Fully Kinetic Hall Thruster Modeling

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A Hall effect plasma thruster was simulated using fully kinetic 2D3V Particle-in-Cell (PIC) and Monte-Carlo Collision (MCC) methodologies. The numerical model included electrons, neutrals, singly and doubly charged ions. Axisymmetric R-Z coordinates were used with a non-orthogonal variable mesh to account for important small-scale plasma structures. Electric field and sheath structures were treated self-consistently. Kinetics of all species were treated on the electron time scale. All important collisions were accounted for. Anomalous diffusion was included via an equivalent scattering frequency. A method for changing the ion to electron mass ratio and retrieving physical results was developed and used. The permittivity of free space was increased to slow plasma oscillations and allow a coarser grid. Goals of the research were to predict performance, particle moments, and the electron energy distribution function, and to study electron transport and instabilities. Results were compared with theory, experiments. The original design and several variations were simulated.

Introduction

This paper describes a fully kinetic numerical model developed at MIT and its application to a 50 Watt Hall thruster, a type of electric propulsion. The research was the topic of the first author’s Ph.D. thesis [36]. The simulation can be run on an ordinary workstation because it uses an artificial mass ratio and artificial permittivity. Other modifications to the physical parameters are introduced in order to retrieve physical results.

Electric Propulsion

Spacecraft require on-orbit propulsion systems for tasks such as station-keeping, orbit re-phasing, and orbit transfer. The fuel efficiency of these systems is exponentially dependent upon the mean exhaust speed \(< v >\) of the propellant according to the rocket equation,

\[ M_p = M_i [1 - \exp\left(\frac{-\Delta V}{< v >}\right)]. \quad (1) \]

Here \(M_p\) is the propellant mass required for a given velocity increment, \(\Delta V\), and \(M_i\) is the vehicle mass before the maneuver. The exhaust speed may be described by specific impulse, \(I_{sp} = < v > / g_o\), where \(g_o\) is the gravitational acceleration at the earth’s surface.

The \(I_{sp}\) of a chemical rocket is limited by the internal energy stored in its propellant. Typical bipropellant systems operate below 300-sec. Advanced cryogenic Hydrogen-Oxygen systems achieve about 470-sec.

Electric propulsion bypasses the internal energy limit by using an outside energy source to accelerate the propellant. Almost unlimited exhaust speeds can be ob-
Figure 1: The acceleration zone of a typical Hall thruster. The geometry is nominally axisymmetric. Electrons are drawn from a cathode to a highly biased anode through which propellant is injected. Along the way, they drift azimuthally (the Hall current) and ionize the propellant, which is ejected axially.

Many electrostatic systems far exceed $I_{sp} = 1000$-sec. Propellant savings can be massive.

Hall Thrusters and Ion Engines

Hall thrusters ionize and accelerate ions by applying a potential difference across a region of high magnetic impedance into which electrons are introduced (see Figure 1). Ion engines are similar in principle but require grids to create the accelerating electric field. Ion engine beams are well collimated, but thrust densities are space-charge limited by the Child-Langmuir equation. Hall thruster beams are more divergent, but thrust densities are not space-charge limited. Typically, Hall thrusters operate near $I_{sp} = 1600$-sec while ion engines operate closer to $I_{sp} = 3000$-sec.

Particle-In-Cell Modeling

The Particle-In-Cell (PIC) plasma simulation methodology involves applying forces to charged super-particles which reside within a computational grid. Each super-particle represents $10^6$ or so plasma particles. Figure 2 is a generic flow chart of the method.

Monte Carlo Collision Methodology

Scattering of one species off a background species is often handled through a Monte Carlo Collision (MCC) algorithm. For a given particle, one first finds the total cross section, which yields an expected collision frequency. This frequency, the time-step, and a random number are then used to determine whether a scattering event takes place. If so, cross sections for different events (e.g. excitation, ionization, elastic scattering) are compared to determine what type of event occurs.

Goals and Methodology

Goals of the research were to predict performance, particle moments, and the electron energy distribution function (EEDF) of a real thruster. To achieve these goals, we developed a complete, self-consistent numerical model of a real discharge. In the process, we studied electron transport and instabilities. The final product can be used to simulate new Hall thruster designs without actually building them.

