Plasma Plume Far Field Analysis

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Near field plume behavior of Hall or ion thrusters, obtained from laboratory measurements or from computational modeling, serves as input for analytical modeling of the free jet expansion in the far field. There, the expansion is determined by ion inertia and by electric fields created due to electron thermal pressure. This analytical approach allows us to describe plume behavior at relatively large distances at a minimal computational burden and to get an insight into the dependence of the expansion on thruster and plasma parameters. A self-similar flow solution, demonstrating a logarithmic expansion at large distances, shows that plume expansion is faster for a larger ratio of electron temperature to ion kinetic energy. Also, the relative increase in plume width due to electron pressure is larger for a narrower near field plume. The validity and limitations of the solution are being investigated.

1. Introduction

Compared to chemical rockets, electric thrusters are characterized by a much larger specific impulse (jet velocity), thus enabling significant propellant mass savings. When used to propel spacecraft, this may result in large cuts in spacecraft mass and consequently in mission costs. Moreover, demanding space missions, requiring large velocity increments which make the use of chemical rockets impractical, become realizable when the use of electric propulsion is considered. As electric propulsion technology matures and is being introduced into more and more near and long term mission projects, spacecraft integration issues are being addressed too. One of the main issues in this regard is that of the jet plume. At least in the cases of two of the leading types of electric thrusters, namely the Hall thruster and the ion engine, it consists of a plasma of energetic (hundreds of eV) xenon ions. Depending on the plume flux distribution, these energetic ions can erode spacecraft surfaces by sputtering, especially those of the solar panels. Damage could also be caused by deposition of sputtered thruster material. In addition, the propagation of electromagnetic waves to and from the spacecraft, for the purposes of communication, telemetry etc., could be disturbed by refraction or reflection effects, depending on the plasma frequency and hence on the plasma density distribution.

Future electrically propelled spacecraft are expected to carry relatively large solar panels extending to a few meters for a small satellite [1] and to tenths and even hundreds of meters for a large interplanetary spacecraft [2]. Moreover, some of the missions envisioned for small satellites require formation flying with distances of tenths of meters between satellites [3]. There is therfore a need to know the downstream flux and density profiles in the plume of electric thrusters to a considerable distance. In recent years, significant ef-

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forts have been devoted, both theoretically and experimentally, to the study of the behavior of the plume of ion and Hall thrusters [4-17]. Laboratory plume measurements however, are limited by vacuum chamber dimensions and by ambient gas pressure and flow effects to a relatively short distance from the thruster. At the same time, numerically intensive theoretical modeling, while allowing to take into account various kinetic effects, face increasing computational difficulties when going to large distances.

An alternative approach is to obtain near field behavior from laboratory measurements or from computational modeling, and to use it as an input for analytical modeling of the free jet expansion in the far field. There, after near field effects such as charge exchange and the residual magnetic field have decayed, the plume expansion is determined in fact by the ion inertia and by the electric fields created as a result of the electron pressure. Such a simplified analytical model, capturing the physics of plume expansion due to electron pressure, can describe plume behavior at relatively large distance at a minimal computational burden and allows to get an insight into the dependence of the expansion on thruster and plasma parameters. At the same time, it could also serve as a test to more elaborate computational intensive models.

Some attempts to solve analytically related flow problems can be found in the literature. Morozov [18] applied a kinetic treatment for the problem of a flat quasi-neutralized ion beam with no initial transversal velocity and obtained a first order solution, demonstrating the importance of the ratio of the electron temperature to the ion kinetic energy. Nevertheless, in this solution the driving electric field was that of the unexpanded beam, which resulted in an unrealistic exponential (hyperbolic cosine) beam divergence. Parks and Katz [19] found a self-similar flow solution for a cylindrical quasi-neutralized ion beam with a constant axial velocity over the whole cross section of the beam. A Gaussian self-similar density profile was obtained. More recently, a general self-similar flow solution for plasma plumes was suggested by Korsun et al. [20,21]. However, when the complicated solution was simplified for the case of the Hall thruster, a conical (straight lines) divergence was obtained.

