A Numerical Model of Thruster-Exhaust Plumes

Ikuya Kameyama,* Jeffery M. Monheiser,† and Paul J. Wilbur‡
Colorado State University
Fort Collins, CO 80523

Abstract

A numerical model that can be used to compute the flux of charge-exchange ions generated in ion-thruster exhaust plumes is described. Because the model does not employ a particle-in-cell algorithm, the effects of changes in operating and design parameters can be evaluated rapidly using a personal-computer-based system. Required beam-ion and neutral densities are calculated from thruster geometry and operating conditions rather than being assumed as they have been in previous studies. Even though some crude assumptions and approximations are used in the model so that the particle-in-cell algorithm can be avoided, the computed beam-plasma potentials appear reasonable and the computed total ion density profile for the UK-10 thruster agrees with measurements.

Nomenclature

\begin{align*}
\textbf{N} & \quad \text{Number of grid-aperture sets} \\
\textbf{N}_h & \quad \text{Neutral flux through an accel-grid aperture} \\
\textbf{n}_0 & \quad \text{Neutral density} \\
\textbf{n}_p & \quad \text{Beam-ion density} \\
\textbf{n}_{CE} & \quad \text{CE-ion density} \\
\textbf{n}_e & \quad \text{Electron density} \\
\textbf{n}_{el} & \quad \text{Electron density at a reference point} \\
\textbf{n}_i & \quad \text{Total ion density} \\
\textbf{n}_{i, \text{max}} & \quad \text{Maximum total ion density} \\
\textbf{n}_{i, \text{min}} & \quad \text{Minimum total ion density}
\end{align*}

\begin{align*}
P_0 & \quad \text{Ambient pressure} \\
r & \quad \text{Radial position} \\
T_0 & \quad \text{Neutral temperature} \\
T_e & \quad \text{Electron temperature} \\
T_{e, \text{in}} & \quad \text{Electron temperature inside the beam} \\
T_{e, \text{out}} & \quad \text{Electron temperature outside the beam} \\
V & \quad \text{Beam-plasma potential} \\
V_0 & \quad \text{Beam-plasma potential at the electron-density reference point} \\
V_e & \quad \text{Electron-source potential} \\
V_r & \quad \text{Radial velocity component} \\
v_i & \quad \text{Thermal velocity of neutral atoms} \\
v_c & \quad \text{Axial velocity component} \\
v_\theta & \quad \text{Azimuthal velocity component} \\
x & \quad \text{Distance parallel to the electric-field direction} \\
z & \quad \text{Axial position} \\
\eta & \quad \text{Propellant utilization efficiency} \\
\theta & \quad \text{Angle measured between the outward normal to an accel-grid aperture and the line connecting the aperture and a mesh point}
\end{align*}

Introduction

Ion thrusters are attracting growing interest among satellite-system designers because of their low propellant-mass requirement for long-term missions.\textsuperscript{1,2} In order for these designers to be confident that the ion thrusters will be compatible with other on-board equipment, it is important to know the extent to which ions in thruster-exhaust plumes will interact with various spacecraft.
surfaces causing sputtering damage and/or contamination. Beam ions exit from a thruster at such a high speed that their deflection due to the relatively small electric fields in the thruster-exhaust-plume plasma will generally be negligible and their current densities at spacecraft surfaces can readily be computed. However, low-energy ions created from propellant neutrals or sputtered metallic atoms in the exhaust plume can be directed onto spacecraft surfaces along trajectories that diverge dramatically from the direction of the exhaust beam.

In previous studies, particle-in-cell (PIC) codes have been used to compute the current densities of low-energy ions onto spacecraft surfaces. These studies used simplified beam-current and neutral-density profiles downstream of thruster grids to estimate production rates of low-energy, charge-exchange (CE) ions. Trajectories of these ions are then tracked in electric fields obtained by solving the Poisson equation coupled with the Boltzmann model of electron density. However, each PIC calculation, which is representative of a single thruster operating point and satellite configuration, typically requires a long computational time.

This paper describes a simple numerical model of exhaust-plume/spacecraft interaction phenomena that can be used on personal-computer-based systems to conduct parametric studies and to evaluate design changes rapidly and accurately. As an example, the codes are applied to compute beam-ion and neutral-atom densities, beam-plasma potentials, and CE-ion fluxes downstream of the UK-10 thruster.

