DEVELOPMENT OF A MHD CODE USING CIP METHOD AND ITS APPLICATION TO A LASER FUSION ROCKET

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Abstract

A magnetohydrodynamics (MHD) code is being developed. CIP (cubic interpolated pseudo particle) method is used to solve hydrodynamics, a moving mesh method is adopted to trace the plasma surface as free boundary, and SSI (symmetrical semi-implicit) method is used to calculate the diffusion term. The code is checked against the standard problems. We found good agreement between the self-similar solutions and the numerical results. An analysis of the plasma behavior in the magnetic nozzle is performed for a fusion rocket. The results of the code show a plasma behavior consistent with the magnetic nozzle concept.

Introduction

Fusion reaction can release a large amount of energy. To give rise to the fusion reaction, two schemes are proposed. One is magnetic confinement fusion (MCF) and the other is inertial confinement fusion (ICF). Use of energy released by fusion could realize a propulsion system that might be more attractive compared with existing systems. A propulsion system driven by a laser-induced fusion, so called ICF rocket, is an attractive candidate. It could provide both large specific impulse and power. Design studies of spacecraft based on ICF have been made for several years by some researchers. [1, 2]

The basic concept of ICF rocket is as follows: ICF rocket has a magnetic nozzle composed of a solenoidal superconducting magnet. When the fusion reaction occurs by laser irradiation of pellet, the resulting plasma as fusion products begins to expand into the external magnetic field. The plasma compresses the magnetic field and deposits its energy to the magnetic field. As the plasma reaches the maximum expansion radius, the compressed magnetic field starts to push back the plasma and redirect it, thus producing thrust.

In order to indicate the validity of the above concept, analysis of the plasma behavior in the magnetic nozzle and the estimation of the thrust conversion efficiency have been performed. For example, Hyde[1] has proposed a conceptual design of ICF rocket and has calculated the thrust conversion efficiency by using MHD code. The efficiency was reported to be 60 % there. However the efficiency depends on the configuration of the magnetic nozzle, i.e., the intensity or shape of magnetic field, fusion reaction point, etc. Thus parametric research is needed to find the optimum configuration of the nozzle and to obtain the thrust conversion efficiency.

A new MHD code is being developed by the present authors. The code uses CIP (Cubic Interpolated Pseudo Particle) method [3] to analyze the plasma behavior in the magnetic nozzle. The CIP method is a universal solver for hyperbolic equation and its characteristics are numerically stable and less diffusive.

When calculating the plasma expansion into a magnetic field in the magnetic nozzle, the plasma cloud initially occupies a small region of the whole calculation domain and then it expands rapidly outward. This requires a robust and stable computation method. This is the reason why the CIP method is adopted here for the analysis of plasma behavior in the magnetic nozzle.

The plan of the present paper is as follows: In the next section the basic equation of the plasma motion and description of the computational model are presented. In section 3, the code is checked against the well known problems of fluid and MHD dynamics. The analysis of plasma behavior in the magnetic nozzle is presented in section 4 along with discussions. The model adopted here is based on a configuration proposed by Hyde [1]. The conclusion is given in section 5.

Computational Model

Basic Equation

Basic equations solved in the present simulation code are the following one-fluid MHD equations and Maxwell equations in MKS units.

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0 \qquad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho}(\nabla p - \mathbf{J} \times \mathbf{B}) = 0 \qquad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T + \frac{1}{\rho c_v}(p(\nabla \cdot \mathbf{u}) - \mathbf{E} \cdot \mathbf{J}) = \frac{1}{\rho C_v}(\nabla \cdot (\kappa \nabla T)) \qquad (3)$$

$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} \qquad (4)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (5)$$

Here, ρ is the density of plasma, $\boldsymbol{u}$ the velocity, $\boldsymbol{p}$ the pressure, $\boldsymbol{J}$ the current density, $\boldsymbol{B}$ the magnetic field intensity, $\boldsymbol{T}$ the temperature, C_v the specific heat, $\boldsymbol{E}$ the electric field intensity, κ the heat conductivity, and μ_0 the permeability of free space. The plasma density n and the pressure p represent $(n_i + n_e)$ and $(p_i + p_e)$, respectively. Furthermore the generalized Ohm's law is used:

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \cdot \mathbf{J} - \frac{\nabla p_e}{n_e} \qquad (6)$$

where η is the electric conductivity. (It is neglected in the computations presented in the following sections.) The subscript i and e attached to p and n denote the components of the ion and electron, respectively.