After it is demonstrated in section 2 that charge exchange effects are small and limited to the near field in the case of free jet Hall thruster plumes, the far field model is presented in section 3. First, the model assumptions and equations are described. Then, a modification of the self-similar flow used in [19] is applied to model the plume of the Hall thruster. It is shown that the stream-line solution behaves logarithmically at large distances. In section 4 the analytical solution is used to draw graphs of stream-lines and density profiles for a set of electron temperature and near field cases. The results demonstrate that plume expansion is faster for a larger ratio of electron temperature to ion kinetic energy, and that the relative increase in plume width due to electron pressure is larger for a narrower near field plume.

2. Charge exchange in a free jet plume

The problem of charge exchange in the plume of plasma thrusters is usually regarded as an important collisional process and has been extensively treated in the literature [6,8,9,13,16,17]. Nevertheless, these works in general model the typical situation of a laboratory experiment in a vacuum chamber where accelerated thruster plume ions are neutralized by charge exchange with the ambient gas neutrals. In the case of free jet expansion in space, ambient gas effects are negligible and charge exchange results practically only from the interaction with the flux of neutrals emerging from the thruster. However, as the simple calculation below shows, this effect has any significance only in the very near vicinity of the thruster (near field).

When the ion flux, I_i, expands into a constant ambient density, n_{am}, it decays due to charge exchange according to:

$$I_i(r) = I_{i0} e^{-r/\lambda_{ce}}, \tag{1}$$

where r is the distance from the thruster exhaust, $\lambda_{ce} = (n_{am}\sigma_{ce})^{-1}$ is the charge exchange mean free path and σ_{ce} is the charge exchange cross section.

We neglected here the contribution of the slowly moving ions generated by the charge exchange process. In the case of free jet expansion, we assume the ion flux to expand in a wider plume of slowly moving neutrals. The neutrals have a constant thermal velocity and expand in a solid angle Ω_n. As a result, their density drops according to:

$$n(r) = n_0 \frac{a^2}{r^2}, \tag{2}$$

where n_0 is the neutral density at the thruster exhaust, a is the (spherical) radial distance of the thruster exhaust from the effective origin of the neutral flux, defined by $S_0 = \Omega_n a^2$, and S_0 is the cross sectional area at the thruster exhaust. The ion flux losses due to charge exchange are then described by:

$$\frac{dI_i}{dr} = -\frac{a^2}{r^2}\frac{I_i}{\lambda_0}, \tag{3}$$

where $\lambda_0 = (n_0\sigma_{ce})^{-1}$. The solution to Eq. (3) is given by:

$$I_i(r) = I_i(a)e^{\frac{a}{\lambda_0}(a/r-1)}, \tag{4}$$

where $I_i(a)$ is the ion flux at the thruster exhaust. Unlike the constant ambient neutral density case, in the free jet case the ion flux decays at large distance to a finite value, $I_i(a)e^{-a/\lambda_0}$.

Fig. 1 shows the effect of charge exchange on the ion flux in a free jet emerging from a Hall thruster with an exhaust cross sectional area, S_0, of 25 cm^2, operating at a mass flow rate, $\dot{m}$, of 2.2 mg/s with a propellant utilization, η_p, of 80% and ion velocity, v_i, of 20,000 m/s. The neutrals are assumed to have a velocity, v_n, of 400 m/s and to expand in an angle, θ_n, of 30° corresponding to a solid angle $\Omega_n = 2\pi(1 - cos\theta_n)$. Under these conditions we have: $n_0 = 2.04 \cdot 10^{12}$ cm^{-3} and $a = 5.4$ cm. σ_{ce} was calculated using the formula of Pullins et al. [22]:

$$\sigma_{ce}(Xe, Xe^+) = (142.21 - 23.3 \cdot log_{10}(g)) \cdot 10^{-20} \;\; m^2, \tag{5}$$

where g is the relative ion-neutral velocity. Shown also in Fig. 1 are the effects, on the same ion flux, of constant ambient densities of $3.54 \cdot 10^{12}$, $3.54 \cdot 10^{11}$ and $7.07 \cdot 10^{10}$ cm^{-3}, corresponding respectively to chamber ambient pressures of 10^{-4}, 10^{-5} and $2 \cdot 10^{-6}$ Torr (300°K). As can be seen, unlike the vacuum chamber cases, the charge exchange effect in the free jet case is small and limited to the near vicinity of the thruster.

