One-Dimensional Numerical Simulation of Two-Fluid
Nonequilibrium Plasma Flow in MPD Thrusters

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Abstract

A numerical analysis of one-dimensional flow in an MPD thruster has been done, including nonequilibrium ionization/recombination processes of the propellant gas and velocity slip between charged and neutral particles. The calculated results are noticeably different from that of one-fluid model. The velocity of charged particles near the inlet is much higher than the mean velocity calculated by one-fluid model, and the inlet electron temperature of two-fluid model is much lower than that of one-fluid model. As a result, more realistic current distribution is obtained.

Introduction

Self-field MPD (magnetoplasmadynamic) thrusters are promising as a future propulsion system from a viewpoint of high specific impulse. Experiments on MPD thrusters have been performed by numerous workers [1–3]. The existing experiments explain that the current distribution in a discharge chamber is closely related to the thruster performance. In order to examine the distribution and thrust performance, numerical simulations have been done by assuming one-fluid plasma [2–8]. The calculated results for Ar show that current distributions have peaks near inlet and outlet. However, small currents are measured near the inlet in the experiments [1].

For the purpose of analyzing more realistic flows, the studies based on two-fluid models has been performed [9,10].

In this paper, numerical analyses of a two-fluid plasma flow are done, including nonequilibrium ionization/recombination processes of the propellant gas and velocity slip between charged particles and neutral particles. In this two-fluid model, ions and neutrals have different flow velocities because in an MPD thruster ions are pushed by the Lorentz force and neutrals are pushed by collisions with ions. Flow velocity of ions has special effect on current density profile, so that thrust performance of MPD is influenced by ions velocity. When mass flow rate of propellant gas becomes lower, or total discharge current becomes higher, the difference between the two flow velocity would be larger, thus the two-fluid model is important in those conditions.

In this model, the propellant gas is Argon and a heat conduction model is included. In numerical analysis of one-dimensional plasma flow in MPD, the sonic condition must be applied. Since there are two flows of ions and neutrals in this model, two sonic conditions exist. In this code the calculation of the flowfield and the electromagnetic field are performed independently. The flowfield is calculated with a 2nd-order-accurate explicit MUSCL TVD scheme, whereas the corresponding electromagnetic field is obtained by solving steady electromagnetic solutions.

Assumptions

For the purpose of simplification, the following assumptions are introduced into the present analysis:

1) The propellant gas is Argon.
2) Only the single-ionization and recombination processes are considered:

\[ \text{Ar} + e^- = \text{Ar}^+ + e^- + e^- \]  \hspace{1cm} (1)

3) The plasma is a perfect gas having constant specific heats.
4) The magnetic field is entirely self-induced.
5) The plasma flows one-dimensionally.
6) The charged and neutral particles velocities are different.
7) The plasma has three different temperatures, electron, ion and neutral temperatures.
8) Only the heat conduction is considered as transport phenomena.

Fundamental Equations

Fundamental equations of flowfield

The analysis is based on the one-dimensional two-fluid equations that are written in a conservative form as
Fig. 1. Two-fluid model in MPD thrusters.

Fig. 2. Thruster configuration.

Fig. 3. Cross section of quasi-steady MPD arc channel MC-II used for referred experiment$^3$.

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S} + \mathbf{H},
\]

\[
\mathbf{U} = \begin{bmatrix}
\rho_n \\
\rho_i \\
\rho_n u_n \\
\rho_i u_i \\
e_n \\
e_i
\end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix}
\rho_n u_n \\
\rho_i u_i \\
\rho_n u_n^2 + p_n \\
\rho_i u_i^2 + p_i + p_e \\
(p_n + e_n) u_n \\
(p_i + e_i) u_i
\end{bmatrix},
\]