Description of Model and Method

The code treats the compressible flow as given in equation (1). Cylindrical symmetry is assumed: 2.5 dimensional $(r, z, v_r, v_\theta, v_z; \partial/\partial\theta = 0)$ model is adopted here.

Introducing the magnetic vector potential $(\nabla \times \mathbf{A} = \mathbf{B})$ in the Coulomb gauge, and utilizing the equation (6), we obtain the equation

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times (\nabla \times \mathbf{A}) - \frac{\eta}{\mu_0} \nabla \times (\nabla \times \mathbf{A}) + \frac{\nabla p_e}{n_e} \qquad (7)$$

where the relation $\mathbf{E} = -\partial \mathbf{A}/\partial t$ has been used. Taking the rotation of the equation (6) and substituting the resulting equation into the equation (5), we obtain the following equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \left\{ \nabla \times \left(\frac{\eta}{\mu_0} \nabla \times \mathbf{B}\right) + \nabla \times \left(\frac{\nabla p_e}{n_e}\right) \right\} \qquad (8)$$

Thus B_r and B_z can be obtained from the θ component of equation (7) and B_θ is given by the θ component of the equation (8).

As mentioned previously, the CIP [3] method is adopted here. The CIP method is proposed to solve the fluid equation which has the form,

$$\frac{\partial f}{\partial t} + (\mathbf{u} \cdot \nabla) f + G = H, \qquad (9)$$

where

$$f = (\rho, \mathbf{u}, T), \qquad (10)$$

$$G = \left(\rho \nabla \cdot \mathbf{u}, \frac{1}{\rho}(\nabla p - \mathbf{J} \times \mathbf{B}), \frac{1}{\rho C_v}(p(\nabla \cdot \mathbf{u}) - \mathbf{E} \cdot \mathbf{J})\right), \qquad (11)$$

$$H = \left(0, 0, \frac{1}{\rho C_v}(\nabla \cdot (\kappa \nabla T))\right). \qquad (12)$$

In this method, equation (9) is sequentially solved in time,

$$\frac{\partial f}{\partial t} = H, \quad \text{(diffusion phase)}, \qquad (13)$$

$$\frac{\partial f}{\partial t} = -G, \quad \text{(non-advection phase)}, \qquad (14)$$

and

$$\frac{\partial f}{\partial t} + (\mathbf{u} \cdot \nabla) f = 0, \quad \text{(advection phase)}. \qquad (15)$$

The CIP method is applied to the equations (15): this equation is solved by shifting a cubic interpolated profile with a local velocity $\boldsymbol{u}$. On the other hand, the equations (14) can be solved using the finite difference method. For the equation (13), SSI (Symmetrical Semi-Implicit scheme) method [6] is applied. Furthermore, the same procedure is adopted in solving the equations (7) and (8). When the plasma expands into the magnetic field in the magnetic nozzle, the surface of the plasma faces a vacuum. We adopt here a moving mesh method to treat the plasma surface as a free boundary.

Application to Problems

We checked the code against the well-known problems of the fluid dynamics:

(i) cylindrical shock tube [3],
(ii) converging cylindrical shock [4],
(iii) unsteady flow into a vacuum [5].

For the last problem, it is found that the moving mesh method improves the mass balance considerably, i.e. without it, the system mass decreases with time. In addition to the above problems, we also checked the heat wave propagation for non-linear heat conduction [8] and MHD shock problem [9].

We found good agreement between the self-similar solutions and the numerical results. The comparison is given in Ref. 10.

To check the code against the MHD problems, we take up here a plasma bounce motion in the magnetic field of an ICF reactor [11]. The configuration adopted for one-dimensional, cylindrical calculation is shown in Fig.1. The plasma has a radius of 1m and a mass of 0.05mg. Its density distribution is uniform. The internal energy is 13.76MJ. The superconducting magnet (SCM) is located at 11.5m from the reactor center and it generates a uniform magnetic field with an intensity of 0.435T.

Time evolutions of magnetic field are given in Fig. 2. The plasma expands rapidly at early times, is then decelerated and reaches a position of maximum expansion at 0.4 μsec, followed by compression at still later times. Figure 3 shows the time evolution of various components of the energies in the system. The initial plasma thermal (internal) energy is converted into the kinetic energy and then into the magnetic energy. The converted energy is recovered into the plasma energy at later times, the system total energy being conserved throughout the plasma bounce motion in the magnetic field.