The first step was to devise, write, and test a basic algorithm utilizing full PIC and MCC methodologies. The next step was to model an actual thruster, accounting for the following: electric potential boundary conditions; cathode emission; heavy particle collisions with the walls; scattering collisions; ionization collisions; Coulomb collisions; excitation collisions; charge exchange collisions; anomalous Bohm diffusion. These steps were described in papers, along with preliminary results [37] [38]. Results were validated through parametric studies and comparison with experimental measurements, as reported in the thesis [36]. This paper summarizes the thesis and also reports more recent modeling of doubly charged ions ($Xe^{2+}$), dielectric boundaries, and secondary electron emission at those boundaries.
Literature Survey/Brief Summary of Previous Work

Hall thruster are nominally axisymmetric, although azimuthal oscillations may play an important role in electron diffusion. Typical modeling efforts have been 1-D [21] [22] or 2-D [10], focusing on the RZ plane. At least one effort has captured the $R\Theta$ plane [19]. A 3 dimensional model would be ideal, but this is beyond the capabilities of current workstations.

Measurements of the EEDF

Measurements of the EEDF in some Hall thrusters have shown multiple populations of electrons and gradients in temperature along magnetic streamlines [14]. However, interpretation of the data requires knowledge of the EEDF. It is usually assumed to be Maxwellian.

2-D Hybrid-PIC Modeling at MIT

MIT’s “hybrid PIC” SPT simulation [10] relies upon quasineutrality ($q_e \approx q_i$), Maxwellian electron distributions, assumed levels of Bohm diffusion, and wall effects based on the local electron temperature. This model has been used to predict the performance of the SPT-100 [11] and Busek thrusters [39]. Predicted performance agreed well with experimental measurements when Bohm diffusion coefficients about one quarter of the classical $(1/16B)$ coefficient were used. The code has also been used to predict ionization oscillations [12]. High frequency oscillations and electron transport are sensitive to wall boundary conditions.

Because the hybrid PIC code assumes quasineutrality, it may not be fully applicable to metallic walled thrusters. Secondary emission from metals is much less than from dielectrics. In theory [12], this means metallic walled thrusters lose less heat to the acceleration channel walls, yielding higher electron temperatures and more abrupt ionization layers. These may be just a few electron cyclotron radii wide [13]. To model such layers, a numerical method should allow for non-neutrality. Full PIC methods do this and allow for non-Maxwellian electron energy distributions.

2D PIC Modeling

Our full PIC hall thruster simulation draws from and builds upon previous models. Unique features of our simulation are outlined in the section titled "Numerical Method."

The PIC/Monte-Carlo/DSMC Hall thruster simula-

<table>
<thead>
<tr>
<th></th>
<th>Design</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anode Current(A)</td>
<td>.17</td>
<td>.20</td>
</tr>
<tr>
<td>Anode Power(W)</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>Thrust (mN)</td>
<td>2.2</td>
<td>.86</td>
</tr>
<tr>
<td>$I_{sp}$ (sec)</td>
<td>1600</td>
<td>670</td>
</tr>
<tr>
<td>Efficiency</td>
<td>.32</td>
<td>.048</td>
</tr>
</tbody>
</table>

Table 1: The design performance is presented in the left column. The design mass flow rate was .13 mg/s. Measured performance linearly interpolated to the same mass flow rate is presented in the right column. Propellant: Xenon; Diameter: 4.8 mm; B-field: .5 T; Mass flow rate: .13 mg/s; Anode Potential: 300 V.