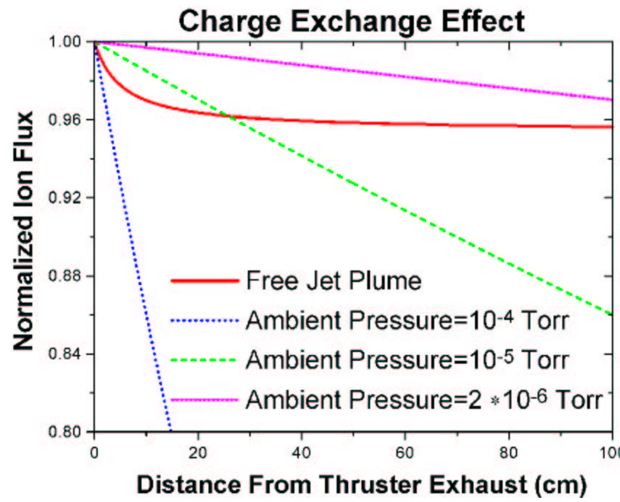

Figure 1: Free jet ion flux losses due to charge exchange for a Hall thruster with $S_0 = 25$ cm^2, $v_i = 20,000$ m/s, $\theta_n = 30°$ and $n_0 = 2.04 \cdot 10^{12}$ cm^{-3}. Shown also are the charge exchange ion flux losses due to vacuum chamber ambient pressures of 10^{-4}, 10^{-5} and $2 \cdot 10^{-6}$ Torr (300°K).

3. Far field analysis

3.1 Model assumptions and equations

The plume plasma consists of collisionless heavy ions (typically xenon) and of light electrons which, under their own thermal pressure, tend to run away from the plume. As a result, an electric field is created which pushes the ions outwards and hence enhances the transversal expansion of the plume.

Neglecting electron inertia ($m_e \to 0$), the electron momentum equation represents a balance between the thermal pressure and the electric field:

$$en_e \nabla\Phi \approx \nabla P_e, \tag{6}$$

where e, n_e and P_e are respectively the electron charge, density and pressure, and Φ is the electric potential. Taking the electron gas to be perfect ($P_e = n_e kT_e$) and isothermal, the electron density and the potential are related by the Boltzmann relation:

$$\Phi = \frac{kT_e}{e} ln(n_e/n_{00}), \tag{7}$$

where T_e is the electron temperature, k is the Boltzmann constant and n_{00} is a reference density.

It has to be mentioned here that the isothermal assumption is not arbitrary but results from the large electron thermal conductivity which dominates heat transfer in the plume and tends to minimize temperature gradients. To see that, we compare conduction along the plume, $-\frac{\partial}{\partial z}(\kappa \frac{\partial T_e}{\partial z}) \sim \kappa T_e/L^2$, with convection, $\rho v \frac{\partial}{\partial z}(C_p T_e) \sim \rho v C_p T_e/L$:

$$\frac{1}{P_{ec}} = \frac{conduc.}{convec.} = \frac{\kappa}{\rho v C_p L} = \frac{\pi}{\dot{m}} \frac{R}{(L/R)} \frac{\kappa}{C_p}, \tag{8}$$

where P_{ec} is the Peclet number, κ and C_p are respectively the electron heat conductivity and capacity, L and R are respectively axial and transversal characteristic lengths of the plume, ρ and v are respectively the flow (ion) density and velocity and $\dot{m}$ is the total ion mass flow emerging from the thruster. Note that Eq. (8) is equivalent to the heat flux criterion in [20]. Using the expression for κ in [23], it can be shown that $\kappa/C_p = 0.133 \cdot T_e^{5/2}/\lambda$, where λ is the coulomb logarithm and the numerical coefficient is for xenon, T_e is given in eV and κ/C_p in CGS units. At the vicinity of the exhaust of a Hall thruster, $L \approx 25$ cm, at typical operating conditions: $\dot{m} \approx 2$ mg/s, $T_e \approx 3$ eV and $L/R \approx 5$, we get $1/P_{ec} \approx 250$, indicating that the isothermal assumption is a good approximation there. Moreover, from (8) we can see that, if the isothermal assumption holds in the thruster vicinity, then neglecting the weak dependence on λ, $1/P_{ec} \sim L$, i.e., the isothermal assumption is more justified downstream.