Overview

Properties in the plasma environment downstream of ion-thruster grids are calculated through a four-step process that involves evaluation of 1) beam current densities, 2) neutral densities, 3) beam-plasma potentials/densities, and 4) detailed charge-exchange ion trajectories. Each process is carried out sequentially as suggested in Fig. 1 over a 2-D, axi-symmetric region downstream and along the axis of the accel grid of the thruster shown in Fig. 2. The electron source used in the model is assumed to be axi-symmetric, but it can be located outside of the calculation region. The neutralization plane in this figure is assumed to be located a few screen-grid aperture diameters downstream of the thruster grids; its location could be prescribed more accurately using the Kaufman model. Beam-current and neutral-atom densities are computed separately from the geometry and operating parameters of the thruster chamber, and the beam-plasma potential is then obtained in the calculation region. Finally, CE ions produced are tracked to the calculation-region boundary using results obtained from all of the other steps. The coordinates and velocity components at which the CE-ions exit would have to be used as input to a program such as the Engineering WorkBench to determine if and how they would impinge on a prescribed spacecraft in a prescribed space-plasma environment.

Beam-Current-Density Calculation

Optics calculations are performed to obtain beamlet-ion-current-density data for representative aperture sets in a thruster grid set using the 2-D ion-optics program written by Arakawa and Ishihara. As shown in Fig. 3, the program yields solutions for the potential field in the vicinity of the aperture sets along with trajectories of ions extracted from the thruster discharge chamber through a sheath located near the screen aperture. The radial profile of the beamlet current density at the downstream boundary of the region is shown on the right of the figure.
Beamlet orientations with respect to the thruster centerline are determined by considering the effects of both grid dishing and compensation strain. Beamlet deflections due to compensation strain in the accel grid are incorporated using a simple beamlet steering model. The beam current density at any location downstream of the neutralization plane is computed next using data like those in Fig. 3 by summing beamlet contributions from all aperture sets in the grids. Each beamlet-current-density profile is projected downstream using divergence angles obtained from trajectory-angle and current-density data at the neutralization plane. The calculations were repeated twelve times with slightly altered aperture coordinates obtained by rotating and shifting the grids by small amounts and the results were averaged in order to generate a smooth beam-ion-density map.

A beam-ion-density map was computed using this code for the UK-10 thruster with the original T5 grid set (screen-, accel-, and decel-grid aperture diameters of 2.15 mm, 1.75 mm, and 2.15 mm, respectively; screen-, accel-, and decel-grid thicknesses of 0.25 mm, 0.75 mm, and 0.75 mm, respectively; screen-accel and accel-decel grid gaps of 0.75 mm and 0.5 mm, respectively; a screen-grid-aperture center-to-center distance of 2.46 mm; a grid dish depth of 6 mm inward; an accel-grid compensation strain of 0.5%; and an active beam diameter of 10 cm) operating at the condition reported in Ref. 10. Typically, the plasma density inside the thruster chamber is not uniform. For this specific example, perveance across the thruster-grid radius was assumed to be a cosine shape with a beam flatness parameter of 0.73 as shown in Fig. 4a. Then, in the calculation, five different beamlet-current-density profiles, corresponding to different radial locations on the grids, were used to approximate this continuous perveance variation. Plasma potential and electron temperature inside the thruster chamber were not reported in Ref. 10 so they were assumed to be 43 V and 3.0 eV, respectively. The resultant beam-ion density is shown in Fig. 4b. Note that the center of accel grid is located at the origin, which is just outside of the calculation region at the left boundary of the figure.