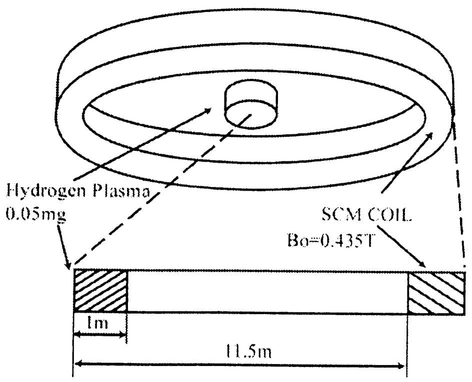

Fig. 1 Configuration for one-dimension calculation

The maximum plasma radius is estimated as 5.5m by considering the energy balance between the plasma and magnetic field, the result being consistent with the calculational result shown in Fig. 2.

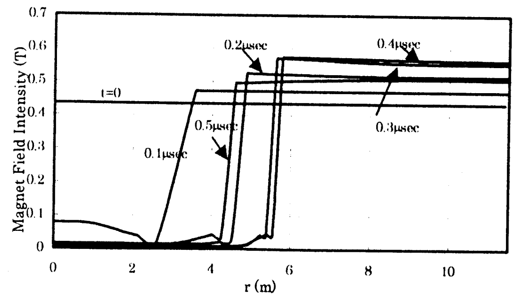

Fig.2 Time evolution of magnetic field intensity **B**

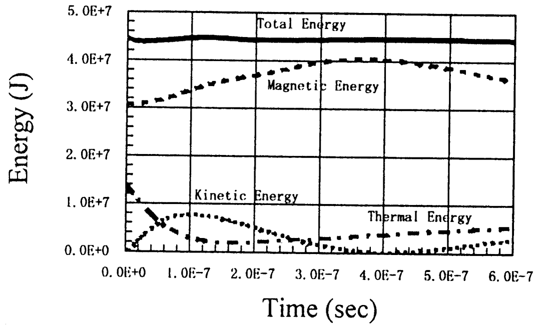

Fig. 3 Energy balance

An instability analysis of rotating plasmas in a magnetic field is being made by a version of the CIP-MHD code [12] modified to include the Hall term. During the course of the study, we checked the Hall MHD code against a test problem of the magnetic field evolution. [13]

Analysis of The Plasma Behavior in The Magnetic Nozzle

The calculational model for the magnetic nozzle adopted here is based on the design of ICF rocket proposed by Hyde [1].

A schematic layout of the magnetic nozzle is shown in Fig. 6. The magnetic field in the nozzle is generated by a solenoidal SCM coil which has a radius of 6.5m, carries a current of 22MA and stores 7978 MJ. An angle θ (cone angle) subtended from the Z axis to the solenoidal coil is taken to be 55°.

The plasma has a radius of 1m and a mass of 8.3g mainly contributed by hydrogen propellant. Its density distribution is uniform. The internal energy derived from fusion reaction is 1300MJ.

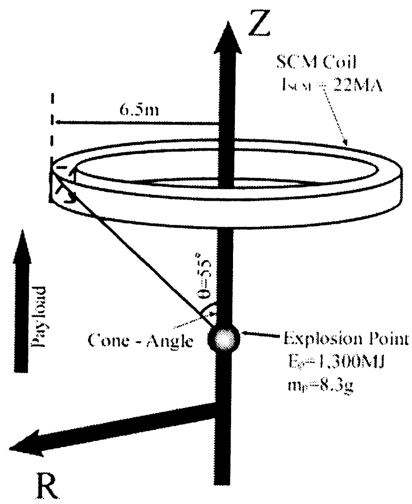

Fig. 6 Schematic of calculation model for magnetic nozzle

In order to extend the CIP code to two-dimensional problems such as this, the code is rewritten in a generalized curvilinear coordinate system [14]. The initial mesh structure adopted here is shown in Fig.7. To accommodate the plasma motion (deformation from the initial spherical geometry) in the magnetic field, two different mesh-grid systems are adopted, i.e. one for the plasma and the other for the ambient magnetic field. The grid structure is reconstructed at every time step by numerically solving the Poisson equation [15].

