COMPUTATION OF HALL THRUSTER PERFORMANCE

L. Garrigues, I.D. Boyd, and J.P. Bocu

Centre de Physique des Plasmas et Applications de Toulouse
Université Paul Sabatier
118 route de Narbonne, 31062 Toulouse Cedex 4, FRANCE
Department of Aerospace Engineering
University of Michigan
Ann Arbor, MI 48109, U. S. A.

Abstract

A quasineutral 1D hybrid model of a Stationary Plasma Thruster (SPT) has been used to predict the performance of a laboratory SPT100 model (SPT100-ML) which is being tested in the French facility PIVOINE. In this model ions are treated as particles, electrons as a fluid, and quasi-neutrality is assumed. We describe the calculated efficiencies, thrust and specific impulse in xenon and krypton and their variations as a function of applied voltage and mass flow rate. Typically, for a voltage of 300 V and an anode mass flow rate of 5 mg/s, the thrust is on the order of 70 mN for xenon and krypton, and the global efficiency is 40% and 30% respectively for xenon and krypton. Although the model is rather simple the predicted trends are in reasonable agreement with the experimental measurements.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area of the Thruster [m²]</td>
</tr>
<tr>
<td>B</td>
<td>Magnetic Field [T]</td>
</tr>
<tr>
<td>d</td>
<td>Dimension of the Channel [m]</td>
</tr>
<tr>
<td>δ</td>
<td>Dirac Function</td>
</tr>
<tr>
<td>Δt</td>
<td>Ion Transport Time Step [s]</td>
</tr>
<tr>
<td>c</td>
<td>Electric Charge Constant = 1.602×10⁻¹⁹ C</td>
</tr>
<tr>
<td>E</td>
<td>Electric Field [V/m]</td>
</tr>
<tr>
<td>E_e, E_h</td>
<td>Electron, Ion Exhaust Beam Mean Energy [eV]</td>
</tr>
<tr>
<td>f_i</td>
<td>Ion Distribution Function</td>
</tr>
<tr>
<td>ϕ_i, Φ_0</td>
<td>Ion Flux, Neutral Flux in the Anode Plane [m⁻³s⁻¹]</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational Constant=9.81 ms⁻²</td>
</tr>
<tr>
<td>γ</td>
<td>Thrust Loss Factor</td>
</tr>
<tr>
<td>I_{sp}</td>
<td>Specific Impulse [s]</td>
</tr>
<tr>
<td>J_i, J_d, J_m</td>
<td>Ion, Discharge, and Neutral Equivalent Current [A]</td>
</tr>
<tr>
<td>k_i, k_m</td>
<td>Ionization, and Momentum Exchange Rate [m⁻³s⁻¹]</td>
</tr>
<tr>
<td>m_e, M</td>
<td>Electron Mass = 9.1×10⁻³¹ kg, Neutral Atom Mass Xe=131.3 uma, Kr=83.8 uma</td>
</tr>
<tr>
<td>m_i, m_j</td>
<td>Propellant Mass Flow Rate at the Anode, and Ion Mass Flow Rate at the Exhaust [kg/s⁻¹]</td>
</tr>
<tr>
<td>μ_e</td>
<td>Axial Electron Mobility [m²V⁻¹s⁻¹]</td>
</tr>
<tr>
<td>n_i, N_a, n_p</td>
<td>Ion, Neutral, and Plasma Density [m⁻³]</td>
</tr>
<tr>
<td>N_i, N_a, N_m</td>
<td>Number of Macro-Particle Created in a Cell i, Simulated, and Minimal</td>
</tr>
<tr>
<td>η, η_a, η_d, η_E</td>
<td>Total, Propellant, Acceleration, and Beam Energy Efficiency</td>
</tr>
<tr>
<td>v_i, v_m</td>
<td>Energy Efficiency</td>
</tr>
<tr>
<td>v_m, v_wall_m</td>
<td>Collision Frequency [s⁻¹]</td>
</tr>
<tr>
<td>v_e</td>
<td>Momentum Exchange Collisions</td>
</tr>
<tr>
<td>v_e</td>
<td>Frequency, Collisions with walls [s⁻¹]</td>
</tr>
<tr>
<td>Ω_B</td>
<td>Electron Energy Loss Frequency [s⁻¹]</td>
</tr>
<tr>
<td>P_d</td>
<td>Electron Angular Cyclotron Frequency [s⁻¹]</td>
</tr>
<tr>
<td>r_i, v_i, w_i</td>
<td>Dissipated Power [W]</td>
</tr>
<tr>
<td>S_i, S_{max}</td>
<td>Position [m], Velocity [ms⁻¹], and weight of a test particle</td>
</tr>
<tr>
<td>S, S_{v_i}</td>
<td>Ion Production Rate in a Cell i, and Maximal Ion Production Rate [m⁻³s⁻¹]</td>
</tr>
<tr>
<td>S, S_{v_j}</td>
<td>Ion Production Rate [m⁻³s⁻¹] and Ion Production Rate in Velocity Space [m⁻³]</td>
</tr>
<tr>
<td>T</td>
<td>Thrust [N]</td>
</tr>
<tr>
<td>V_b</td>
<td>Ion Exhaust Beam Velocity [ms⁻¹]</td>
</tr>
<tr>
<td>V_d</td>
<td>Applied Voltage [V]</td>
</tr>
<tr>
<td>v_e, v_i, v_0</td>
<td>Electron, Ion Mean Velocity, and Neutral Atom Velocity [ms⁻¹]</td>
</tr>
<tr>
<td>x, v_c</td>
<td>Axial Position [m], and Axial Velocity [ms⁻¹]</td>
</tr>
</tbody>
</table>