Neutral-Density Calculation

In these calculations only the density of propellant atoms is considered because the densities and CE cross-sections of non-propellant atoms are small by comparison. It is then assumed that the density at any point can be determined as a sum of a uniform, ambient component, which would be significant only in a ground-test environment, and a grid-system-source component, which results from the flow of unionized propellant. The second component is approximated by superposing the effects of point sources located at the centers of all
accel-grid apertures. A convex accel grid and one of its apertures are shown in Fig. 5 to illustrate the geometry and to define the symbols used to determine the contribution from the aperture to the neutral density at a downstream mesh point. Neutral atoms are assumed to be emitted through the aperture with a squared-cosine distribution. This distribution was chosen to reflect the assumption of collisionless downstream transport and the neutral density measurements of Crofton.11 Representing the neutral flux through the aperture as $N_h$, the distance between an aperture and the mesh point as $l$, the angle between the outward normal to the aperture and the line connecting the aperture and the mesh point as $\theta$, and the thermal velocity of neutral atoms as $v_n$, the density contribution due to this flux at the mesh point is $3N_hl\cos^2\theta/(2\pi l^2v_n)$. Assuming the neutral flux is uniform over the entire grid, the total neutral density at the mesh point is then determined by summing terms given by this expression for all of the grid apertures and the term for the uniform neutral-atom density due to the ambient pressure:

$$n_0 = \frac{P_0}{k_BT_0} + \sum_{\text{accel-grid apertures}} \frac{3N_hl\cos^2\theta}{2\pi lm_n} I^2 \sqrt{\frac{8k_BT_0}{\pi m_n}}$$

(1)

Figure 6 shows a map of neutral density downstream of the UK-10 thruster computed for the operating condition that yielded the beam-ion-density data of Fig. 4.

**Beam-Plasma-Potential Calculation**

Beam-plasma potentials, $V$, are computed using the beam ion density, $n_B$, charge-exchange ion density, $n_{CE}$, and electron density, $n_e$, in the following form of Poisson's equation:

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{\partial^2 V}{\partial z^2} = \frac{e}{\epsilon} (n_e - n_B - n_{CE})$$

(2)

The boundary conditions used in the calculations are indicated in Fig. 7. On the upstream and downstream boundaries and along the centerline, zero electric field components normal to the boundaries are imposed (i.e. $\partial V/\partial z = 0$ on the upstream and downstream boundaries and $\partial V/\partial r = 0$ on the centerline); the potential on outer peripheral boundaries is set to zero and electron-source potential is set at $V_c$. The determination of each of the three densities that appear on the far-right-hand side of Eq. (2) will be addressed next.
(barometric) equation, which is written in the following form, is widely used for this purpose:

$$n_e = n_{e0} \exp \left[ \frac{e(V - V_0)}{k_B T_e} \right]. \quad (3)$$

In this expression, $n_{e0}$ is electron density at a reference location where the potential is $V_0$ and $T_e$ is the electron temperature. The Boltzmann equation can be derived by imposing a steady-state momentum balance between electric field and pressure force associated with electrons described by a Maxwell-Boltzmann distribution at any location with uniform electron temperature.

In a typical beam plasma, however, electron densities are sufficiently low ($\sim 10^{8-10} \text{ cm}^{-3}$) that electron mean-free-paths for momentum transfer through coulomb collisions with each other (the shortest mean-free-paths) are of the order of $10^3 \sim 5 \text{ cm}$, and this is greater than a typical beam diameter. Under such conditions, electrons would not be expected to form a Maxwell-Boltzmann distribution as confirmed for the UK-10 thruster by de Boer. Some experiments were conducted using a SERT-II thruster modified so that it operates with xenon propellant in order to address the validity of the Boltzmann equation. Details of these experiments and their results are described in the appendix and a modification of the equation is suggested. In a sample calculation for the UK-10 thruster, however, Eq. (3) was used without changes for simplicity even though the electron mean-free-path is much greater than the size of a typical calculation region ($\sim 30 \text{ cm}$) or the mesh size ($<1 \text{ cm}$) for the case.

In previous studies a non-uniform electron temperature was incorporated as a variation of the Boltzmann model and energy-conservation equations were applied. Additional experimental results described in the appendix suggest one electron temperature prevails in the beam plasma where electron densities are greater and another outside of the beam where they are lower. Invoking the assumption of equal electron and ion densities enabled use of the following description of electron-temperature variation with density and the more complex and time-consuming, energy-conservation approach was avoided:

$$T_e = T_{e\text{out}} \frac{1 - \cos \left( \pi \frac{\log n_i - \log n_{i\text{max}}}{\log n_{i\text{max}} - \log n_{i\text{min}}} \right)}{2} \quad (4)$$

In this expression, $n_i$ is the total ion density (i.e. the sum of the beam and charge-exchange ion densities) and $T_{e\text{in}}$ and $T_{e\text{out}}$ are electron temperatures inside and outside the ion beam, respectively.