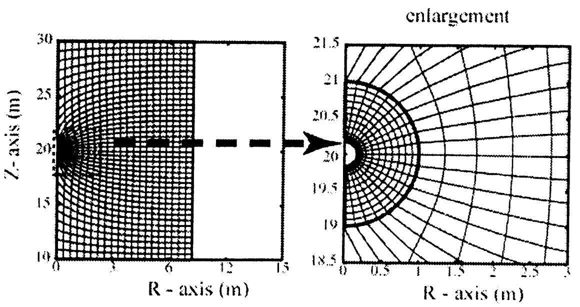

Fig. 7 Initial mesh-grid structure

The preliminary results by the code are shown in Figs. 8~10. Figures 8 and 9 give temporal changes of magnetic field distribution and density distribution, respectively. The external magnetic field is compressed by the expanding plasma and then redirects it. A density peak is seen in Fig. 9 (at 10μsec) in the coil direction due to the push-back from the external magnetic field. The temporal change of mesh-gird structure is shown in Fig.10. As the plasma expands, the mesh intervals are compressed in the coil direction. Figure 11 shows the time evolution of the outermost mesh of the expanding plasma. The plasma expands isotropically in the early stage while compressing the external magnetic field and then it is reflected by the magnetic field.

The results of the code show a plasma behavior which is consistent with the magnetic nozzle concept mentioned in section 1. These results are also consistent with those of MHD calculation reported by Hyde [1].

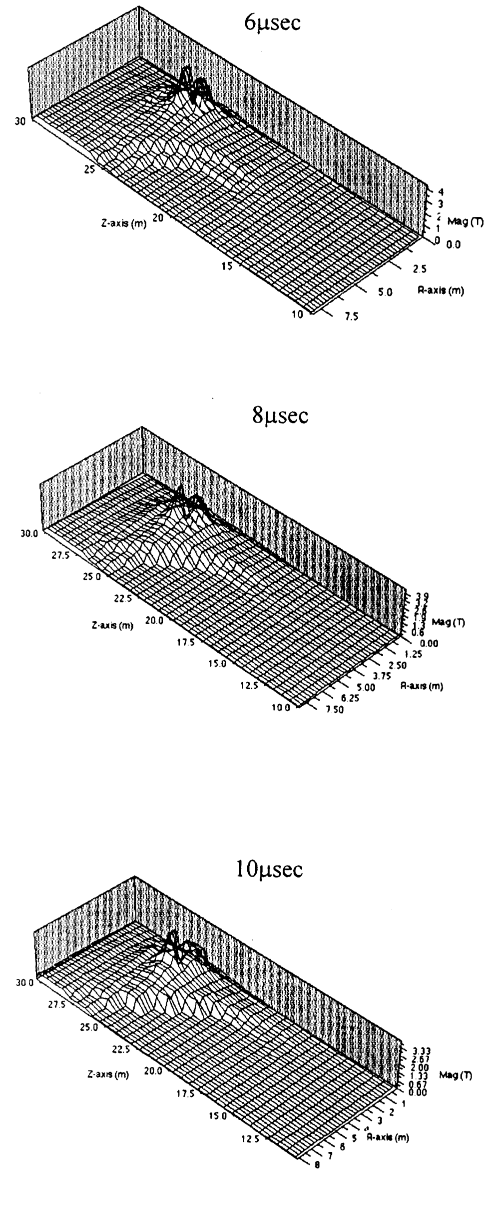

Fig. 8 Temporal change of magnetic field distribution

The thrust efficiency η in terms of momentum is calculated as follows:

$$\eta = \frac{\sum M v_z}{\sum M \mid v_0 \mid} \tag{16}$$

where M is the mass of ion, v_z the z-component of its velocity and $|v_0|$ the absolute value of initial velocity. The sum (Σ) is carried over the all plasma regions. The results are shown in Fig. 12 as a function time. The value of η increases rapidly from 6μsec.