The ion motion due to the electric field is described by the ion momentum equation:

$$\mathbf{v} \cdot \nabla \mathbf{v} \approx -\frac{e}{M} \nabla \Phi \tag{9}$$

where M is the ion mass. Quasi-neutrality is assumed also, $n_e \approx n_i \equiv n$, implying that $\Delta n \ll n$, where n_i is the ion density and $\Delta n = n_i - n_e$. Then using Eq. (7), Eq. (9) can be written as:

$$\mathbf{v} \cdot \nabla \mathbf{v} = -c^2 \nabla (ln\rho), \tag{10}$$

where $c = \sqrt{kT_e/M}$ is the ion acoustic velocity, and $\rho = Mn$. Eq. (10) together with the continuity equation,

$$\nabla \cdot (\rho \mathbf{v}) = 0, \tag{11}$$

form the set of equations describing the plume flow.

The isothermal and the quasi-neutrality assumptions are commonly used in plasma plume modeling. On the other hand, the cold fluid assumption for the ion flow (Eq. (9)), instead of a kinetic or particle approach, is the main simplification that enables us to look for analytical solutions. In the case of a vanishing ion temperature the justification of this approach is straightforward. There are however experimental indications that in the case of the Hall thruster the ion flow, emerging from the thruster exhaust with a typical energy of 250 eV, has an energy spread of 20-40 eV (see for example [6,13]). Nevertheless, far enough from the thruster the angle by which it is seen is narrow. It is assumed then that the spread in the directions of individual particles, arriving from the thruster and passing through a point at the far field, diminishes, and as a result we can use there the cold fluid approximation for the ions.

3.2 Self-similar flow model for the Hall thruster plume

In the case of the Hall thruster, we use a cylindrical coordinate system and start to look at the flow from a plane, $z = 0$, located a distance d from the thruster exhaust. At that distance, it is assumed that all near field effects, including the annular plume profile, the residual magnetic field, charge exchange and ion-neutral collisions, have decayed. Moreover, vacuum chamber measurements of ion kinetic energy distribution in the plume of Hall thrusters [6,13] have indicated that the energy is almost independent of the flow direction over a wide range of angles. It is therefore assumed that at the plane $z = 0$ the ion velocity amplitude, v_0, is independent of the direction as seen from the center

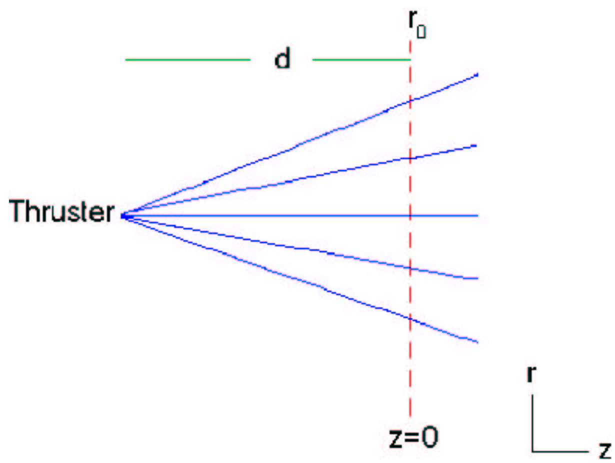

Figure 2: Schematic of the boundary conditions of the flow model for the Hall thruster far field plume. The analysis starts from the $z = 0$ plane, located a distance d from a point representing the thruster exhaust, from which the flow is assumed to emerge.

of the exhaust plane (see Fig. 2). Then, the axial and radial components of the velocity at $z = 0$ are given by:

$$v_{z0} = \frac{v_0}{[1 + (r_0/d)^2]^{1/2}}; \qquad v_{r0} = v_{z0}\frac{r_0}{d}, \tag{12}$$

where r_0 is the radial coordinate at the plane $z = 0$.