The mini-TAL Thruster

The preliminary subject of the our PIC simulation is a 50-Watt thruster built at MIT [20] and diagrammed in Figure 3. A preliminary design included dielectric channel walls. The final design (as built) uses metallic walls. Design and measured performance are found in Table 1. Experimental performance is plotted in Figure 4 [20]. (Note: These results differ from those reported in [38], which were preliminary). The actual variables
measured were thrust, anode current, and mass flow rate. The thrust efficiency is calculated from thrust, $T$, anode current, $I_d$, and mass flow rate, $\dot{m}_m$:

$$\eta_t = \frac{T^2}{2\dot{m}_m I_d V_d}.$$  

(2)

The utilization efficiency is calculated assuming all ions exit having seen the entire anode potential difference;

$$\eta_u = \frac{\dot{m}_i}{\dot{m}_m} = \frac{T}{v_i \dot{m}_m}$$  

(3)

where $v_i = \sqrt{2(\Phi - \Phi_o)/\dot{m}_m}$ and $(\Phi - \Phi_o) = \Phi_d = 300V$.

The thrust efficiency of this thruster is low, suggesting a magnetic field that is too weak or improperly aligned. Post-operative testing showed that the permanent magnets experienced only minimal damage [20]. Therefore, Ref. [20] concludes, a temporary change in the magnetic field during operation seems likely. Inadequate cooling of the iron center pole could allow the temperature to rise far enough for the magnetic permeability to approach zero. This would change the B-field strength and shape, possibly allowing some streamlines to cross directly to the anode, in effect “short circuiting” the discharge. Our research tends to support these conclusions; the magnetic field was, at least, misaligned.

Figure 3: Diagram of MIT’s 50W thruster with numerically predicted ion densities for $M/m = 960$, $\gamma = 5$, and $\dot{m} = .1mg/s$ [36].

Collision Cross Sections

A full PIC simulation requires cross sections as a function of relative kinetic energy. Such cross sections are available in the literature. Here we review some of the available data and explain how they are used in our simulation.

Figure 4: Performance of the MIT thruster at 300 Volts [20].

$Xe^{+}+Xe$ Charge Exchange

For the resonant charge exchange process, $Xe^{+} + Xe^0 \rightarrow Xe^0 + Xe^{+}$, the cross section typically used is that of Rapp and Francis [27] [32] [23] [40]. However, this cross section is known to be erroneous.

In Ref. [29], Sakabe and Izawa graphically compiled all experimental data of symmetric charge-transfer cross sections studied since the 1930’s. Then, starting from the Shroedinger equation, they calculated cross sections and presented them alongside the experimental data. The calculated cross sections (for all non-transition elements) were given in tables. In Ref. [30], Sakabe and Izawa presented a universal formula for the charge exchange cross section;

$$\sigma(v) = |A - B \log_{10}(v)|(I/I_o)^{-1.5} \text{cm}^2$$  

(4)

where $v$ is the relative speed in cm/s, $A = 1.81 \times 10^{-14}$, $B = 2.12 \times 10^{-15}$, $I$ is the ionization potential (12.1 eV for singly charged Xenon) and $I_o$ is the ionization potential of hydrogen (13.6 eV). Equation 4 predicts slightly lower $\sigma$ than the tabular cross sections. The simulation uses this formula.

Figure 5 shows the Rapp and Francis cross section, Sakabe and Izawa’s tabular data, and the cross section predicted by Equation 4. For relative energies $1 < \epsilon < 500$ eV, Equation 4 predicts cross sections about thirty percent larger than the Rapp and Francis cross sections. Boyd has used Equation 4 in some of his modeling [5]. Boyd [6] has also used the cross sections of Ref. [24].
Further refinement is needed in this area. This simulation assumes the $Xe^{2+}$-$Xe^0$ charge exchange cross section is 1/2 of the equivalent cross section for $Xe^+-$Xe$^0$ calculated from Equation 4. The justification is found in Ref. [6], in which Boyd states the cross section for $Xe^{2+}$ is approximately one half that for $Xe^1+$, citing the data of Hasted and Hussain [15]. Note, however, that Equation 4 itself with $I = 32$ eV (the second ionization potential of Xenon) yields a value just 23 percent of the value for $Xe^{1+}$. A fit to the data of Hasted and Hussain [15] is not used because the data is for relative energies in excess of 500 eV; these energies are more typical of an ion engine plume. Ref. [24]. Note, however, that Equation 4 itself with $I = 32$ eV (the second ionization potential of Xenon) yields a value just 23 percent of the value for $Xe^{1+}$. A fit to the data of Hasted and Hussain [15] is not used because the data is for relative energies in excess of 500 eV; these energies are more typical of an ion engine plume.

