp}} \qquad (2)$$

In most of the analyses, payload mass is maximized which is evaluated in this model as,

$$m_{pl} = m_0 - \frac{1}{1-\epsilon_e}\sum_i |\dot{m}_i| \Delta T_i - \alpha P - m_{th} \quad (3)$$

where the first term on the right-hand side is vehicle mass at departure, the second the sum of propellant and its structure in a heliocentric flight, the third power plant and propulsion system mass and the last term is thruster mass.

Since longer thruster operating time tends to give larger payload mass, the following constraint on the lifetime of a thruster is necessary for electric propulsion mission given as

$$\sum_i \Delta T_i \le \Delta T_{\text{limit}} \quad (4)$$

Boundary conditions

The boundary conditions at departure are expressed by the following conditions

$$x = x_d(t_d) \quad (5)$$

$$v = v_d(t_d) + \sqrt{c_3} \quad (6)$$

$$m = m_0 \quad (7)$$

where c_3 is a characteristics energy given 0 for spiral departure. For impulsive departure such as using chemical propulsion, the following relationship is used to calculate c_3 as

$$\Delta V = \sqrt{c_3 + \frac{2\mu_d}{r_d}} - \sqrt{\frac{\mu_d}{r_d}} \quad (8)$$

where r_d is the radius at the departure.

Arrival boundary conditions are given for rendezvous condition,

$$x = x_a(t_a) \quad (9)$$

$$v = v_a(t_a) \quad (10)$$

In the case that the vehicle approaches the planet with a relative velocity v_r, Eq.(10) is replaced by

$$v = v_a(t_a) - v_r \quad (11)$$

or

$$|v - v_a(t_a)| \le v_r \quad (12)$$

Other arrival conditions such as soft landing or insertion to the orbit around the planet are given by properly calculating the v_r term on the right hand side of the above equation. The positions and velocities of the departure and arrival planets are determined by solving the Kepler's equation.

One more constraint is applied if a flight time is given as an input parameter.

$$t_a - t_d = T_f \quad (13)$$

Method of Solution

The method of solution is very important in solving optimization problems. Recent advance in optimization methods provides efficient and robust algorithms, such as Sequential Conjugate-Gradient Restoration Algorithm (SCGRA)[6] and Direct Collocation with Nonlinear Programming (DCNLP).[7,8,9,10,11] In each method the trajectory is divided into a lot of segments with a node on each side and the constraints are evaluated on the node. The SCGRA and DCNLP's superior point is, contrary to the classic variation methods, the precise initial estimate of variables is not necessary and a lot of parameters and constraints on them can be easily incorporated. The authors have been using the SCGRA and DCNLP for solving low-thrust trajectory optimization problems and found each method has both merits and demerits in using electric propulsion trajectory optimization.

The SCGRA has the following properties,

- Easy to implement.
- Low computation cost in time and memory usage, therefore, a large number of nodes is easily taken to have a high-accuracy solution.
- Convergence depends on the initial estimate of the solution.

The DCNLP has the following properties,

- Robust convergence characteristic even against a poor initial estimate.
- Equality and inequality conditions on state variables such as Eq. (12) are easily implemented
- Computation time and memory size significantly increase with number of nodes; in a large-scale problem, sparse matrix operation technique is necessary, which makes the implementation of a code difficult.

From these characteristics, the following method was used to solve the problem in a practical basis. Firstly, run the DCNLP code with a small number of time steps to obtain a rough estimate, then, run the SCGRA code using it as a starting value with a large number of nodes reassigned to get a final estimate. (The calculation cost of a DCNLP code depends on how to handle large size matrices and their sparse patterns. The algorithm solving sparse nonlinear programming problem in this study is based on the work in Ref. [9].)

Sample Calculation

The low thrust trajectory optimization on an Earth-Nereus transfer is shown as the example of the effectiveness of the method. The Earth-Nereus mission is proposed by the ISAS doing sample return from the Nereus in the year 2002 to 2004 using ion thruster system.[13]

The calculation has been done on the first half of the flight with parabolic velocity departure from the Earth and rendezvous to the Nereus. Assumed calculation condition was as follows, vehicle mass at departure 365 kg with characteristic energy $c_3 = 17.7\,\mathrm{km}^2/\mathrm{s}^2$. Ion thruster performance was set Isp = 3120 s, $P = 0.25$ kW and $\eta = 0.5$. Flight time was fixed to 604 days and departure date was selected in January 2002. In the calculation, Eqs. (1) to (13) are evaluated at discrete points, i.e., the trajectory is divided into a lot of segments having nodes on each side. The number of nodes selected in this calculation was 16 in the DCNLP and 896 in the SCGRA.