In order to simplify, and since we are interested mainly in the transversal expansion of the plume, we will neglect at this stage changes in the axial velocity, i.e., we take $v_z = v_{z0}$ and solve the radial component of the equation of motion:

$$v_z\frac{\partial v_r}{\partial z} + v_r\frac{\partial v_r}{\partial r} = -c^2\frac{\partial ln(\rho)}{\partial r}. \tag{13}$$

We look for a self-similar flow of the type $r = r_0 f(z)$, where $f(z)$ is the stream-line function. It follows that $v_r = d \cdot v_{r0} df/dz$. The same type of self-similar flow was used by Parks and Katz [19] to obtain a solution for the problem of a cylindrical quasi-neutralized ion beam. The main difference is that in [19] $v_z = v_0$, while in our model the boundary velocity components are given by Eq. (12). In a sense, our problem can be regarded as a generalization of the one treated in [19] to include the wide "beam" case (plume), a modification which more realistically describes the plume of Hall thrusters.

3.3 Solution

The self-similar flow $r = r_0 f(z)$ can be regarded as a coordinate transformation: $(r, z) \to (r_0, z)$. From the continuity equation we get then: $\rho = \rho_0/f^2(z)$. Plugging this relation and the self-similar flow in the momentum equation leads to variable separation and solution for the density and the stream-line function. The expression for the self-similar density is:

$$\rho = \frac{\rho_{00}}{f(z)^2}[1 + \frac{r^2}{d^2 f(z)^2}]^{-d^2/2R^2}, \tag{14}$$

from which the potential is derived using Eq. (7):

$$\Phi = -\frac{kT_e}{e}[2ln(f(z)) + \frac{d^2}{2R^2}ln(1 + \frac{r^2}{d^2 f(z)^2})]. \tag{15}$$

It is interesting to note here that when d becomes very large, the density profile described by Eq. (14) converges to the Gaussian profile obtained in [19] for the cylindrical beam case.

The stream-line function is given implicitly by the relation:

$$\alpha z = \int_1^f \frac{dx}{[(\alpha d)^{-2} + ln(x)]^{1/2}}, \tag{16}$$

where

$$\alpha = \frac{\sqrt{2}c}{v_0 R} = \sqrt{\frac{kT_e}{\frac{1}{2}Mv_0^2}}\frac{1}{R},$$

and R is a length characterizing the plume width at $z = 0$. The radial velocity is then given by:

$$v_r = v_{r0}[1 + \alpha^2 d^2 ln(f(z))]^{1/2}. \tag{17}$$

For large z, $f(z)$ can be approximated by:

$$r = r_0 f(z) \approx r_0\frac{z}{d}[1 + \alpha^2 d^2 ln(\alpha z)]^{1/2}, \tag{18}$$

where the error goes as $ln^{-1}(\alpha z)$. Eq. (18) indicates that the deviation of the stream-line from straight line (no far field interaction) is logarithmic. In Fig. 3 we can see a comparison between the exact and approximate solutions for the stream-line

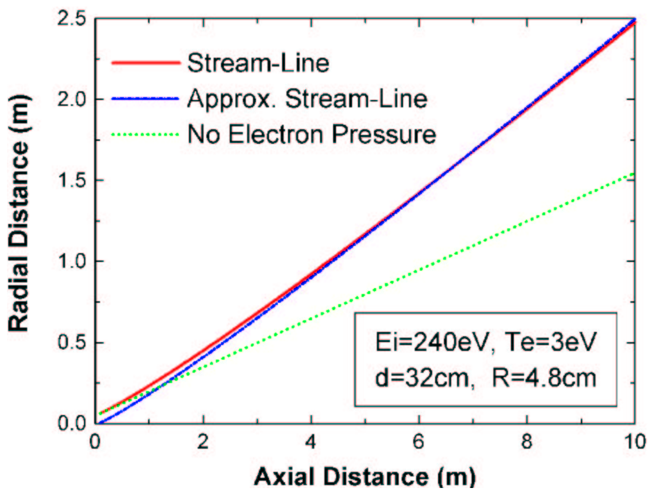

Figure 3: A comparison between exact and approximate solutions for the $r_0 = R$ stream-line. $E_i = 240$ eV, $kT_e = 3$ eV, $d = 32$ cm and $R = 4.8$ cm. The $kT_e = 0$ case is shown also.

