Solar-Electric Propulsion: The “Flying Carpet” Concept

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Abstract

A new concept is described for propelling deep-space scientific probes to very high velocities of escape from the solar system, far greater than what is possible with chemical propulsion and multiple planetary flybys. The concept depends on the development of very thin photoelectric materials, most likely GaAs, deposited on a plastic film such as Kapton. As an example, a mass-to-area ratio of the photoelectric membrane of 135 grams/m² and an area of 185 m² are assumed. (This compares to 905 grams/m² for the Hubble Space Telescope at the blanket level, not counting the structural supports). In the described concept, in place of structural supports, the membrane is tensioned, stabilized and attitude controlled by ion engines at the corners of the membrane. These engines also provide the thrust for the vehicle. The voltage supplied to the ion engines is constant, but the mass flow rate of the Xenon propellant is controlled to vary proportional to the solar energy input, i.e., indirectly proportional to the square of the distance from the sun. A concept is also described for the deployment, aided by the ion engines, of the membrane in a micro gravity environment. It is also noted that the area of the photoelectric membrane, as described, is far smaller than for a solar sail with the same thrust. Thus, for the parameter values assumed in this paper, the area of the photoelectric membrane is smaller by a factor of 1600 compared with a solar sail.

Introduction

The concept is illustrated in Fig.1. A large, low weight membrane consisting of solar electric elements deposited on a plastic film supplies the electric power to operate ion engines or Hall thrusters. The largest of the four engines shown in the figure is an integral part of the spacecraft. Three smaller engines, with thrust vector control, are used to tension, stabilize and attitude control the membrane. They also provide a portion of the total thrust. No structural members are used to support the membrane, hence the designation of Flying Carpet, which is taken from the tale in Thousand and One Nights.
The needed ion engines are close to the current state of the art. The membrane and its control by the ion engines are not. For the purpose of the example shown below, a membrane with a mass-to-area ratio of 135 grams/m², solar-electric conversion efficiency of 0.20 with gallium arsenide and an area of 185 m² is assumed. Some brief consideration is also given to amorphous Si with a conversion efficiency of 0.020. The generated voltage, and with it the net accelerating potential of the ion engines and the specific impulse, are kept constant, independent of the distance from the sun. However the mass flow rate of the propellant (xenon), and hence the ion beam current, are controlled to be proportional to the solar radiation input, hence indirectly proportional to the square of the distance from the sun.

An earlier paper\(^{(1)}\) was based on a rough approximation of a straight line trajectory. In this paper, an exact trajectory for escape from the solar system is calculated. This trajectory is assumed to start from the minimum (parabolic) Earth escape velocity and is based on the thrust being at all times parallel to the path.

The energy difference between the minimum Earth escape velocity and the final total energy of the vehicle at infinity obtained with the flying carpet concept, can be several times larger than the energy difference for reaching minimum Earth escape starting from LEO. For this reason, only a rough estimate will be made of the additional propellant loading, were the vehicle to start from LEO.

*Analysis*

A short analysis of the energetics, the trajectory, the ion engine voltage and of the needed beam current is made.

If \( r \) is the distance from the sun, \( r_0=1 \text{ AU} \), \( J_0 \) the solar constant=1360 W/m² and \( A_e \) the area of the photoelectric membrane, assumed to be oriented perpendicular to the line of sight to the sun, then the incident solar power \( P_h \) is given by

\[
P_h(r) = A_e J_0 (r_0/r)^2
\]  
(1)
Also, if $\eta_e$ is the solar to electric conversion efficiency of the membrane, and $\eta_i$ the ion engines' input to beam-power efficiency, the ion beam power (combined for all engines) $P_i$ is given by

$$P_i(r) = \eta_e \eta_i A_0 J_0 (r_0/r)^2 = \frac{1}{2} m^*(r) u_i^2 = \frac{1}{2} u_i^2 dM/dt$$ (2)

where $m^*$ is the combined mass flow rate of the engines, $u_i (= \text{const.})$ the ion beam velocity relative to the spacecraft, and $M(t)$ the mass of the vehicle.