\[ \sigma_{1+,+2} = A \frac{\ln(\frac{E}{\epsilon_1})}{E \epsilon_2}, \]  

where $E$ is the relative energy in eV and $\epsilon_{12} \approx 19.9$-eV is the difference between the first and second ionization potentials. Comparison with data in Ref. [1] shows that $A = 2.7 \times 10^{-17}(meV)^2$ fits the data to within about fifteen percent, which is about the same error as the data. Many other references address elastic and inelastic electron impact scattering cross sections, including [25], [26], [8], and [16]. Our polynomial curve fits for elastic scattering and excitation are shown in Figure 3 of Ref. [38].

\[ Q_{1n} = \frac{8.28072 \times 10^{-10}}{c_r} \text{cm}^2, \]  

where $c_r$ is the relative speed between species 1 and species 2 in cm/s. We use this cross section. Boyd’s DSMC plume models typically use the cross sections of Delgarno [9].

**Electron Impact Ionization and Scattering**

The cross sections for single and double ionization of Xenon neutrals, $e + Xe^0 \rightarrow Xe^{+n} + (n + 1)e$, were taken from Ref. [1] and fit to a polynomial, shown in the bottom graph of Figure 6. Also plotted is the semi-empirical formula of Müller as described in Ref. [31].

Figure 5: Charge exchange cross sections for singly charged Xenon.

**Ion-Neutral Scattering**

For scattering of ions off a neutral background, Oh’s DSMC plume model [23] uses the formula of Banks [2] to arrive at

\[ Q_{1n} = \frac{8.28072 \times 10^{-10}}{c_r} \text{cm}^2, \]  

where $c_r$ is the relative speed between species 1 and species 2 in cm/s. We use this cross section. Boyd’s DSMC plume models typically use the cross sections of Delgarno [9].

Figure 6: Cross sections for electron impact ionization of Xenon. Top: Total ionization cross section (data). Second: $Xe^+ + e \rightarrow Xe^{2+} + 2e$. Third: $Xe^+ + e \rightarrow Xe^{2+} + 3e$. Bottom: $Xe^+ + e \rightarrow Xe^{2+} + 2e$. Shown is a sampling of data points (sans error bars) along with polynomial curve fits and Equation 6.
Numerical Method

Figure 7 is a flow chart of the code. All super-particles (each representing \( \sim 10^6 \) particles) are treated kinetically; their trajectories are followed as they move within a computational grid. Electric and magnetic forces are implemented via the PIC method. Collisions are implemented primarily via the MCC method.

The simulation uses a cylindrical coordinate system. Two coordinates in space are tracked, the R (radial) coordinate and the Z (axial) coordinate. Three coordinates in velocity are tracked, \( v_z, v_r, \) and \( v_\theta \), where \( \Theta \) represents the azimuthal direction. At each time-step, particles actually move in all three directions, but the azimuthal coordinate is always discarded. See Figure 1 for visualization.

The simulation region is shown in Figure 8. Note that parts of the anode and center pole are embedded inside the computational grid. This is a significant upgrade to versions of the code reported in Refs. [38] and [37], both of which treated the anode as a vertical boundary. Plasma can now exist inside of the anode.

The following methods speed the simulation:

- The mass ratio \( M/m_e \) is decreased, speeding up heavy particles.
- The permittivity constant is increased; \( \varepsilon_0' = \varepsilon_0 \gamma^2 \). This increases the Debye length, allowing a coarser grid, and slows plasma oscillations, allowing a longer time-step.

To preserve densities and mean-free-paths when heavy particles travel too fast, we scale cross sections and flow rates as described in Refs. [38] and [37]. Essentially, we increase the electrical conductivity of the plasma perpendicular to the magnetic field in proportion to the increase in ion velocity (due to its artificial mass), thus preserving internal scales such as the width of the ionization region. Mass flow rate, thrust, and \( I_{sp} \) must be re-scaled at the thruster exit in order to plot the performance.