Introduction

In a SPT, the ions are created and accelerated in a plasma channel between two concentric dielectric cylinders. Xenon (or krypton) is injected through the anode which is localized at one end of the channel. The cathode is outside the channel and provides the electron current entering the channel through the other end of the channel (exhaust). Because of the low gas pressure in the channel and the small dimensions of the device, a plasma can be created in the channel
only if the electrons are confined by a magnetic field. A radial magnetic field is created in the channel by external coils and a magnetic circuit. The radial magnetic field is large in the exhaust region and decreases toward the anode. The large value of the radial magnetic field in the exhaust region induces a drop of the axial electron velocity. In order to maintain a large enough plasma conductivity, the axial electric field must increase in the exhaust region. Electron heating in this field leads to strong ionization of the neutral flow. The ions, which are not sensitive to the magnetic field and are collisionless in the channel, are accelerated by the electric field in the exhaust region. The thrust and specific impulse of Hall Thrusters are well adapted for north-south station-keeping of Geostationary Earth Orbit telecom satellites.

The French Space Agency - CNES will launch an experimental telecom satellite STENTOR with two platforms of PPS-1350 and SPT-100. New programs involving SPT’s are also under consideration. First, the Hall Thrusters are considered for telecom constellations in Low Earth Orbit (SKYBRIDGE programme). The use of electric propulsion is also under consideration for interplanetary missions. The ESA 2000 scientific observation programme concerns the launch of small satellites by Ariane 5. The SPT is a solution for orbit transfer of the observation satellites of Near Earth Objects like the Moon or comets.

For these new applications both larger and smaller devices are required. In parallel to experimental investigations, numerical models can help explain and optimize the physical processes in Hall Thrusters. A one-dimensional transient hybrid model of the SPT-100 acceleration channel has been developed recently. The primary components of the model include the continuity equation for neutrals with a constant velocity and ionization loss term, a microscopic description of the ion transport and a fluid description of electrons. Charged neutrality is assumed, and the axial distribution of the radial magnetic field profile is given. The model successfully predicted the plasma oscillations observed in this device. The predictions of this simple quasi-neutral hybrid model have been qualitatively validated with a more accurate 1D PIC MCC model.