Figure 1 shows the optimized trajectories yielded by each method (calculation proceeded from Fig. 1 (a) by the DCNLP to Fig. 1 (b) by the SCGRA.) Arrows were written to show the thrust direction on each node. Figure 1 (a) is a calculation result by the DCNLP. As seen in the figure, the trajectory is composed of only 16 nodes which is low in its accuracy. Therefore the SCGRA was employed to improve the accuracy of the solution by using it as the starting value of the SCGRA calculation. The trajectory in Fig. 1 (b) is a final estimate obtained by the SCGRA with a large number of time steps assigned. The vehicle mass at arrival was calculated 352 kg and thruster operating time 562.3 days. The poor accuracy of the solution in the DCNLP is improved by the SCGRA phase such as seen in the thrust direction of the orbit and the coasting position.

The convergence characteristics of the combined method is in Fig. 2. The line shows a calculation time history for the combined method in terms of the payload mass normalized by its final value. The DCNLP started calculation from a starting trajectory that simply connected departure and arrival points (only Eqs. (5) and (9) were satisfied.) The half of the calculation time in the DCNLP (denoted by NLP*) was consumed for finding the trajectory which satisfied all constraints and then the DCNLP proceeded to yield the trajectory which satisfied all the constraints and maximized payload mass. After the DCNLP's solution reached in a steady state, the SCGRA followed to calculate an improved accuracy solution.

As seen in Fig.2 the calculation time of the DCNLP with 16 nodes was comparable to the one in the SCGRA with 896 nodes. Computational characteristics suggest that calculation cost increases linearly in the SCGRA and increases quadratically or more dramatically in the DCNLP. Therefore the idea only to use the DCNLP with many nodes does not work well in the trajectory calculation considered here since the penalty in computation cost is quite large compared to the benefit of the improved accuracy of the solution. Neither the idea only using the SCGRA does not work at all since the starting value required for the SCGRA convergence is very hard to guess. Thus the combined method using the DCNLP as the initial value solver for the SCGRA is very effective.

Thrust Misalignment

Thrust misalignment of a vehicle slightly deviates the trajectory of the vehicle from the planned one, i.e., the arrival position and velocity change. Since not enough time is left for electric propulsion to recover its error it must be corrected by chemical thrusters. Thus the error estimation of the trajectory is important in analyzing mission performance and the development of its calculation technique is necessary.

In the calculation procedure, the upper limit of the thrust misalignment angle θ is given as the input parameter and within this angle a small perturbation $\delta f(t)$ is added to the thrust direction of the planned orbit. The following functional which is defined as the difference of the velocity from the one of the planned orbit is introduced

$$I(t) = |v(t) - v_{\text{plan}}(t)| \tag{14}$$

and its value at the arrival point is maximized to estimate the error.

Then the problem is stated as follows; find the control history $f(t)$ that maximizes the functional $I(t_a)$ with the following conditions

$$f(t) = f_{\text{plan}}(t) + \delta f(t) \tag{15}$$

where

$$f(t) \cdot f_{\text{plan}}(t) \leq \cos\theta \tag{16}$$

Method of Solution

The SCGRA was used for the optimization method since the solution was slightly deviated from the planned trajectory and no possibility that the code fails convergence.

Sample Calculation

The Earth-Nereus transfer mission was taken for a sample calculation. The calculated solution in the previous section for the Earth-Nereus transfer was used as the planned trajectory in solving the thrust misalignment problem. The calculation condition was same to the ones in the sample calculation of the previous section.

Shown in Fig. 3 is a required ΔV correction as a function of thrust misalignment angle from the optimal control direction, where required velocity correction linearly increases. The Earth-Nereus mission plan[13] estimates the required mid-course ΔV correction is 50 m/s, therefore, thrust misalignment angle larger than 0.5° has the possibility to affect the mission performance.

In Fig. 4 the trajectories which give largest velocity correction for the 0.5° thrust misalignment angle case are shown, where the dotted arrow points the optimal thrust direction at each time of the orbit and the solid one points the direction that gives largest velocity correction at arrival point (thrust misalignment angle is 10 times magnified.)

