A MODEL FOR THE CHARACTERIATION OF PLASMA INSIDE THE STATIONARY PLASMA THRUSTER

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Abstract

Three-fluid model of plasma in Stationary Plasma Thrusters (SPT) is developed. The model is based on the assumption of electric neutrality of plasma and on the 1-D description of particle transport in the discharge channel. The purpose of this paper is to discuss the role of electron energy balance, since in our previous papers the electron temerature axial distribution were taken arbitrarily following the results of measurements. In frame of such a model we determine the stationary plasma parameter distributions along the axis of discharge channel, which appear to be consistent with the expected plasma parameter axial profiles in SPT100.

Introduction[1]

Low density of plasma in Stationary Plasma Thrusters (SPT) impose kinetic description as the most adequate. However, because of its large computational effort, there appear the necessity of developing simpler approximate plasma models. First steps in this direction were done by formulating hybrid models[1-4], where electrons were described as fluid while ions and atoms in terms of kinetic equation. Beside of it, other approximations of electron dynamics, like assuming Bohm diffusion[1] or simple Ohm's law for electron current[2,3,4] were made in this models. Replacing the electron momentum balance by Ohm's law removes the problem of passing the ion sound barrier by ions from the analysis.

Therefore applying the purely fluid models[5-9] without reducing the electron momentum balance became reasonable. The main approximation in this case, as compared with hybrid models, stays in neglecting the ion velocity dispersion resulting from that, the ions are created at the positions of different potential values. However, in first versions of these models, another approximations were like assuming the arbitrary axial distributions of: - electron azimuthal drift[5] or – electron temperature[6,7,8]. This reflects the main dificulties in searching the stationary solutions in frames of such models : - passing the ion sond barrier and -. the rapid decrease of electron temperature close to exit, where the Joule heating is not balanced by electron collisional energy loss. In our previous paper[8] we were trying to overcome first of these problems by introducing the solutions discontinous at $v_i = c_s$. However, it has produced another arbitrariness – choosing the position of discontinuity. The idea of Fruchtman and Fish[9] of exploring the analogy between SPT and Laval nozzle and hence postulating the smooth transition of the ion sound barrier appers then as the proper way to overcome this difficulty in stationary problems. We were testing it introducing arbitrary temperature axial distributions in[6,7]. But except some general features of temperature distributions coming from measurements made on smaller devices (SPT50)[10], there was no way for reasonable guess of temperature distributios for given operating conditions. The aim of this paper is to analyze the electron energy balance in frame of fluid model in order to find simple approximate way of determining electron temperature in plasma column in terms of other plasma at given position.

We first repeat the formulation of three-fluid model of plasna in SPT. Next we discuss the approximatiom providing the local relation between temperature and other plasma parameters and the general properties of stationary solutions in this approximation.

Three-fluid model of plasma in SPT

Governing equations

Most of the fluid models (as well as the hybrid models) are based on the assumption of the electric neutrality. This assumption is reasonable when applied to plasma volume in SPT except the Debye sheath near the isolating walls. The ion and atom velocities are assumed to be approximately purely axial remembering that the ion Larmor radius is much greater than the linear dimensions of plasma column. However, the presence of sheath is equivalent to nonvanishing radial component of electric field and hence to the ion flow to the walls. But, remembering that the sheath thickness is of the order of Debye length, we assume that the radial components of electric field and ion velocity are nonvanishing in the narrow boundary layers only. Hence we will describe the ion and atom motion averaging the respective fluid equations over the cross section of the plasma column. The same averaging procedure will be applied to the electron transport equations remembering that the electron mean free paths along magnetic field are much greater than those transverse to magnetic field direction. The effect of sheath will then appear only via boundary conditions on the isolating walls. In case of ions and atoms we will apply also the « cold fluid » approximation neglecting the ion velocity dispersion which appeared as a result of creating new ions at the positions of different electric potential values.

Then denoting plasma species (ions, atoms, electrons) with indices {*i, a, e*} we are faced with the equation system:

- *ion continuity and momentum equations* :

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i V_i)}{\partial z} = \beta n_a n_e - \sigma_i n_i \equiv Q_i n_i \qquad (1)$$

$$n_i \frac{\partial V_i}{\partial t} + n_i V_i \frac{\partial V_i}{\partial z} = \frac{eEn_i}{M} - Q_i(V_i - V_a) \qquad (2)$$

where: β is the ioniztion rate, E - the electric field, and

$$n_i \sigma_i \equiv \frac{2}{R_2 - R_1}\left(n_i V_{ir}^+ - n_i V_{ir}^-\right)\Big|_{walls} \qquad (3)$$

is the ion loss rate on the walls;