In this paper, we focus on the prediction of the performance of a SPT100 with a 1D model. We describe in the first part the approximations and governing equations of the model, while the numerical techniques and data are discussed in the second part. In the third part, we define the performance of the Hall Thruster. The calculations of the performance as a function of voltage and mass flow rate in xenon and krypton of a SPT100 are presented in detail and compared with experimental results in the fourth part. The last part is dedicated to some brief conclusions.

Approximations and Governing Equations

The 1D hybrid model is based on the same assumptions as the model of Ref. [5] except that ion transport is described by following the trajectory of a sample of ions (PIC model) rather than by solving numerically the Vlasov equation. These assumptions are: 1) quasi-neutrality of the plasma, 2) ion transport is described with a 1D Vlasov equation, 3) the electric field is obtained from the current density, using the electron momentum equation, 4) the electron mobility perpendicular to the magnetic field is classical and includes the effects of electron-neutral collisions as well as electron-wall collisions (in a phenomenological way), and 5) the electron distribution function is Maxwellian, the electron mean energy being obtained from a simplified energy equation.

The hybrid model is therefore based on the following equations:

The effect of the magnetic field is negligible for the ions, so the ion Vlasov equation is given by:

\[
\frac{\partial f_i(x,v_x,t)}{\partial t} + v_x \frac{\partial f_i(x,v_x,t)}{\partial x} + \frac{e}{M} F(x,t) \frac{\partial f_i(x,v_x,t)}{\partial v_x} = S_{v_x}(x,v_x,t)
\]

(1)

where

\[ S_{v_x}(x,v_x,t) = n_i(x,t)N_e(x,t)\delta(v_x - v_0) \]

(2)

\[ = n_i(x,t)\delta(v_x - v_0) \]

All the ions are created with the neutral atom velocity. The ions density is deduced from the integration of the ion distribution function in velocity space:

\[ n_i(x,t) = \int f_i(x,v_x,t)dv_x \]

(3)

The axial mean velocity of the ions is:

\[ \bar{v}_x(t) = \int v_x f_i(x,v_x,t)dv_x \]

(4)

The plasma density is obtained with the assumption of quasi-neutrality:

\[ n_p(x,t) = n_i(x,t) \]

(5)

The equation of continuity for the neutral atoms with constant velocity \( v_0 \) and a loss term due to ionization gives the neutral density:

\[ \frac{\partial N_p(x,t)}{\partial t} + v_0 \frac{\partial N_p(x,t)}{\partial x} = -n_p(x,t)N_e(x,t)k_b(x,t) \]

(6)

The neutral atom density in the anode plane is:

\[ N_p(0,t) = \frac{\Phi_0}{v_0} \]

(7)

where \( \Phi_0 \) is the neutral flux at the anode plane deduced from the gas mass flow rate.

Since the plasma is supposed to be quasi-neutral, the electric field is not obtained from Poisson's equation. The electric field is deduced from the electron momentum equation (with only the drift term) and from the current continuity.
\[ E(x,t) = -\frac{\varphi(x,t)}{n_p(x,t)\mu_e(x,t)} + \frac{J_T(t)}{n_p(x,t)e\mu_e(x,t)} \]  

(8)

The ion flux comes from the eq. (1), the relation:
\[ V_d = -\int_0^d E(x,t)dx \]

(9)
gives, with eq. (8), the total current.

The electron mobility is supposed to be the classical mobility in a transverse magnetic field:
\[ \mu_e(x,t) = \frac{e}{m_e} \frac{v_m(x,t)}{v_m^2(x,t) + \omega_B^2(x)} \]

(10)

When the electrons ionize the neutrals in the region near the exhaust of the thruster, the neutral density decreases to zero. The momentum exchange frequency decreases to zero and so does the electron conductivity. We do not obtain a stable solution for the eq. (8) if no other assumption is made on the electron transport.