\[
\mathbf{S} = \begin{bmatrix}
-\dot{\rho}_n \\
-\dot{\rho}_i \\
-\dot{m}_n \\
jB + \dot{m}_i \\
-\dot{e}_i \\
jB u_i + \dot{e}_i \\
\frac{i_0}{\sigma} - \dot{e}_i \dot{n}_i
\end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix}
0 \\
0 \\
-\frac{F_{in}}{\gamma} \\
-\frac{F_{in}}{\gamma} \\
-\frac{F_{in}}{\gamma} \\
-\frac{F_{in}}{\gamma} \\
\end{bmatrix},
\]

where

\[
e_n = \frac{1}{2} \rho_n u_n^2 + \frac{p_n}{\gamma - 1}, \quad e_i = \frac{1}{2} \rho_i u_i^2 + \frac{p_i}{\gamma - 1}, \quad e_e = \frac{p_e}{\gamma - 1}.
\]

In the equations, the subscript $n$ represents the neutral, $i$ the ions, $e$ the electrons respectively. The pressure of $j$-species $p_j$ is given as

\[p_j = \rho_j R_j T_j.\]

The values in the vectors $\mathbf{S}$ and $\mathbf{H}$ are given as follows. The rate of production $\dot{\rho}_i$ is given by the difference between ionization and recombination as

\[\dot{\rho}_i = m_n \dot{n}_n - m_e \dot{n}_e\]

where

\[\dot{n}_i = k_i n_n n_e, \quad \dot{n}_e = k_e n_n^2.\]

The rate of coefficient for ionization reaction $k_i$ is conventionally expressed in the form:

\[k_i = \frac{76.8}{T_e^2 \exp\left(\frac{-\varepsilon}{k T_e}\right)},\]

while the rate coefficient for recombination reaction $k_e$ is given as

\[k_e = 4.0 \times 10^{-21} T_e^{-\frac{3}{2}}.\]

The rate of ion momentum production by ion production $m_i$ is given as

\[m_i = m_n u_i \dot{n}_n - m_e u_i \dot{n}_e\]

and the rate of ion energy production $\dot{e}_i$ is given as

\[\dot{e}_i = \frac{e_e}{n_e} \dot{n}_e - \frac{e_e}{n_n} \dot{n}_n.\]

The momentum transfer $F_{in}$ is given by

\[F_{in} = \frac{1}{2} m_n (u_i - u_n) n_n \nu_n.\]

The energy transfer from ions to neutral particles $F_{in} u$ is given by

\[F_{in} u = \frac{1}{2} m_n (u_i - u_n) n_n \nu_n u\]

where $u$ is the mean velocity defined as

\[u = \frac{\rho_n u_n + \rho_i u_i}{\rho_n + \rho_i}.\]
The energy interaction between \( j \)-species and \( k \)-species \( Q_{nk} \) is given by
\[
Q_{nk} = \frac{3m_j}{m_k} n_k k(T_j - T_k) u_{nk}.
\]  
(16)

**Fundamental equations of electromagnetic field**

The equations describing the electromagnetic field are written as
\[
j = \sigma (E - u_i B),
\]  
(17)
\[
\frac{dB}{dz} = -\mu_0 j.
\]  
(18)

**Numerical Analysis**

**Numerical Procedure**

Time-integration of the flowfield is done by using a 2nd-order accurate explicit MUSCL TVD-upwind scheme\(^\text{12,13}\), whereas the electromagnetic field is given by the steady electromagnetic equations. The time-integration proceeds until the time-dependent flowfield calculation converges to a steady solution. The calculation sequence of plasma flow is shown in Fig. 4.

**Inlet Conditions**

The inlet physical properties needed for the calculation are densities \( \rho \), velocities \( u \), temperatures \( T \), and electric field \( E \). The conditions for determination of these eight properties are:
1) Given discharge current \( J \),
2) Given mass flow rate \( \dot{m} \),
3) Subsonic character,
4) Sonic condition for charged particles,
5) Sonic condition for neutral particles,
6) Fixed stagnation temperature \( T_0 = 300 \text{ K} \),
7) Fixed inlet temperature of neutral \( T_\infty = 300 \text{ K} \),
8) Fixed inlet temperature of ions \( T_i = 300 \text{ K} \).