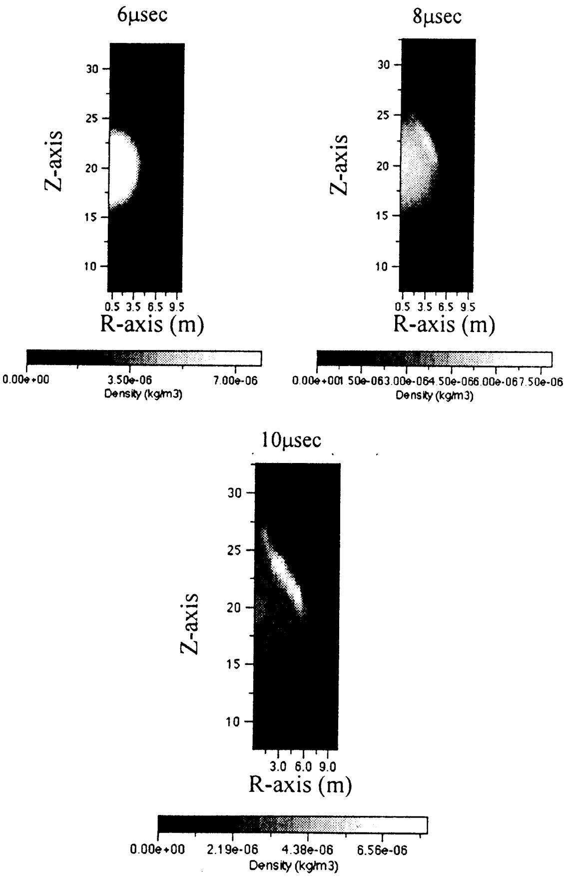

Fig. 9 Temporal change of density distribution

A later times the CIP-MHD code fails to calculate the plasma motion due to the severe deformation of the mesh-grid structure as shown in Fig. 10. Thus, further study is needed to optimize the mesh structure and to obtain the final results of the thrust efficiency.

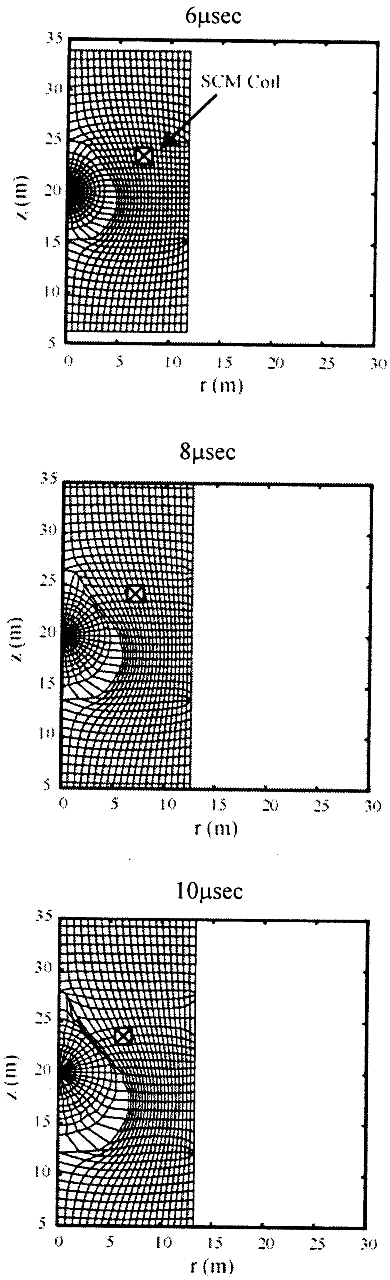

Fig. 10 Temporal change of mesh-grid structure

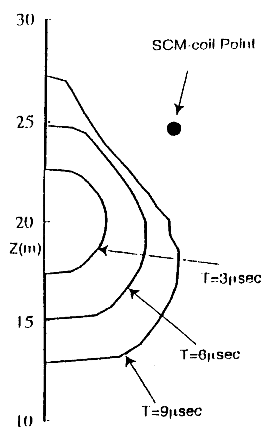

Fig. 11 Outermost mesh of the expanding plasma

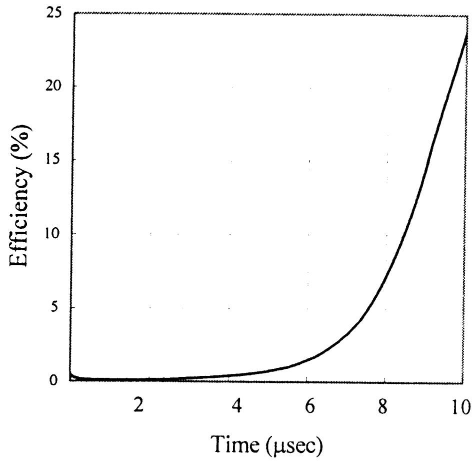

Fig. 12 Thrust efficiency

The authors have also been developing a three-dimensional hybrid code where ions are treated as particles and electrons as inertialess fluid [16]. The two different approaches- hydrodynamic and particle-would enhance the understanding of the plasma behavior and the mechanism to produce thrust in the magnetic nozzle.

Conclusions

We have developed a MHD code using the CIP method. In order to trace the plasma surface effectively, the moving mesh method is adopted. The SSI method is used to solve the diffusion equation. The code is checked against the well-known problems of fluid and MHD dynamics.

An analysis of plasma behavior in the magnetic nozzle is performed. The results of the code show a plasma behavior consistent with the magnetic nozzle concept. However, at later times the code fails to calculate the plasma motion due to the severe deformation of the mesh-grid structure. Thus, further study is needed to optimize the mesh structure and to obtain the final results of the thrust efficiency.

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