In order to make possible the electron transport in the direction perpendicular to the magnetic field even when the neutral density goes to zero, we added to the electron collision frequency a contribution due to collisions with the walls. This "near wall conductivity" was suggested by the Russian school\(^6\) to explain the "anomalous" electron transport in the SPT. We account for electron-wall collisions in the simulation by introducing a supplementary constant collision frequency\(^5\).

The electron mean energy is obtained from the following simplified energy equation:
\[ \frac{5}{3} \frac{dE_e(x,t)}{dx} = -eE(x,t) + \frac{v_e(x,t)}{v_e(x,t)} - E_e(x,t) \]

(11)

with an energy gain term due to the electric field and a loss term due to electron-atom collisions and to the electron secondary emission by electron impact\(^5\).

We discuss below the numerical techniques and the data we used to solve this set of equations.

Data and Numerical Techniques

We need the excitation, ionization and momentum exchange cross section by electron impact on xenon and krypton atoms. The inelastic cross sections (excitation, ionization) are included in the electron energy loss frequency of eq. (11). The ionization cross sections are used to estimate the ion production rate of eq. (2) and the loss term of neutral atoms in eq. (6). The momentum exchange cross section is used in the electron mobility (eq. (10)). The inelastic cross sections come from the paper of Puech and Mizzi\(^9\) for and from data of Date et al.\(^10\) for electron impact on krypton atom. We have considered only excitation and ionization from the ground state. Ionization and energy loss rates are obtained by assuming that the electron distribution function is Maxwellian. A comparison of the ionization and energy loss rates \(k_i\) and \(k_e\) in xenon and krypton is given in Fig. 1.

![Fig. 1: Ionization and Energy Loss Rates calculated with a Maxwellian electron distribution function in xenon and krypton.](image)

The momentum exchange rate \(k_m\) is estimated to be respectively 2.5\times10^{-14} and 2.0\times10^{-13} m^3 s^{-1} for xenon and krypton. It is difficult to estimate the electron-wall momentum interaction frequency \(v_{wall,m}\). We adjusted this frequency to obtain a calculated current in reasonable agreement with the experimental results\(^11\). In the results presented in the fourth part, the electron-wall frequency is constant in all the channel and has a value of 2\times10^3 s^{-1}.

Eq. (11) is solved using a classical 4th order Runge-Kutta method\(^12\). We use an upwind scheme to solve eq. (6)\(^12\). In the model of Ref. [5], eq. (1) was solved with an upwind scheme and a time-splitting method. The upwind scheme is a simple method but rather dissipative\(^14\). In the calculations presented here, we used a Monte Carlo Particle method to describe the collisionless ion transport.

The channel is divided into cells. As in the PIC technique\(^15\), we consider macroparticles, where each macroparticle represents a large number of ions (more than 10^3). The trajectory of a test particle number \(l\) is integrated using Newton's law:
\[ \frac{dv_i}{dt} = v_i \]
\[ M \frac{dv_i}{dt} = -eE \]

(12)

The classic leap-frog scheme is used for the resolution of these equations\(^16\).

At each time step \(\Delta t\) of the ion motion, some macroparticles are created in each grid cell depending on the ion production rate \(S_i\). The number of test particles created in each cell is determined as follows:
\[ N_{i} = \frac{S_i}{S_{\text{max}}} N_{x} + N_{m} \]

(13)

The first term is directly proportional to the production rate. The difficulty comes from the difference between the maximal and the minimal production rate.
(a factor of $10^4$ or more). The second term permits to maintain a minimum number of particles in each cell and to avoid null ion density in a cell. A weight is affected to the macro-particles $l$ proportionally to the ion production rate $S_l$, and inversely proportional to the number of simulated ions created in the cell $N_l$:

$$w_l = \frac{S_l \Delta t}{N_l} \quad (14)$$

Random numbers are used to distribute the test particles uniformly in the cell. The velocity of the macro-particles created by ionization is $v_{il}$.