At the inlet boundary, the flow is assumed as subsonic and there are two sonic conditions of charged and neutral particles downstream as discussed in next section. Therefore, three of properties should be determined from downstream. In the present work, inlet velocities \( u_i \), \( u_n \), and ion temperature \( T_i \) are determined from downstream by the 1st-order extrapolation. Then we alter inlet ionization fraction \( \alpha \) until \( T_i \) coincides 300 K. Using this \( \alpha, u_n \), and \( u_i, \rho_i \), and \( \rho \) are obtained for a given mass flow rate. \( T_i \) is determined from the condition that stagnation temperature \( T_0 = 300 \text{ K} \) using the following equation,
\[
\left[ \frac{\gamma - 1}{\gamma - 1} m_e (\rho_n T_n + \rho_i T_i + \rho T_e) + \epsilon n_i \right] + \frac{1}{2} \rho u_i^2 + \frac{1}{2} \rho_i u_i^2) / (\rho_n u_n + \rho_i u_i) = \frac{\gamma k}{\gamma - 1} m_e T_0.
\]  
(19)

The only property of electromagnetic field \( E \) is determined from given discharge current \( J \).

**Sonic Conditions**

Because the two-fluid model is considered in this work, there are two different sonic conditions for charged particles and neutral particles\(^\text{12-13}\). From the conservation equations of one-dimensional flowfield,
\[
\frac{du}{dz} = X
\]  
(20)
\[
\frac{d}{dz} (\rho u^2 + p) = Y
\]  
(21)
\[
\frac{d}{dz} \left( \frac{1}{2} \rho u^3 + \frac{\gamma}{\gamma - 1} \rho u_1 \right) = Z
\]  
(22)

the velocity gradient is obtained as
\[
\frac{du}{dz} = -\frac{\gamma - 1}{M^2 - \frac{1}{a^2}} (Z - \frac{\gamma}{\gamma - 1} u Y + \frac{\gamma + 1}{\gamma - 1} \frac{u^2}{2} X),
\]  
(23)
where
\[
M \equiv \frac{u}{a}, \quad a \equiv \frac{\gamma}{\rho}.
\]  
(24)

In order to get continuous solution at a sonic point \( M = 1 \) in Eq.(23), the denominator on the right hand of Eq.(23) should be equal to zero. Therefore, the sonic conditions for charged particles and neutral particles are given as follows.
1) Sonic condition for charged particles:
At \( M_i = 1 \), we have

\[
E = \frac{r-1}{r-1} \frac{u_i}{j} (jB - F_{in} + \dot{m}_i) \\
+ \frac{1}{2} (F_{in} + Q_{en} + Q_{in} + \varepsilon \dot{n}_i - \dot{e}) \\
- (\gamma + 1) \dot{\rho} u_i^2 \\
2(\gamma - 1) j.
\]

(25)

2) Sonic condition for neutral particles
At \( M_n = 1 \), we have

\[
\frac{1}{2} \left( \frac{r+1}{r-1} \dot{\rho} u_n^2 + \frac{r}{r-1} u_n (F_{in} - \dot{m}_n) \\
- (F_{in} u + Q_{en} + Q_{in} - \dot{e}) \right) = 0,
\]

(26)

where

\[
M_i \equiv \frac{u_i}{a_i},
\]

(27)

\[
M_n \equiv \frac{u_n}{a_n},
\]

(28)

\[
a_i^2 \equiv \frac{\gamma P_i}{\rho_i},
\]

(29)

\[
a_n^2 \equiv \frac{\gamma P_n}{\rho_n},
\]

(30)

Downstream Conditions
At the downstream boundary, each property is extrapolated from the adjacent inner point, due to its supersonic character.