**Beam-Ion Density ($n_B$)**

Because potentials observed experimentally in ion beams are relatively uniform and quite low compared with net accelerating voltages, it is assumed that the effects of these potentials on beam-ion motion are negligible. Also, it is assumed that CE ions created in the beam do not reduce the beam-ion current densities significantly because the charge-exchange-collision rate is typically small compared to the beam current. Hence, beam-ion densities calculated for a sum of beamlet current densities are used directly for $n_B$.

**Charge-Exchange-Ion Density ($n_{CE}$)**

The CE-ion production rate in the beam is determined using a computed neutral density along with the beam-ion density and velocity. In this model, bulk motion of the CE ions created at a point in the beam is assumed to be in a direction parallel to the local electric field at that point. These ions are assumed to have an isotropic velocity distribution with a magnitude of $(3k_B T_i/m_{Ce})^{1/2}$ at their point of creation. Assuming the electric fields were uniform from this point, they would spread within an envelope like that shown in Fig. 8 until they reach a calculation-region boundary. The full width of the envelope at

![Fig. 8 Geometry of CE-Ion Flux used in Beam-Plasma-Potential Calculations](image-url)
any downstream location is the maximum normal
distance between the extension of the electric field
vector and the trajectory of the outermost ion. For
a constant electric field, $E$, a simple 2-D analysis
of ion motion yields an envelope full-width given
by the expression

$$d = 2 \sqrt{\frac{3k_BT_0}{2eE}} \left( \frac{3k_BT_0}{2eE} + x \right)$$  \hspace{1cm} (5)

where $x$ is the distance parallel to the electric-field
direction. The contributions of the CE ions created
inside the ion beam to the densities at all mesh
points within the envelopes except the origin point
are computed. It is assumed that the flux density is
uniform across the width of each envelope at each
downstream location and that the ion speeds are
determined by the potential differences from their
origins. By using this approximation, time-
consuming iterations between beam-plasma-
potential and CE-ion-trajectory calculations can be
avoided.

The flux of CE ions that are about to stall due
to adverse potential gradients is reduced by the
factor $\exp[e\Delta V/(k_BT_0)]$. Also, CE-ion densities and
local electric fields are averaged with values from
eight adjacent mesh points to improve
computational stability.

**Beam-Plasma-Potential Solution for UK-10
Thruster**

Figure 9 shows maps of beam-plasma potential
computed for the UK-10 thruster operated at the
condition of Figs. 4 and 6. The plasma potential
shows a sudden drop corresponding to the edge of
ion beam and decreases slightly with downstream
position. The neutralizer was placed outside of the
region and is not shown in the figure. The total
ion density profile computed 7.5 cm downstream of
the accel grid is compared with the experimental
results of de Boer$^{10}$ in Fig. 10.

**Charge-Exchange-Ion-Trajectory Calculation**

Using the results of preceding calculations, the
motions of CE ions produced at various locations
downstream of an ion thruster can be computed. It
is assumed that CE ions move in the computed axi-
symmetric, beam-plasma potential field, which
reflects the space-charge effects of beam and
charge-exchange ions and of electrons, without

---

The initial position of a CE ion ($r$ and $z$
coordinates) is selected to match the probability of
CE-ion production, which is proportional to the
product of the neutral-atom and beam-ion densities.
The magnitudes and directions of the initial
velocities of the CE ions are selected randomly
assuming that they have a Maxwell-Boltzmann
distribution characterized by the neutral
temperature, $T_0$. Then, the equation of motion for
each singly-charged CE ion in the potential field is computed using numerical integration.