The ion density is calculated on the nodes of the grid by a simple linear weighting scheme. In order to limit the total number of ions in the simulation we use the following procedure. The $N_l$ ions are created during a time interval $\Delta t$ which does not necessarily coincide with the integration time step of the ion trajectories but can be larger. When the total number of simulated particles is larger than a given limit, the $\Delta t$ for ion generation is chosen large enough so that the total number of simulated ions leaving the device during this time interval is slightly larger than the total number of simulated ions generated by ionization. This avoids the random elimination of low weight macro-particles presented in this paper, typically 20,000 test particles are used. The computational time is 2 hours to simulate 600 $\mu$s (vs. 15 $\mu$s when the Vlasov equation is solved with a finite difference method instead of a particle method).

Comments on the boundary conditions

As mentioned in Ref. [5] the results of the model are dependant on the boundary conditions for the ion density at the anode (actually we impose a non zero ion flux at the anode, and an ion velocity equal to $v_{i0}$). In the results presented in these paper we impose an ion density of $10^{13}$ cm$^{-3}$ next to the anode. If a much lower ion density at the anode is used a large anode sheath electric field develops in the anode region and the model does not give realistic results. This problem is partly due to the quasineutrality assumption and to the fact that the calculated electric field (see eq. (8)) goes to infinity when the plasma density goes to zero. Work is underway to improve the model description of the anode region.

Hall Thruster Performance

The thrust, the specific impulse and the total efficiency characterize the performance of the SPT. The thrust is the product of the ion mass flow rate at the thruster exhaust by the ion mean velocity at exhaust. If we express the ion mass flow rate as a function of the propellant mass flow rate at the anode, we obtain:

$$T = \dot{m}_i V_{th} = \dot{m} \eta_T V_{th}$$

The specific impulse is deduced from the thrust:

$$I_{sp} = \frac{T}{\dot{m} g}$$

and the total efficiency of the thruster, with $P_d = J_d V_d$, can be written as:

$$\eta = \frac{T^2}{2 \dot{m} P_d}$$

We can also describe the global efficiency with the expression below:

$$\eta = \eta_a \eta_d \eta_E T^2$$

where the propellant efficiency is:

$$\eta_a = \frac{J_i}{J_{i0}}$$

The neutral equivalent current is $J_{i0} = \epsilon \Phi_{i0} A$.

The acceleration efficiency is given by:

$$\eta_d = \frac{J_d}{J_{i0}}$$

and the beam energy efficiency:

$$\eta_E = \frac{V_{th}}{V_d}$$

The thrust loss factor $\gamma$ includes exhaust beam divergence and profile losses.

Results and Discussion

We performed calculations to study the effects of the nature of the gas (xenon vs krypton), of the applied voltage, and of the mass flow rate on the performance of a Hall thruster. The simulated device is the SPT100-ML. This SPT100 Hall Thruster has been tested in the French facility PIVOINE. For more details on the design of this thruster and on the PIVOINE facility, the reader can refer to the paper by S. Béchu et al.

The axial profile of the radial magnetic field we use in the code for a coil current of 4.5 A is shown in Fig. 2.

![Fig. 2: Axial distribution of the radial magnetic field used for the simulations. The anode is at x=0, and the exhaust at x=d.](image-url)
The neutral atoms are injected with a velocity of \(3 \times 10^4\) cm/s in the case of xenon (\(3.76 \times 10^4\) cm/s in the case of krypton, in order to keep the same thermal energy of the emitted atoms).

The SPT produces spontaneous oscillations of the current\(^6\). The performance of the thruster also oscillates in time and all the results presented in this paper are averaged in time.

**Calculations for xenon**

We present in Table 1 the efficiencies for applied voltage in the range [150-400 V] for a xenon anode mass flow rate of 5 mg/s.