Summary

This paper introduced the low-thrust trajectory optimization code developed for electric propulsion research and its enhancement to the thrust misalignment problem.

The code uses the combined method that takes advantage of the robust convergence of the DCNLP and the low calculation cost of the SCGRA. The code makes trajectory analysis easier and helps electric propulsion system study.

The code can be used for various purposes, such as Earth-Asteroid transfer trajectory optimization and the evaluation of thrust misalignment on a mission performance. In the thrust misalignment calculation, the error caused by it may have a penalty in the mission performance.

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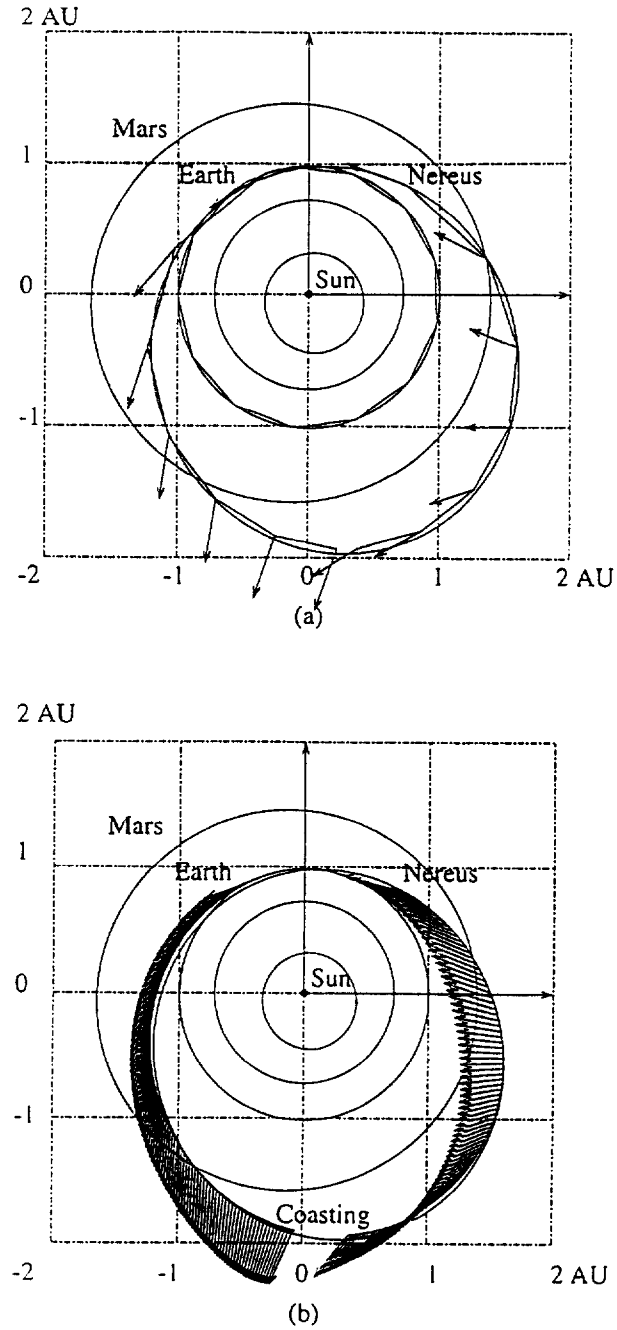


Figure 1: Optimized trajectories for Earth-Nereus transfer; (a) the trajectory after the DCNLP calculation (starting value for the SCGRA calculation), (b) the trajectory after the SCGRA calculation (final estimate)