The actual trajectory calculation is made in each mesh cell. In each mesh cell, the beam-plasma potential $V$ is interpolated using a bi-linear function (i.e., a product of two linear functions, each of which depends only on $r$ or $z$), $V_i$, from the potential values obtained at the four points which define the cell. Then, the trajectory of each CE ion is determined by solving the following differential equations using a 1st-order, forward-marching, finite-difference scheme:

$$\frac{d}{dt} \frac{dr}{dt} = \frac{dV_r}{dt} - \frac{h^2}{r^3} \frac{e}{m_x} \frac{d\phi}{dr},$$

$$\frac{d}{dt} \frac{dz}{dt} = \frac{dV_z}{dt} = -\frac{e}{m_z} \frac{d\phi}{dz}.$$

Here, $v_r$ and $v_z$ are the velocity components in the $r$ and $z$ directions, respectively, and $h$ is the angular momentum of the CE ion (i.e., $r \times v_\theta$), which is conserved since the CE ions are assumed to move in an axi-symmetric potential field. This constant angular momentum is used to compute $v_\theta$ at the final point. The time step used in the calculations is chosen so that 100 steps would transport the ion across the cell if the electric field in the cell were zero. Figure 11 shows CE-ion current densities in the calculation region for the condition corresponding to the data of Figs. 4, 6, and 9.

**Conclusions**

A simple model has been developed to compute plasma properties and trajectories of charge-exchange ions in the exhaust plume of an ion thruster using the computational power of conventional personal computers without a PIC algorithm. Even though some crude assumptions and approximations are used in the programs, the computed beam-plasma potentials appear reasonable and the computed total ion density profile for the UK-10 thruster agrees with measurements.

**Acknowledgment**

This work was supported under Advanced Electric Propulsion Research Grant NAG3-1801 from the NASA Lewis Research Center. The authors would like to thank David G. Fearn of the British Defense Research Agency and Mark W. Crofton of the Aerospace Corporation for information regarding the UK-10 thruster.

**References**

9. Private communication with Fearn, D.G., of DERA Farnborough

Appendix

In order to address the question of Boltzmann-equation validity, experiments were conducted in which plasma potential, electron temperature and electron density distributions were measured across diameters of an ion beam produced by the modified SERT II thruster equipped with a 15-cm-diameter, small-hole accelerator-grid set. The nominal thruster operating conditions were: thruster-cathode and neutralizer flow rates of 46 mA eq. Xe, main-chamber flow rate of 458 mA eq. Xe, screen and accel grid voltages of 800 V and -500 V, respectively, and a neutralizer keeper potential near vacuum tank ground (-2 to 0 V). At these conditions, the nominal beam current was 430 mA and the ambient pressure was 1.8 \times 10^{-5} \text{Torr} (2.4 \times 10^{-5} \text{Pa}). A spherical Langmuir probe was used to measure electron energy distributions on several axial planes at three radial positions (one in the beam and two beyond the edge of the beam, near and far from the neutralizer). The probe-voltage-current traces were analyzed using numerical differentiation to obtain energy distributions. Electron densities, \( n_e \), were then estimated from these plots as the area under the curves while electron temperatures were computed on the basis of corresponding mean energies determined from the data. These results suggested electron temperatures fell into two groups with values corresponding to positions inside and outside of the beam. The experiments were conducted with the neutralizer grounded at a standard beam current (nominal case), with it biased sufficiently negative of ground so its keeper potential was around \(-8 \text{ V}\) at the standard beam current (neutralizer negative case), and with the neutralizer grounded at lower beam currents (reduced beam current cases).

The validity of the Boltzmann equation (Eq. (3)), in which electron density is only a function of the term \( e(V-V_0)/(k_BT_e) \), was then evaluated. In the experiments, the most positive surface in contact with the plasma was the grounded vacuum tank and, therefore, \( V_0 \) was taken to be zero and electron densities were plotted against the resulting parameter, \( eV/(k_BT_e) \).

Figure A1 gives the data collected for all of the cases considered. This figure shows the expected correlation on a semi-log plot, but the slope of the line through the data was found to be 0.57, which is significantly less than unity (the value associated with the Boltzmann equation).

Additional results obtained from the experiments and shown in Fig. A2 are used in the calculations to reflect changes in electron temperature within the beam where the electron densities were greater and outside of the beam where they were lower (Eq. (4)).

![Fig. A1 Correlation between Electron Density and Plasma-Potential/Electron-Temperature Ratio](image1)

![Fig. A2 Correlation between Electron Temperature and Density](image2)