As shown in Table 1, the thrust efficiency increases with the applied voltage, with a peak for a voltage of 300 V. In the 1D model, neither the ion beam divergence nor the ions losses to the wall are taken into account and the \(\gamma\) factor is therefore set to 1 (see eq. (19)). Most of the ions are created in the region just before the acceleration region at the end of the thruster. In the acceleration zone where the magnetic field is large, the electric field must increase to compensate for the low electron mobility. The ions are accelerated by this electric field toward the exhaust\(^7\).

It is clear that, in order to minimize ion recombination on the walls, the acceleration channel should be short enough. This aspect can however not be properly described by our model since ion losses to the walls are not considered. The ion loss on the wall chamber in a SPT is relatively low compared with the ion losses on the grid of an ion engine thruster\(^9\).

<table>
<thead>
<tr>
<th>(V_d (V))</th>
<th>(\eta_d(%))</th>
<th>(\eta_a(%))</th>
<th>(\eta_E(%))</th>
<th>(\eta(%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>44.6</td>
<td>95.6</td>
<td>53.9</td>
<td>23.0</td>
</tr>
<tr>
<td>200</td>
<td>69.3</td>
<td>96.0</td>
<td>49.5</td>
<td>32.9</td>
</tr>
<tr>
<td>250</td>
<td>82.4</td>
<td>95.8</td>
<td>48.8</td>
<td>38.5</td>
</tr>
<tr>
<td>300</td>
<td>88.4</td>
<td>95.3</td>
<td>46.1</td>
<td>38.8</td>
</tr>
<tr>
<td>350</td>
<td>92.2</td>
<td>94.6</td>
<td>44.3</td>
<td>38.6</td>
</tr>
<tr>
<td>400</td>
<td>93.1</td>
<td>94.0</td>
<td>41.0</td>
<td>35.9</td>
</tr>
</tbody>
</table>

**Table 1 : Calculated efficiency of the SPT100-ML for 5 mg/s of xenon and for different discharge voltages.**

We can note that the acceleration and beam energy efficiencies are relatively insensitive to the applied voltage. The total efficiency depends strongly on the propellant efficiency. When the voltage is low, the ionization of the neutral flux is incomplete. For large values of the applied voltage the fraction of the neutral flux which is ionized by electron can be as large as 93% in the calculations (see table 1 where \(\eta_d\) is 93.1% for a channel voltage of 400 V). The strong influence of the propellant efficiency on the total efficiency has been experimentally observed on a Russian and a Japanese Hall Thruster\(^{10, 21}\).

**Fig. 3 : Total Efficiency as a function of discharge voltage. The xenon mass flow rate is 5 mg/s. Comparisons between measurements\(^{10}\) and calculations. The results from the hybrid model using a Finite Difference Method (FDM) and Particle Method (PM) for the ion Vlasov equation are shown.**

Figure 3 shows a comparison between calculations and experimental results. Note that no adjustment of the magnetic field, or others external parameters was made to optimize the global efficiency of the SPT100-ML in the experiment of Fig. 3. One of the reasons for the lower calculated efficiency could be due to the neglect of the doubly charged ions in the simulation. We have also neglected the dissipated power in the magnetic coils and in the heating of the hollow cathode. This power dissipation could be non negligible at low voltages. Figure 3 also shows the results from the same hybrid model, but for a finite difference method of solution of the ion Vlasov equation instead of a particle. The difference between the two models are only due to inaccuracies in the numerical method (the finite difference solution is less accurate). All the other simulations results in this paper correspond to the particle method for the description of ion transport.

The thrust efficiency is maximum for a channel voltage of 300 V with a value of 50 % in the experiments and typically 40 % in the simulation. For larger voltages, the engine efficiency decreases due to the decrease of \(\eta_E\).