Scale Lengths, Collisions, and Transport

The simulation uses a normalized unit system. The unit of distance is the nominal Debye length, the unit of time is the nominal inverse plasma frequency, the unit of potential is the nominal electron temperature in eV, and so on. Nominal values are set or estimated at program initialization. The nominal electron temperature is usually set to \( T_e = 50 \) eV. The nominal plasma density at \( \dot{m} = .13mg/s \) is \( n_e \approx 7 \times 10^{14} \), resulting in a Debye length of \( \lambda_D \approx .0018cm \) and a plasma frequency of about \( 1.7 \times 10^{11}sec^{-1} \). The nominal gyro radius is about three nominal Debye lengths.

A mean free path analysis justifies modeling some collisions and ignoring others (see Table 2). Electrons are magnetized (trapped on magnetic streamlines) which extends their path lengths such that each is expected to undergo many collisions. The thruster wouldn’t work otherwise. Coulomb collisions may be important for low energy electrons. Charge-exchange collisions are important for plume modeling and erosion studies. Ion-neutral elastic scattering is included because the cross section is similar to the charge exchange cross sections. Neutral-neutral scattering is ignored; the effect should be minor,
Table 2: Collisions included in the simulation.

<table>
<thead>
<tr>
<th>Type</th>
<th>Included</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron-Neutral Elastic Scattering</td>
<td>X</td>
</tr>
<tr>
<td>Electron-Neutral Excitation</td>
<td>X</td>
</tr>
<tr>
<td>$e + Xe \rightarrow 2e + Xe^+e^+$</td>
<td>X</td>
</tr>
<tr>
<td>$e + Xe \rightarrow 3e + Xe^{2+}$</td>
<td>X</td>
</tr>
<tr>
<td>$e + Xe^+ \rightarrow 2e + Xe^{2+}$</td>
<td>X</td>
</tr>
<tr>
<td>Electron-Electron Coulomb</td>
<td>X</td>
</tr>
<tr>
<td>Electron-Ion Coulomb</td>
<td>X</td>
</tr>
<tr>
<td>Ion-Neutral Charge Exchange</td>
<td>X</td>
</tr>
<tr>
<td>Ion-Neutral Scattering</td>
<td>X</td>
</tr>
<tr>
<td>Ion-Ion Coulomb</td>
<td>X</td>
</tr>
<tr>
<td>Neutral-Ion Scattering</td>
<td>X</td>
</tr>
<tr>
<td>Neutral-Neutral Scattering</td>
<td>X</td>
</tr>
<tr>
<td>Electron-Ion bulk recombination</td>
<td>X</td>
</tr>
<tr>
<td>Electron-Ion wall recombination</td>
<td>X</td>
</tr>
</tbody>
</table>

and doing otherwise would properly require DSMC techniques. Doubly charged Xenon is included.

Possible mechanisms for electron transport toward the anode include classical diffusion, Coulomb scattering, wall effects such as secondary electron emission, azimuthal plasma waves, and $\vec{E} \times \vec{B}$ drift associated with the $\Theta$ fields of azimuthal plasma waves. Since the model is axisymmetric, azimuthal wave effects are not directly modeled. However, anomalous Bohm type electron diffusion is included through an equivalent scattering frequency. This effect may be turned on or off through a Boolean switch in the header file. We ignore secondary emission at metallic walls, but include it at dielectric walls.

**Unique Features**

Among full PIC Hall thruster simulations, this one is unique in the following ways:

- The geometry is that of a real thruster, for which some experimental data is available.
- The numerical grid is non-orthogonal.
- A novel method for accelerating classical diffusion and retrieving physical results is implemented.
- The free space permittivity constant is changed.
- Anomalous Bohm type electron diffusion is included via an equivalent scattering frequency.
- Multiply charged ions are included.
- Ion-neutral charge exchange and scattering collisions are modeled.
- The metallic wall potential is allowed to float. It is computed self-consistently.
- Cathode electrons are injected at the rate required to preserve quasineutrality at the free-space (plume) boundary.
- Coulomb collisions are modeled as a diffusion process in velocity space based on a Langevin formulation of the Fokker-Planck equation.

