PHYSIC-MATHEMATICS MODEL OF A SELF-HEATED HOLLOW CATHODE ELECTRIC DISCHARGE EVOLUTION

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Abstract

It is well known that one from most radical method of any electric propulsion system dynamic characteristics improvement is the self-heated hollow cathode (SHC) application. This device has minimum time of ignition, and start signal is escorted by almost instant (up to 100 ns) appearance of a discharge current.

It is obviously that reliable start is one from most important attributes for SHC. In particularly this is first stage – initialization, development and transition to quasi-stationary regime of electric parameters. This paper consecrates to researching of this problem.

Physic-mathematics model

The scheme of a cathode experimental model, which now is tested, has shown in fig. 1 and fig. 2. Main assumptions are:
- discharge plasma consists from electrons, ions and neutral atoms;
- external ionization is not equal for all discharge length;
- length of a electrons and ions free path is lesser than electric field changing length in near-electrode area:

\[ \lambda_{+,-} < < E \left( \frac{\partial E}{\partial x} \right) \]  

(1)

Taking account only volume stage and without questions of stability, we could describe main processes of a discharge evolution with help of Poisson and continuity equation for electric field. For 1d case we have:

\[ \frac{\partial n_+}{\partial t} + \frac{\partial (n_+ v_+)}{\partial x} = \alpha n_+ - \beta n_+ n_+ + Qe; \]

\[ \frac{\partial n_-}{\partial t} - \frac{\partial (n_- v_-)}{\partial x} = \alpha n_- - \beta n_- n_+ + Qe; \]

\[ \frac{\partial E}{\partial t} = \frac{1}{2e} (q_- - q_+); \]

\[ v_- = \mu_- E; \]

\[ v_+ = \mu_+ E; \]

\[ \int E(x) dx = U(t). \]

(2)

It is necessary to add border conditions for these equations:

\[ n_-(0,t)v_-(0,t) = n_+(0,t)v_+(0,t), \]

\[ n_+(d,t)v_+(d,t) \]

(3)

In the system (2) and border conditions (3) we have made some designation:
- \( n_-, n_+ \) - the electrons and ions concentrations,
- \( v_-, v_+ \), \( \mu_-\mu_+ \) - the electrons and ions drift velocity and mobility, \( E \) – electric field, \( U \) – the voltage between electrodes, \( d \) – the distance between electrodes, \( Q \) – gas ionization velocity, \( \gamma \) - the coefficient of secondary emission electrons from a cathode, \( \alpha, \beta \) - coefficients of collision ionization and dissociation recombination, \( \varepsilon \) - the constant, \( e \) – the electron charge. In the system (2) were not taking into account the electric particular diffusion. Ionization and recombination defines volume bereavement of electrons and ions. If we add in to system (2) the Ohm’s equation for full circuit:

\[ J = \frac{\varepsilon}{R_{in} + r_{ext}} \]

(4)

we have closed equations system, that describes very first stage of a discharge evolution.

Fig.1 Photo of the SHC.

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Fig. 2 The cathode experimental model.
IB – the ignition block;
EPSS – the electric power supply system.

**Main simplicities and approximations**

It is known that in a discharge distribution of electric field intensity \( \vec{E} \) is very non-similarity in a arc length. Usually the \( \vec{E} \) intensifies to thin near-electrodes layers and in a discharge column \( \vec{E} \) is almost constant (fig. 3a).

This picture is right but only for simply cases. Researches of SHC that has operated in working regimes shown significant differences from simple case (fig. 3b).

So the diaphragm is defining significant different pressure of propellant gas areas: form emitter up to diaphragm is almost constant and sharp transition with supersonic leap on diaphragm and than free molecular flow of plasma in to environment.

![Diagram](image)

**fig. 3 Distribution of \( \vec{E} \) in a arc length.**

a) simple case of a glow discharge; b) in SHC.

So distribution of gas propellant pressure we can approximate by expression like this:
\begin{align*}
P &= \begin{cases} 
  p_0, & 0 < x < l_\theta \\
  p_0 \cdot f(\text{diaphragm geometry}), & l_\theta \leq x < l_\theta' \\
  p_0' \cdot f(P_0 \cdot f(\text{diaphragm geometry})), & l_\theta' < x \leq x_k
\end{cases}
\end{align*}

The gas propellant pressure is a secondary parameter. First definition parameter — is a concentration of gas propellant, which is defined from temperature \( P = nkT \). That is why summary equation would be more difficult which would need some experimental data about gas temperature distribution.

Next aspect of a model is approximation of first initializing impulse. It is obviously that electrons and ions generation in a discharge, which is initialized, is time function, and main model of breakdown in this case is a streamer model. Even estimation calculation of a electrons generation velocity in this approximation is very difficult. Because of that we consider all energy of first

**The results of computation**

Results of a one computation variant for the system (2)-(4) are shown in the fig. 5. In this case conditions discharge is to be for gas propellant ionization, and real nature of electrons generation velocity was changed by rectangular function (fig. 4).

```
<table>
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<td>Real distribution</td>
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<td>( \tau )</td>
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**Fig. 4 Approximation dependence and real distribution of electrons generation velocity.**

of a SHC transition to the quasi-stationary regime of current are reviewed. High coefficient of transvoltage, term and power of a IB impulse allows this.

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<td>10²</td>
</tr>
<tr>
<td>10⁰</td>
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<tr>
<td>10⁻²</td>
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| \( \tau, \text{ ns} \) |
| 100 |
| 200 |
| 300 |
```

**Fig. 5 Experimental data of a discharge current transition.**

**The experimental data**

Dynamic measurement of a discharge evolution were carried out by high-working device, that were set by 2 of 10-digit ADC (analog to-digit converter) with frequency 60 MHz, which parallel loaded to 2 RAM 128 kb each. Block of conjugation allows galvanic isolation of measurement block from a computer up to 2 kV. The results of a discharge ignition measurement have shown in Fig. 6. Quality correspondence of theoretical and experimental data in a time range from start up to 300 ns is obviously.

Further nature of a discharge evolution is more difficult. In the fig.7 You can see that a voltage and current significant oscillation escorts the discharge evolution. Approximately on 8 millisecond for the high voltage form requires for arcing supporting. In this case significant influence to a EPS thrust characteristics is possible. More difficult picture is existed in a discharge evolution of time range up to 3 second (fig.8). Beside of high-voltage (up to 250 V) form of a discharge the existence of arcing transition regimes (up to 100 V) is obviously. Influence of electric power system in also obvious.
Fig 6 results of a discharge ignition measurement.

fig. 7 Further nature of a discharge evolution. a)
Fig. 7 Further nature of a discharge evolution. a)

Fig. 7 Further nature of a discharge evolution. b)
Fig. 8 Discharge evolution up to 3 s.

Conclusion

In the paper results of theoretical and experimental data of first discharge evolution stage – ignition. Comparison of it is presented. Possible calculation errors are shown.