Results

Numerical Results are given for a fixed \( J = 8.1 \) kA and different \( \dot{m} \). These parameters along with the geometric parameters are given in Fig. 2. The parameters are chosen to be as close as possible to the referred experiment\(^3\) (see Fig. 3).

Analyses of flowfield

Calculated properties are shown in Figs. 5-10. For the purpose of comparison between the one-fluid and two-fluid models, calculated properties of one-fluid model are also shown in Figs. 5, 8-10. Figure 5 shows the distributions of flow velocities of charged and neutral particles for \( \dot{m} = \) (a) 0.481 g/s and (b) 0.368 g/s. The velocity of charged particles increases rapidly by Lorentz force near the inlet where the ionization fraction is low (see Fig. 6). Thus, the velocity is much higher than the velocity of neutral particles and mean velocity of one-fluid model in this region. The velocity difference between the charged and neutral particles remains to the downstream end. Figure 5 also explains that the difference becomes stronger for a lower mass flow rate.

Fig. 5. Distribution of velocities of two-fluid model and mean velocity of one-fluid model for \( \dot{m} = \) (a) 0.481 g/s and (b) 0.368 g/s.

Figure 7 shows the distribution of temperatures of electrons, ions, and neutral particles for \( \dot{m} = 0.481 \) g/s. There is a noticeable difference among these three temperatures. The temperatures of ions and electrons increases rapidly downward from the inlet. Figures 8 and 9 shows the temperature of heavy particles and that of electrons respectively. The temperatures of heavy particles of two-fluid and one-fluid models differ strongly in Fig. 8. In contrast, there is not clear difference between the temperatures of electrons of two-fluid and one-fluid models except the inlet values in Fig. 9. The inlet temperature of electrons of two-fluid model is much lower than that of one-fluid.

As a result of introduction of two-fluid model, the acceleration of charged particles becomes strong near the inlet and the inlet temperature of electrons decreases remarkably. Because the current distribution depends strongly on the velocity...
Comparison of current fraction with experimental results

Figure 10 shows the comparison between the current densities of two-fluid and one-fluid models. Both currents concentrate near the inlet and downstream end. However, there are a noticeable difference between the concentration profiles near the inlet. The current density of one-fluid model increases monotonously in this region. On the other hand, the current density of two-fluid model has a peak and decreases upward to the inlet. Theses differences are caused by the difference of velocity of charged particles and temperature of electrons.

Figure 3 shows the cross section of quasi-steady MPD arc channel MC-II of referred experiment3). Current distributions on the electrodes can be measured with the segmented anode and cathode. Figure 11 shows the measured current fractions on the
anode and the cathode of MC-II and the calculated current fractions of one-fluid model and two-fluid model for different mass flow rates. The current fractions of calculation are obtained by integrating the calculated current distribution on each segment. The calculated fraction on segment No.1 is much higher than that on the middle segments. In contrast, the measured fraction does not differ strongly from the middle segments. The simplification in numerical models causes these difference between the measurements and the calculation results. The two-fluid model of this work leads to a slight decrease in current fraction on segment No.1, as seen in Fig. 11. This fact implies that a velocity nonequilibrium plays an important role in current distributions in an MPD thruster.

**Summary**

Based on nonequilibrium two-fluid plasma model with geometric parameters taken as close as possible to the referred experiments, numerical analyses of one-dimensional MPD thruster has been done, enabling us to achieve the steady flowfields. In the vicinity of inlet, the difference between the velocity of charged particle of two-fluid model and the mean velocity of one-fluid model is observed. In addition, it is found out that the inlet electron temperature becomes very low in the case of two-fluid model. This difference and that of temperatures of
electrons at the inlet leads to the improvement of current distribution profile near the inlet.

Therefore, it is concluded that this two-fluid model can be utilized as the numerical model for multidimensional flow analyses.

References