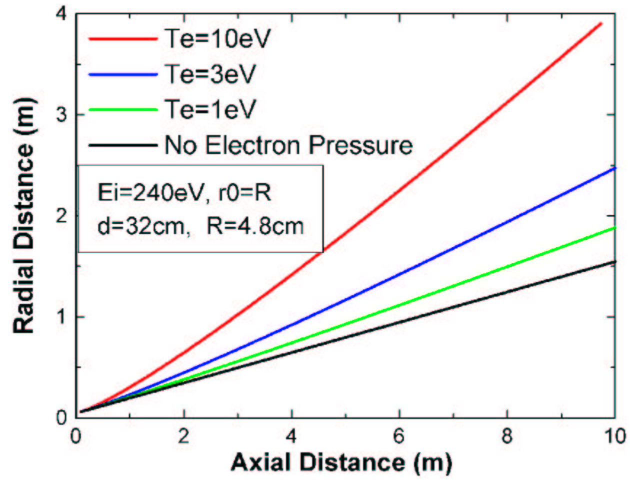

Figure 4: The stream-line $r_0 = R$ for $kT_e = 0$, 1 eV, 3 eV and 10 eV. $E_i = 240$ eV, $d = 32$ cm and $R = 4.8$ cm.

$r_0 = R$, for $E_i \equiv \frac{1}{2}Mv_0^2 = 240$ eV, $kT_e = 3$ eV, $d = 32$ cm and $R = 4.8$ cm. As can be seen, Fig. 3 demonstrates a very good agreement. Shown also (dotted green line) is the $kT_e = 0$ (no electron pressure) case for the same parameters.

4. Results & discussion

The self-similar solution described by Eqs. (14) and (16) was used to investigate the effects of the electron temperature and the near field plume width on the far field plume expansion. In this investigation, d and R were taken to be 32 cm and 4.8 cm respectively, corresponding to the distance at which probe plume measurements were performed with a SOREQ Hall thruster [24] and the width obtained by fitting the measured plume density profile there to the functional dependence of Eq. (14) for $\rho(z = 0, r)$. The kinetic energy of the ions at thruster exhaust was assumed to be 240 eV, corresponding to a thruster operating voltage of 300 V and a typical voltage utilization of 80%.

In Fig. 4, the stream-line $r_0 = R$ is plotted for the first 10 meters and for four electron temperature cases: $kT_e = 0$ (no electron pressure), 1 eV, 3 eV and 10 eV. The widening effect of the electron temperature is clearly visible. The effect of the electron temperature is also demonstrated in Figs. 5 and 6 showing the self-similar density profiles at axial distances of 2 m, $\rho(z = 2m, r)$, and 10 m, $\rho(z = 10m, r)$, respectively for the same four temperature cases. By comparing Figs. 5 and 6 one can see also that the relative differences between the different temperature cases increase with the distance.

Fig. 7 demonstrates the effect of the near field plume width. It compares the stream-line $r_0 = R$ for $R = 4.8$ cm, $E_i = 240$ eV and $kT_e = 3$ eV between two cases: $R/d = 0.15$ ($d = 32$ cm) and $R/d = 0.075$ ($d = 64$ cm). Shown also (dotted) are the $kT_e = 0$ stream-lines, representing in fact the near field widths in the two cases. As can be seen, the relative increase in the plume width is larger for the narrower plume ($R/d = 0.075$). This behavior can be understood to be a result of the fact that the narrower is the plume the larger is the electron pressure gradient (electric potential gradient), leading to a larger transversal force (see Eq. (13)). This result indicates that if the near field plume width could be reduced, for example by modifying the thruster, the electron pressure far field effect