- *neutral atom continuity and momentum equations*:

$$\frac{\partial n_a}{\partial t} + \frac{\partial(n_a V_a)}{\partial z} = - \; Q_i, \qquad (4)$$

$$n_a \frac{\partial V_a}{\partial t} + n_a V_a \frac{\partial V_a}{\partial z} = 0, \qquad (5)$$

where one can easily deduce $V_a = const \equiv V_0$;

- *electron continuity, momentum and energy equations*:

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e V_{ez})}{\partial z} = Q_i \qquad (6)$$

$$n_e \frac{\partial V_{ez}}{\partial t} + n_e V_{ez} \frac{\partial V_{ez}}{\partial z} + \frac{1}{m}\frac{\partial(n_e T_e)}{\partial z} = -\frac{eEn_e}{m} + n_e \omega_B V_{e\theta} - n_e \nu_e V_{ez} \qquad (7a)$$

$$n_e \frac{\partial V_{e\theta}}{\partial t} + n_e V_{ez} \frac{\partial V_{e\theta}}{\partial z} = - n_e \omega_B V_{ez} - n_e \nu_e V_{e\theta} \qquad (7b)$$

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} + \frac{3}{2} n_e V_{ez} \frac{\partial T_e}{\partial z} + n_e T_e \frac{\partial V_{ez}}{\partial z} = n_e \nu_e m\left(V_{ez}^2 + V_{e\theta}^2\right) - \beta n_a n_e \gamma E_{ion} - \beta n_a n_e\left(3T_e/2 + (m/2)\left(V_{ez}^2 + V_{e\theta}^2\right)\right) - n_e \nu_{ew} \tau\left(2T_e + (m/2)\left(V_{ez}^2 + V_{e\theta}^2\right)\right) \qquad (8)$$

where: T_e is the electron temperature, ω_B - electron cyclotron frequency, ν_e - electron collision frequency, γE_{ion} - the effective energy cost of ion and τ - the ratio of electron energy lost in the collisions with walls to the total electron energy. Here ν_e is the sum of electron collision frequencies with other particles and with the walls:

$$\nu_e = \nu_{ei} + \nu_{ea} + \beta n_a + \nu_{ew},$$

and

$$\nu_{ew} n_e V_{ez,\theta} \equiv \frac{2}{R_2 - R_1} \times \left(n_e V_{er}^+ V_{ez,\theta}^+ - n_e V_{eir}^- V_{ez,\theta}^-\right)\Big|_{walls} \qquad (9)$$

On the RHS of (8) we distinguish the characteristic source terms:

- (i) first - describing Joule heating;
- (ii) second - the electron energy losses in the inelastic collisions with atoms and
- (iii) third - the electron energy losses in the inelastic collisions with the walls.

Assuming electric neutrality ($n_e = n_i$) we replace electron continuity equation by charge conservation:

$$n_i(V_i - V_{ez}) = I(t)/eS, \qquad (10)$$

where $I(t)$ is the discharge current and S - cross section of plasma column. This will allow us to eliminate V_{ez} and in the next step $-E$, by combining eqs. (2) end (6a).

Collisions with the walls

The effect of electron and ion interaction with the walls was introduced here via boundary conditions on the isolating walls while perfoming the averaging of fluid equations over the plasma column cross section. This averaging procedure cancels all the radial velocity components in plasma volume. In the definitions (3) and (9) there appeare their values at the walls only, where we have also distinguished the incoming $(\ldots^{+})$ and outgoing $(\ldots^{-})$ flow. In this model we will assume, following the results of K. U. Riemann, that the sheath exists on the isolating walls and therefore $V_{ir}^{+}\Big|_{wall} = \sqrt{T_e/M} \equiv V_B$. Moreover, we also assume that number of ions reflected from the walls is much less than that of absorbed or recombined at the walls - $V_{ir}^{-}\Big|_{wall} = 0$. Hence

$$\sigma_i = 2V_B/(R_2 - R_1)$$

In case of electrons we assume, following A. I. Morozov hypothesis of near wall conductivity, that the velocity distribution of reflected and eimtted electrons is isotropic in velocity space:

$$V_{ez}^{-} = V_{e\theta}^{-} = 0$$

However, when evaluating $n_e V_{er}^{+}/_{wall}$ we have to take into account that above assumption holds only for those electrons which pass the potential barrier of sheath (it is implicitely assumed that the electron reflection from the sheath is specular and therefore does not produce any loss of azimuthal and axial momentum components). Hence the incoming electron flow has to be evaluated as:

$$n_e V_{er}^{+} = (1/4) n_{e0} \sqrt{8T_e/\Pi m}\, \exp(e\Phi_s/T_e),$$

where : n_{e0} is the electron density at the sheath edge and Φ_s – the electric potential at the wall with respect to plasma potential at this section of plasma column. Φ_s has to be determined from the condition that the electric current density in radial direction at the walls is equal to 0. Assuming that the ionization within the sheath is negligible and that the magnetic field has no axial component we get

$$n_{i0} V_B = n_{e0} \sqrt{T_e/2\Pi m}\, \exp(e\Phi_s/T_e) \times (1 - \sigma_e(T_e))$$

where $\sigma_e(T_e)$ is the mean number of electrons scattered back from the wall per one incident electron. Hence the sheath is created if $\sigma_e(T_e) < 1$. Otherwise, analysing the boundary layer one has to include other effects like ion sputtering. Since in our model $n_{i0} = n_{e0}$ (at the sheath edge)

$$\nu_{ew} = \frac{2\sqrt{T_e/2\Pi m}}{R_1 - R_2} \times \frac{\sqrt{2\Pi m/M}}{1 - \sigma_e(T_e)}.$$

In further consideration we assume that the factor

$$\exp(e\Phi_s/T_e) = \frac{\sqrt{2\Pi m/M}}{1 - \sigma_e(T_e)} \equiv \alpha$$

is slowly varying with T_e and can be thought as the parameter of the theory. In further computations we will put $\alpha = 0.2$. Electron having kinetic energy ε at the sheath edge will approach the wall with energy $\varepsilon + e\Phi_s$ less than ε, since $\Phi_s < 0$. Having in mind the properties of energy spectrum of secondary electrons we assume that in most of collisions with the wall electrons loose approximately all their energies and hence reappear at the sheath edge with energy $\varepsilon_l = - e\Phi_s$. Therefore the parameter τ appearing in electron energy balance (8) can be estimated as :

$$\tau = \frac{-e\Phi_s}{(3/2)T_e + (m/2)(V_{ez}^2 + V_{e\theta}^2)}$$

and since at least approximately : $-e\Phi_s \propto T_e$ we take τ as the next parameter of the theory putting here $\tau = 0.2$.

Stationary problem

In stationary problem analysis the mass conservation allows us to eliminate n_a by simple relation:

$$n_i V_i + n_a V_0 = \dot{m}/_{SM} \equiv q, \qquad (9)$$

where: $\dot{m}$ is the mass flow rate. This (together with charge conservation and electric neutrality) reduces the governing equation system to:

$$\frac{dJ}{dz} = Q_i \times (J/V_i), \qquad (10)$$

$$\left(C_s^2 - \left(1 + \psi^2\right)V_i^2\right)\frac{dV_i}{dz} = Q_V^{+} - Q_V^{-} \qquad (11)$$

$$(J - I)\frac{\partial V_{e\theta}}{\partial z} = -\omega_B (J - I) - n_e \nu_e V_{e\theta} \qquad (12)$$

$$(3/2)V_i \frac{dC_s^2}{dz} + C_s^2 \frac{dV_i}{dz} = Q_T^{+} - Q_T^{-}, \qquad (13)$$

where now:

$J = n_i V_i$ - is the ion current;

$I - J = n_e V_{ez}$

$\psi = (m/M)^{1/2}\ (I/J - 1);$

$C_s = (5T_e/3M)^{1/2}$ - the ion sound velocity.

$$Q_i \equiv (\beta/V_0)(q - J) - \sigma_i \quad (14)$$

$$Q_T^+ \equiv Q_i \frac{J}{I-J}\left((3/2)C_s^2 + \gamma E_{ion}\right) + Q_i C_s^2 + 2\nu_{ew}\tau \frac{J}{I-J} C_s^2 \ ; \quad (15a)$$

$$Q_T^- = \nu_{eff}\left(\psi V_i^2 + (m/M)\left(V_{e\theta}^2/\psi\right)\right) \quad (15b)$$

$$Q_V^+ \equiv Q_i\left(V_i^2 - \psi^2 V_i^2 - V_i V_0\right) + (2/5)Q_T^+ \quad (16a)$$

$$Q_V^- \equiv (2/3)C_T^- + (m/M)V_i \times (\omega_B V_{e\theta} + \nu_e(I/J - 1)V_i) \quad (16b)$$

and here :