**Fig. 4 shows the variations of the thrust for discharge voltages between 150 and 400 V. As expected, the thrust increases with the applied voltage. Results obtained by the model under-predict the thrust of the SPT100-ML (typically 20 mN). We have neglected in our model ionization mechanisms other than direct ionization. For low voltages, the ionization of excited states could have a non negligible effect on the ion current and on the propellant efficiency. For high voltage, the doubly charged ions could affect the propellant efficiency.**
SPT’s are attractive because they are efficient within the optimum range of specific impulse for station-keeping of satellites (1000-2000 s)\(^9\). We obtain a specific impulse of 1510 s for an applied voltage of 350 V in the simulation, and 1700 s is obtained in the experiments.

Finally, Fig. 5 shows the total thrust efficiency as a function of the thrust. The maximum calculated efficiency is about 10% less than the measured one, and is obtained for a thrust around 60 mN, i.e. 20 mN less than the thrust at maximum efficiency in the experiment. For high thrust, the global efficiency decreases, as observed in other SPT’s\(^8\).

We also studied the influence of the mass flow rate on the thruster performance, for a constant channel voltage of 300 V (see Table 2). The engine efficiency increases with the mass flow rate as expected. The acceleration efficiency is not very sensitive to the increase of the mass flow rate of xenon.

Figure 6 represents the thrust and the input power for a xenon mass flow rate between 3.0 and 7.0 mg/s. We can see a quasi-linear variation of the thrust with the mass flow rate. This linear behavior of the thrust as a function of the mass flow rate has been observed experimentally for different Hall Thrusters (small SPT\(^2\) and for a Laboratory Hall Thruster studied in Israel\(^3\) for example).

![Discharge Power and Thrust as a function of mass flow rate for 300 V.](image)

For \(m = 7\) mg/s, the thrust reaches 100 mN, but we also see that the discharge power is rather large, on the order of 1.6 kW.

**Calculations for krypton**

Table 3 shows the performance of the SPT100-ML with a 5 mg/s flow rate of krypton and can be compared with Table 1 corresponding to xenon. The total efficiency is also represented as a function of voltage in Fig. 7 for krypton.

<table>
<thead>
<tr>
<th>(V_{th} (V))</th>
<th>(\eta_a(%))</th>
<th>(\eta_i(%))</th>
<th>(\eta_E(%))</th>
<th>(\eta(%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>31.9</td>
<td>96.3</td>
<td>47.4</td>
<td>19.7</td>
</tr>
<tr>
<td>250</td>
<td>50.8</td>
<td>96.7</td>
<td>45.5</td>
<td>27.6</td>
</tr>
<tr>
<td>300</td>
<td>62.7</td>
<td>96.7</td>
<td>41.6</td>
<td>29.8</td>
</tr>
<tr>
<td>350</td>
<td>72.0</td>
<td>96.4</td>
<td>39.4</td>
<td>30.7</td>
</tr>
<tr>
<td>400</td>
<td>75.3</td>
<td>96.1</td>
<td>35.9</td>
<td>28.1</td>
</tr>
</tbody>
</table>

**Table 3 : Calculated efficiencies for 5 mg/s of krypton.**

As in the xenon case, the voltage influences strongly the propellant efficiency \(\eta_a\). The global efficiency increases with the voltage. The maximum efficiency obtained with krypton is about 30% (compared with 40% for xenon). The higher efficiency of xenon is related to its larger ionization rate. Brophy et al.\(^9\).
obtained a global efficiency of 38% for a SPT-100 with a voltage of 300 V and a flow rate of 4.23 mg/s. Experimental results of Komurasaki and Arakawa\textsuperscript{21} show that the propellant efficiency of xenon is about two times larger than that of argon.

Fig. 7: Total efficiency as a function of discharge voltage in the SPT100-ML in krypton. Comparisons between measurements\textsuperscript{20} and calculations. The krypton mass flow rate is 5 mg/s.