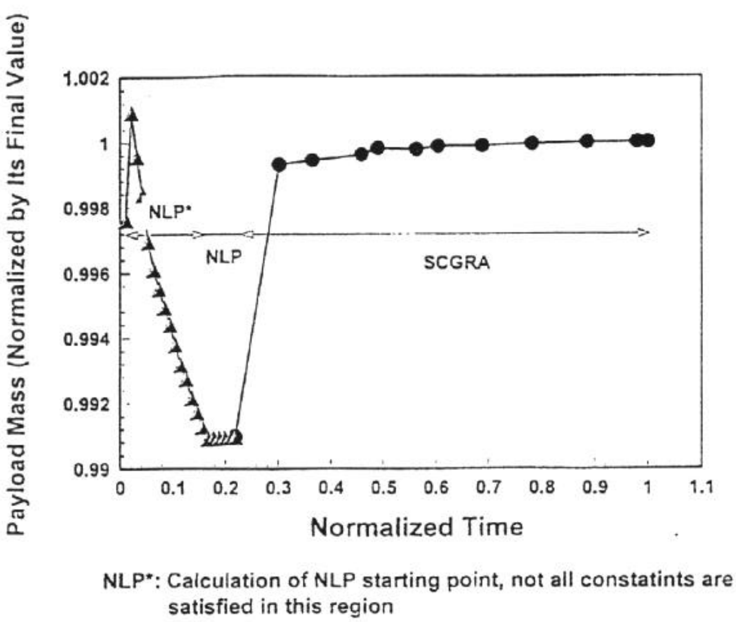
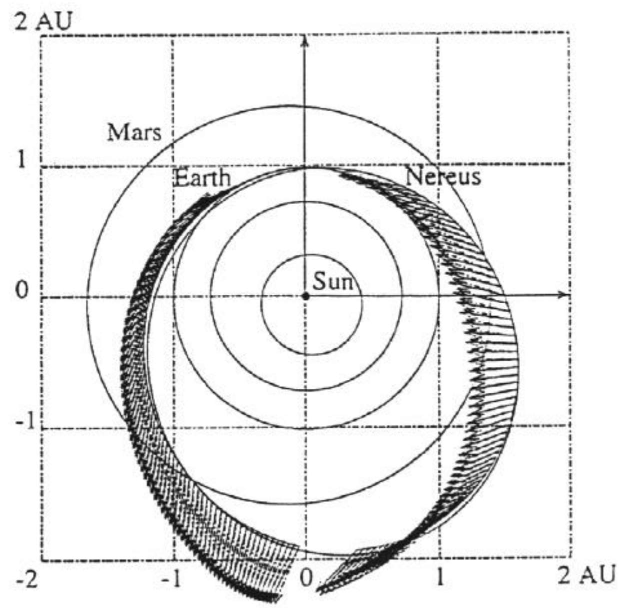


Figure 2: Convergence characteristics of the combined method



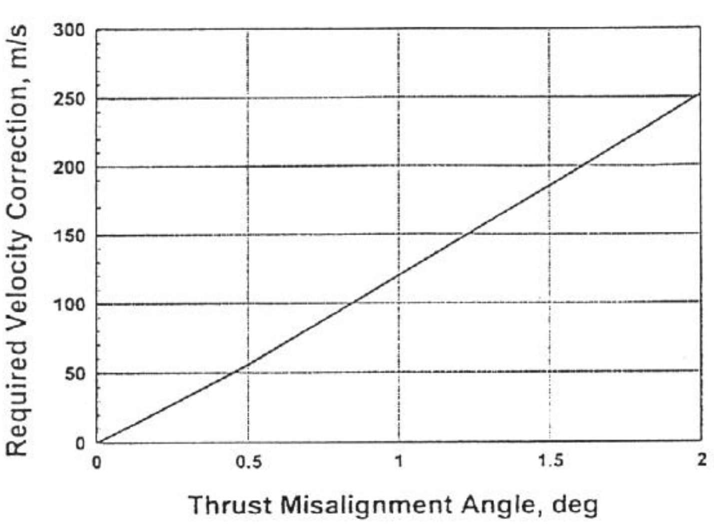


Figure 3: Required ΔV correction as a function of thrust misalignment angle

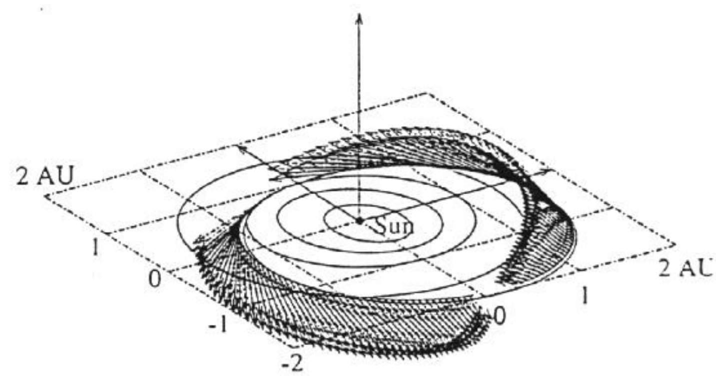


Figure 4: Trajectories for the 0.5° thrust misalignment angle case (thrust misalignment angle is 10 times magnified.)