**Important Details**

The following details are also important:

- Neutrals are simulated directly. Beidler did this, but Hirakawa pre-simulated neutrals using DSMC.
- A half-Maxwellian is assumed for neutrals entering the system at the anode and cathode electrons entering the system at the free space boundary. Neutrals are given an initial temperature of $0.1$-eV, while cathode are given a temperature of between $0.2$-eV and $2.5$-eV, depending on where they enter the simulation region.
- Low energy secondary electrons are created in the center of the plasma through ionization events.
- Neutrals are recycled.
- The standard leapfrog method [4] is used to time-step the particles forward.
- Bilinear interpolation [34] is used to weight particles to the grid nodes, where the field equations are solved, and to weight the fields back to the particles.
- Each time-step, the electric potential is recalculated via SOR using finite differences and the integral form of Gauss’s Law (CGS).
- The magnetic field is assumed to be static. The current densities resulting from this simulation can be used to estimate the induced B-field, thereby showing that it should not be a significant factor for this thruster [36].
- The user may choose (via a header file) to model Coulomb collision using an MCC algorithm instead of the Fokker-Planck algorithm previously mentioned. Or, the user may choose no Coulomb collisions at all.
- Secondary emission is assumed to take the form $\sigma = A(E)^B$, where $\sigma$ is the ratio of secondary to primary electrons, $A=.141$, $B=.576$, and $E$ has units
of electron volts \[7\]. For each impacting electron, we create a maximum of 5.6 secondary electrons in a Maxwellian distribution at the wall temperature.

- Charge is collected at the dielectric wall and used to compute the potential there. The dielectric constant for BN is taken to be 4.4; the permittivity is thus \(\varepsilon = 4.4\varepsilon_0\).

Facilities

The code is written in C and run on a PC, e.g.

CPU: AMD 1.3 GHz Athlon
Main Memory: 512 MB DDR Sdram
Operating System: Windows 2000
Compiler: Microsoft Visual C++ 6.0

With an \(87 \times 49\) \((\gamma = 10)\) grid and \(60K - 70K\) particles of each species, the simulation completes about 2000 iterations per hour. At \(M/m \approx 320\), convergence on the neutral time-scale requires a couple of days. More physical simulations (e.g. \(\gamma = 5\)) are predictably slower.

Code Profile

Table 3 shows the relative amount of CPU time used by each element of a recent version of the code. Most effort is spent pushing particles. Although the SOR Poisson solver takes 400 iterations, it consumes only \(\approx 20\) percent of the CPU time.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle moments</td>
<td>17</td>
</tr>
<tr>
<td>Electron pusher</td>
<td>35</td>
</tr>
<tr>
<td>Poisson solver</td>
<td>22</td>
</tr>
<tr>
<td>Ion pusher</td>
<td>9.9</td>
</tr>
<tr>
<td>Beginning overhead</td>
<td>7.8</td>
</tr>
<tr>
<td>Neutral pusher</td>
<td>2.3</td>
</tr>
<tr>
<td>Other</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Table 3: Profile of the code when run twice for 50 iterations in release mode using Langevin Coulomb collisions. Conditions included 60-70K plasma particles of each species, over 100K neutrals, and a \(87 \times 49\) grid (suitable for \(\gamma = 10\)). Beginning overhead is large due to the length of the run.

Validation Tests

The accuracy of the potential solver was tested on a function which is periodic in \(z\) and \(r\). The leapfrog algorithm was tested by tracking single particles with electric field only, magnetic field only, and a combination of the two. Cyclotron motion and ExB drift were successfully observed to high levels of accuracy. A cold plasma was created and oscillations at the plasma frequency were observed. Momentum conservation, energy conservation, and numerical heating were observed by creating an initial thermal distribution, closing the boundaries such that no particles could escape, and letting the plasma oscillate, assuming specular reflection at all boundaries. Energy conservation of single particles was tested by tracking the energies and positions in time of a few chosen electrons as the simulation ran, with the electric potential fixed in time, and with the electric potential allowed to oscillate. Similar tests showed magnetic bottling and conservation of magnetic moment. The general methodology of Maxwellian particle injection, the initial particle distribution, and the methodology for finding the EEDF and other moments were tested simultaneously by analyzing the initial EEDF. Parametric tests were performed to assure that effects of capacitance, mass ratio, artificial permittivity, and inclusion or exclusion of various collisions were understood. The ultimate test was a comparison of predicted results with experimental results.