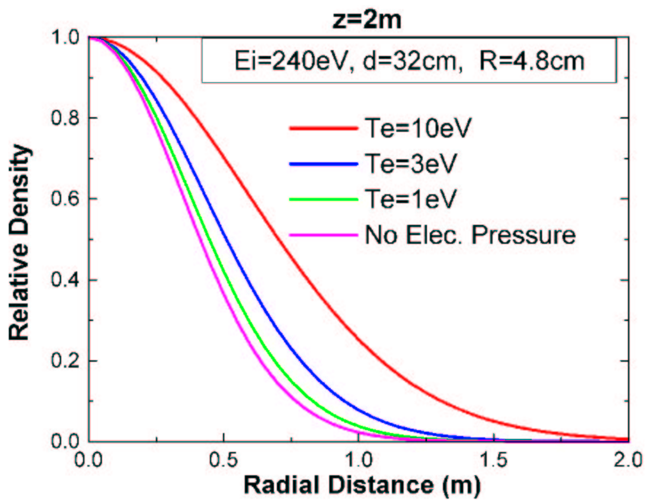

Figure 5: The self-similar density profiles at $z = 2$ m for $kT_e = 0$, 1 eV, 3 eV and 10 eV. $E_i = 240$ eV, $d = 32$ cm and $R = 4.8$ cm.

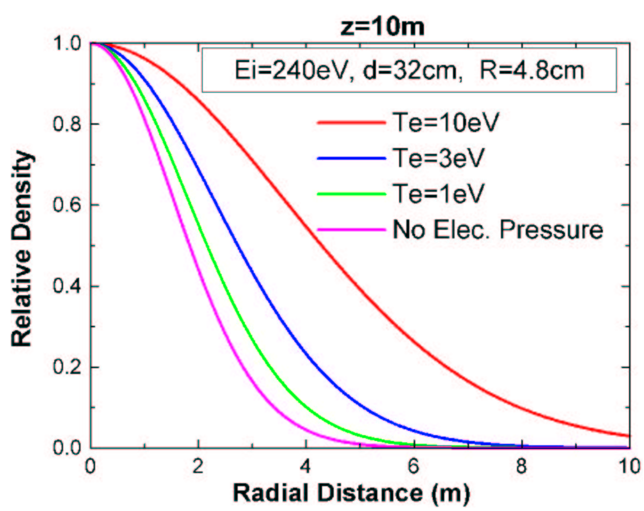

Figure 6: The self-similar density profiles at $z = 10$ m for $kT_e = 0$, 1 eV, 3 eV and 10 eV. $E_i = 240$ eV, $d = 32$ cm and $R = 4.8$ cm.

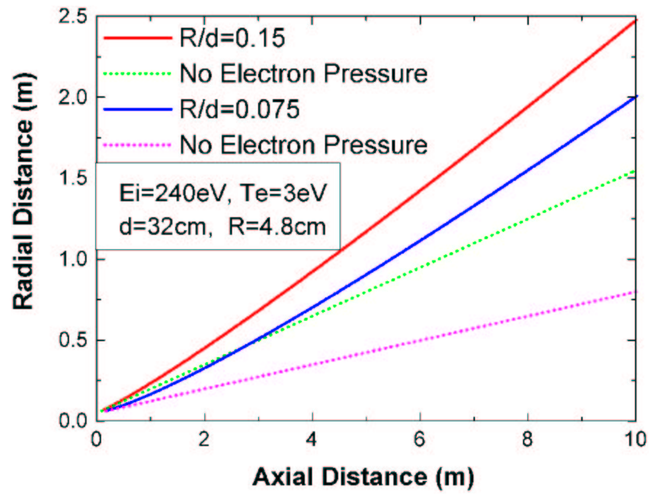

Figure 7: The stream-line $r_0 = R$ for $R/d = 0.15$ ($d = 32$ cm) and $R/d = 0.075$ ($d = 64$ cm). $kT_e = 3$ eV, $E_i = 240$ eV and $R = 4.8$ cm. The $kT_e = 0$ stream-lines, representing the near field widths are shown also (dotted lines).

would become more important.

5. Conclusions

An analytical model, describing the far field behavior of plasma plumes of electric thrusters, was presented. A self-similar solution was obtained, demonstrating a logarithmic expansion at large distances. It was used to calculate stream-lines and density profiles for a set of electron temperature and near field cases. The results demonstrate that plume expansion is faster for a larger ratio of electron temperature to ion kinetic energy, and that the relative increase in plume width due to electron pressure is larger for a narrower near field plume.

While these results indicate the usefulness of the model in describing plume behavior at large distances at a minimal computational burden and in getting an insight into the dependence of the expansion on thruster and plasma parameters, the validity and limitations of the model, especially with regard to the isothermal and the quasineutrality assumptions and the behavior at very large angles, are being investigated.

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