$\nu_{eff} = (1/2)(m/M)^{1/2}(2\,\nu_e - Q_i\ -\ \nu_{ew}\,\tau).$

Equations (10- 13) are subject to the boundary conditions :

$$V_i(0) = V_0;\quad J(0) = J_0;\quad T_e(L) = T_1;$$
$$\omega_B V_{ez}(L) + \nu_e V_{e\theta}(L) = 0; \quad (17)$$

However, in stationary case, J_0 could not be choosen free, since searching for the solution passing smooth the ion sound barrier we have to take its value such, that: $Q_V^+ = Q_V^-$ for $V_i = C_s\big/\sqrt{1+\psi^2}$ (see discussion in [7,9])

Electron energy balance

In equations (10-13) the RHS were splitted in split into two terms « gain » and « loss ». In equations (12) and (13) one can estimate that the « gain » and « loss » terms taken separate are much larger than the respective value of LHS.
This allows us to apply the « adiabatic » approximation :

$$\omega_B(I - J) - \nu_e(J/V_i)V_{e\theta} = 0;$$
$$Q_T^+ - Q_T^- = 0; \quad (18)$$

which will serve to determine $V_{e\theta}$ and T_e (C_s) by solving the respective algebraic relations. First of the relations (18) is the usual ralation betwen the Hall - and axial currents. The second of the is equivaletnt to neglect the z-derivatives of $(T_e^{3/2} V_i)$. Choice of this quantity as an adiabatic invariant can be qualitatively justified by observing that close the exhaust of SPT channel the large increase of V_i coexists with rapid decrease of T_e.

Here we present an example verifying this hypothesis aplied to the results derived in [13], where the fluid model of SPT plasma was solved under assumption of simple Ohm's law for electron current, but with electron energy equation the same as (13) in this paper. The parameters of electron wall collision used in [13] were the following:

$$\alpha = 0.2;\quad \tau = \exp(-20.0eV/T_e);$$

geometrical characteristics like in SPT100 i.e.:

$$R_1 = 3cm;\quad R_2 = 5cm;\quad L = 4cm;$$

the ionization rate were determined by applying Drawin's formula[14], γ factor relating the energy cost of ion to ionization energy was taken according[15] as: $\gamma = 2.5$,

and the operating conditions were:

$$\dot{m} = 4.5mg/s\,(Xe);\quad U_{disch} = 200V;$$
$$I = 2.4A;\quad B_{\max} = 200Gauss;$$

the magnetic field distribution were aproximated by:

$$B = B_{\max}\exp\left(-16\left(z/L - 1\right)^2\right);$$

Using the V_i and J values obtained in[13] we computed the electron temperature T_e by solving equation:

$$Q_T^+(V_i, J) = Q_T^-(V_i, J) \quad (19)$$

and compared it with T_{el} derived in[13].

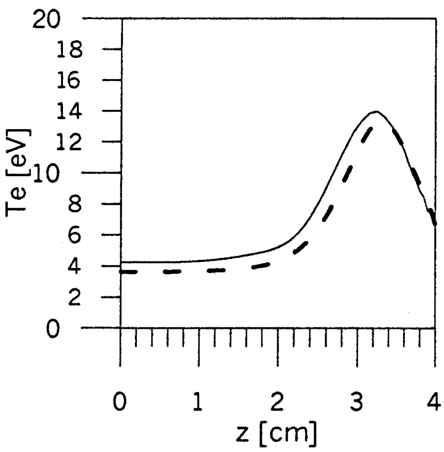

Fig.1 Comparison of T_e (comntinous line) with T_{el} (dashed line)

Fig.1 illustrates that (at least in case presented above) the approximation (19) can quarrantee the reasonable approximation of electronenergy balance in SPT plasmas. Its application reduce the stationary problem to solving the system of two differential equations:

- ion continuity (10)
- ion momentum (11)

which evidently illustrates the analogy of this problem with one-dimensional Laval nozzle theory. There remains the question of boundary condition for T_e at $z = L$. However, since in practice we do not how to determine from the operating conditions the electron temperature value at the exit, we can postulate that electrons entering the channel after passing few times the distance between the walls would achieve the average kinetic energy (excluding the energy of drift motion) equivalent to: $Q_T^+(V_i, J) = Q_T^-(V_i, J)$ at $z = L$.

Conclusion

The 1-D fluid model of SPT plasma was formulated and discussed in context of electron energy balance. It was pointed out that analysing the equation describing this balance it is reasonable to apply simple approximation which reduce this equation to relatively simple algebraic equality. This makes the 1-D fluid model of SPT plasma completely analogous to that met in the 1-D theory of Laval nozzle.

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