The floating wall and capacitance were used to test the electron injection method. The wall potential should not vary much, if at all, as the capacitance varies. However, if too many or too few electrons are created at the free space boundary, the overall balance is upset and the wall potential changes significantly with capacitance, skewing results. If \(L_e = L_a\) is imposed, this happens. However, if \(n_e \approx n_i\) is used to inject electrons, the wall potential does not change much with capacitance. This is, therefore, our final configuration.

Results

MIT’s 50W thruster was simulated under various operating conditions and the results were examined. One such result, the ion density, is shown to approximate scale with the thruster in Figure 3. Convergence of the simulation is determined by the magnitude and rate by which mean values of interest change.

Conservation of Magnetic Moment

A magnetic stream function \(\psi\) was constructed from \(\vec{B}\); contours of \(\psi\) are parallel to \(\vec{B}\). Streamlines equally spaced in \(\psi\) are shown in Figure 9. Without collisions, electrons would be trapped on these streamlines. With collisions, transport arises, as shown in Figure 10.
collision is elastic, kinetic energy and magnetic moment are conserved.

Figure 9: Contour plot of the magnetic stream function, $\psi$. Contours of $\psi$ are streamlines of the magnetic field.

Alternate Geometries

The geometry of the thruster may be changed numerically. After simulating the nominal (as built) design, we (numerically) shortened the inner portion of the hollow anode by .35 mm such that the face is more parallel to magnetic streamlines. Subsequently, we (numerically) lined the inner and outer walls of the acceleration channel with Boron Nitride dielectric material (see Figure 11). Plasma parameter were effected by both changes.

Figure 10: A single electron diffusing across the magnetic field. Particle begins at (*), undergoes a collision at diamond, and ends at square.

Predictions

The simulation can be used to analyze many aspects of the discharge. Unless otherwise indicated, the predictions listed below assume metallic channel walls.

Our newest results include the following:

- A typical performance plot is shown in Figure 19, which assumes $\gamma = 5$, $M/M' = 750$ ($M/m = 320$), and $\dot{m} = 1676$ mg/s. Coulomb collisions were turned off. Predicted thrust for this set of conditions is close to 1.6 mN, while measured thrust is closer to 1.2. More realistic mass ratios and $\gamma$ factors tend to reduce predicted performance. The simulation shown is not fully converged on all timescales; the neutral flow is still developing toward a quasi-steady state. However, time averaged plasma moments and performance do not usually change much in this latter stage of development.

- Figures 12, 13, and 14 show the singly charged ($Xe^{1+}$), doubly charged ($Xe^{2+}$), and charge exchange ($Xe^{1+} + Xe^{2+}$) ion densities predicted for the original geometry at $\dot{m} = 1676$ mg/s using $\gamma = 5$ and $M/M' = 750$ ($M/m = 320$). These figures were generated by converging the simulation shown in Figure 19 to $t = 16,000$ and then time averaging results for 1000 iterations.

- The predicted ratio of doubly to singly charged Xenon is 5-10 percent, by number density. See Figures 12 and 13.
- The density of ions produced through charge exchange is similar to the density of doubly charged Xenon, though it is much higher near the wall. See Figures 13 and 14.

- The frequency of the long period oscillations evident in Figure 19 corresponds well to the expected ionization oscillations at $\nu = \frac{V_s V_i}{2\pi L}$ where $V_s$ and $V_i$ are bulk velocities of neutrals and ions in the discharge, and $L$ is the approximate width of the discharge region. If $L = .05$ cm (see Fig. 12) the formula yields (for predicted bulk velocities) $P = 1/\nu \approx 2000$.