The variations of the thrust with voltage displayed in Fig. 8 show that krypton and xenon exhibit similar behavior. Brophy et al.\textsuperscript{19} give similar (experimental) results for a SPT100 in krypton at 300 V, 4.23 mg/s: 65 mN thrust and 1580 s specific impulse. It is also interesting to note that, in the range of voltage studied, the thrust does not decrease at high voltage. This is due to the fact that the propellant efficiency is still increasing with the voltage in the considered range (see Table 3).

Fig. 8: Thrust as a function of discharge voltage. Comparisons between measurements\textsuperscript{20} and calculations. The krypton mass flow rate is 5 mg/s.

The influence of the anode mass flow on the calculated performances of the SPT100-ML in krypton can be seen in Table 4. The propellant and engine efficiencies in krypton are smaller than in xenon as expected. The thrust increases linearly with the mass flow rate, with a value of 24 mN for $m = 3$ mg/s up to 102 mN for 7 mg/s. For $m = 7$ mg/s, the discharge power is larger than in xenon (2.1 kW vs 1.6 kW for xenon).

<table>
<thead>
<tr>
<th>$m$ (mg/s)</th>
<th>$\eta_p$ (%)</th>
<th>$\eta_p^d$ (%)</th>
<th>$\eta_p^e$ (%)</th>
<th>$\eta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>46.1</td>
<td>96.6</td>
<td>42.9</td>
<td>19.1</td>
</tr>
<tr>
<td>4.0</td>
<td>62.7</td>
<td>96.8</td>
<td>39.9</td>
<td>24.2</td>
</tr>
<tr>
<td>5.0</td>
<td>62.7</td>
<td>96.7</td>
<td>41.6</td>
<td>29.8</td>
</tr>
<tr>
<td>6.0</td>
<td>81.9</td>
<td>96.6</td>
<td>41.4</td>
<td>32.8</td>
</tr>
<tr>
<td>7.0</td>
<td>85.5</td>
<td>96.5</td>
<td>42.3</td>
<td>34.9</td>
</tr>
</tbody>
</table>

Table 4: Efficiencies obtained with the 1D hybrid model for krypton as a function of mass flow rate for a discharge voltage of 300 V.

Comments on the role of doubly charge ions

We tried to include doubly charged ions in the simulations. This group of ions was treated with a Monte Carlo simulation using a creation source term corresponding to direct electron impact double ionization from the ground state of xenon atoms. The results (not presented in this paper) showed that the flux of doubly charged ions at exhaust was about 5% of the total current. The influence of these ions on the performance was not significant. The results of the simulations and the cross-section data show however that the contribution of stepwise ionization (electron impact ionization of $\text{Xe}^+$ leading to $\text{Xe}^{2+}$), which was not included in this preliminary study, is probably very important. This process is probably dominant especially in the exhaust region where the electron energy is large and where the $\text{Xe}^+$ density may be as large as the neutral atom density. The contribution of this process will be studied in future work.

Conclusions

We have used a 1D hybrid model to calculate the performance of a SPT100 and its variations as a function of applied voltage and gas mass flow rate in xenon and krypton. The model predicts correctly the trends. The quantitative agreement is reasonably good in spite of the simplicity of the model. On the other hand we have shown elsewhere\textsuperscript{21} that this model can reproduce a number of qualitative properties of the SPT such as the existence of different regimes of oscillations.

The model predicts a thrust of around 65 mN in typical conditions (350 V, 5 mg/s) for a SPT100 in xenon, a value about 20 mN lower than the measured one. The calculated thrust in krypton is on the order of 70 mN in the same conditions. The calculated efficiencies are on the order of 40% and 30% in xenon and krypton respectively, in reasonable agreement with the measured values (which are about 10% higher).
Acknowledgements

This work was performed in the frame of the Groupe de Recherche CNRS/CNES/SEP/ONERA no 1184 “Propulsion à Plasma pour Systèmes Orbitaux”. The authors want to thank the Laboratoire d’Aéronique, CNRS-UPR9020 in Orléans for sharing the experimental data.

References