- With metallic walls at $\dot{m} = .1676$ mg/s and $\gamma = 10$, the revised anode increased thrust and $I_{sp}$ by about fifteen to twenty percent.

- As expected, the introduction of dielectric walls (with the original anode) changes the potential distribution. In the anode region, the field becomes more axial. However, thrust and $I_{sp}$ do not change much at $\gamma = 10$, $\dot{m} = .1676$ mg/s, and $\Phi_d = 300V$. An electric potential profile for the dielectric walled thruster is shown in Figure 15. The corresponding electron temperature is shown in Fig. 16. For comparison, potential and electron temperature profiles for the metallic walled thruster are shown in Figures 17 and 18. Figs. 15-18 were generated by time averaging fairly well converged solutions over 1000

Significant results outlined in the thesis include the following:

- The magnetic field of this particular thruster was designed incorrectly. In the near anode region, we
discovered an axial component (see Figure 9) which partly explains the poor thrust, specific impulse, and propellant utilization.

- With metallic walls at .1 mg/s, $\gamma = 10$, the modified anode geometry increased predicted thrust and specific impulse increased by about sixty percent.

- At $\Phi_d = 300$ V, the bulk energy of the ion beam is predicted to be around 200 eV. Hence, the actual "utilization" of the neutral propellant may be significantly better than predicted by Eq. 3. which assumes a bulk energy of $\Phi_d$.

- The model predicts a non-isotropic electron energy distribution function. The distribution is better represented by a two-temperatures, one each for directions parallel ($T_k$) and perpendicular ($T_\perp$) to $\vec{B}$.

- The model has also predicted non-Maxwellian electron populations in the perpendicular direction near the anode. The following explanation is proposed: The electric field (nominally $\perp\vec{B}$) increases the energy preferentially. Electrons do not have time to fully thermalize. See Refs. [38] and [36] for further discussion.

Other interesting results from the thesis include the following:

- The introduction of anomalous Bohm type diffusion proportional to $\frac{1}{6\gamma B}$ increased thrust and specific impulse by about ten percent.

- Thrust, specific impulse, and propellant utilization increased with mass flow rate.

- The numerical plasma extended to the interior of the hollow anode under certain conditions (see Figure 12), but this did not seem to be an essential feature of the flow for effective operation. This effect is less dramatic at more realistic mass ratios (e.g. $M/m = 1000$).

- The simulation has predicted some gradients in electron temperature along magnetic streamlines.

- High frequency oscillations in current and plasma density are predicted. Some of these may correspond to theoretical modes.

- Monte Carlo Coulomb collisions changed ion thrust, propellant utilization, and specific impulse by less than ten percent.
Langevin type Coulomb collisions changed thrust and specific impulse by only about one percent. However, the shape of the electron distribution near the anode was more Maxwellian.

With regard to the numerical method, we can say the following:

• Changing $\epsilon_o$ was shown to be a viable method for saving computational resources, although charge separation can be a problem in the limit.
• A method for changing the Xe/e mass ratio and retrieving physical results was developed and used throughout. This method speeds convergence.
• Finer grid resolution is desired to reduce numerical heating and associated errors.

Conclusions

The success of this research shows that the full-PIC Monte-Carlo method is a viable alternative for investigating small scale plasma flows in a Hall thruster. Applied to MIT’s 50W thruster, the code predicted most performance figures to within 30 percent of what was measured experimentally. The code was used to numerically redesign the anode; better performance was predicted with the new design. However, no plans are in place to rebuild and retest the thruster.

The simulation is now being applied to Busek geometries. Results will be compared to experimental measurements. Computational power is an issue; larger thrusters require proportionally finer grids and longer run-times. Additional methods of accelerating the physics and numerics are under consideration.
Figure 19: Performance plot for $\gamma = 5$, $M/M' = 750$, $\dot{m} = 1.676$ mg/s. No Coulomb collisions. This figure shows a typical simulation approaching convergence. Time averaged performance does not change much (on these time-scales) after the initial surge of ionization at program initiation. Thrust and $I_{sp}$ are under-represented here by a few percent.
References