The combined thrust of the engines is designated by $F$ and is given by

$$F(r) = u_i m^*(r) = -u_i dM/dt$$ (3)

The forces acting on the vehicle are illustrated in Fig.2. Here, $v$ is the vehicle's velocity in the heliocentric reference frame, $\rho$ the radius of curvature of the path, $\beta$ the angle between the normal to the path and the sun line, and $\mu_s$ the solar gravitational parameter, $\mu_s = 1.32712 \times 10^{11} \text{ km}^3 \text{s}^{-2}$. The path will be described by polar coordinates $r$ and $\phi$, with the origin in the sun. Differentiation with respect to $\phi$ will be designated by a prime $'$. 

It follows that $\tan \beta = r'/r$, hence $\cos \beta = [1 + (r'/r)^2]^{-1/2}$. If $ds$ is the increment of path length, the corresponding time increment $dt$ is given by

$$dt = ds/v = rv^{-1}[1 + (r'/r)^2]^{1/2} d\phi$$

Hence from (2)

$$M' = -2 \eta_e \eta_i A_0 r_0^2 u_i^2 r^{-1} v^{-1}[1 + (r'/r)^2]^{1/2}$$ (4)

If, as postulated, the thrust is applied in the direction of the path, then $\rho$ is determined by $v^2/\rho = \mu_s r^2 \cos \beta$, as indicated in Fig.2. In polar coordinates, the curvature is given by

$$1/\rho = (r^2 + 2 r'^2 - r r'')(r^2 + r'^2)^{-3/2}$$

The equation of motion therefore becomes, from the expression (3) for the thrust,
\[ M \, \text{d}v/\text{d}t = - u_i \, \text{d}M/\text{d}t - M \, \mu_h \, r^{-2} \, \sin \beta \]

hence

\[ v' + u_i \, M^{-1} \, M' + \mu_h \, r^{-2} \, v^{-1} \, r^t = 0 \]  \hspace{1cm} (5)

In order for the parallel component of the vehicle’s, acceleration to be positive along the path, it is necessary that the thrust exceed the parallel component of the solar gravity. This inequality is illustrated in Fig.2.

The remaining equation that is needed is obtained from equating the normal component of the solar gravitational acceleration to the centrifugal acceleration. Together with the expression for the path curvature, this results in

\[ r'' + (\mu_h \, r^2 \, v^{-2} - 2/r) \, r'^2 - r + \mu_h \, v^{-2} = 0 \]  \hspace{1cm} (6)

Hence, (4), (5) and (6) are three coupled differential equations for \( r, v, \) and \( M \) as functions of \( \phi \). The system is of order four. Starting from the orbital path of the Earth, with the minimum Earth escape velocity \( v_0 \) and mass \( M_0 \), the initial conditions are

at \( \phi = 0 \):

\[ r = r_0 \quad r' = 0 \quad v = v_0 \quad M = M_0 \]  \hspace{1cm} (7)

Following the integration of the system, the other quantities of interest are easily obtained: Thus, if \( m_x \) is the mass of the Xenon ion and \( e \) the universal electric charge, the net acceleration potential, \( V_i \), of the ion engines is

\[ V_i = \frac{1}{2} \, (m_x/e) \, u_t^2 = \text{const.} \]  \hspace{1cm} (8a)

The ion current \( I_i = P_i/V_i \) is

\[ I_i = \eta_e \eta_i A_c J_0 (r_0/r)^2 \left[ \frac{1}{2} \, (m_x/e) \, u_t^2 \right]^{-1} \]  \hspace{1cm} (8b)

The mass flow rate \( m^* \) and thrust \( F \) follow from

\[ m^* = - v \, r^{-1} \left[ 1 + (r'/r)^2 \right]^{-1/2} \, M' \]  \hspace{1cm} (8c)

\[ F = - u_i \, v \, r^{-1} \left[ 1 + (r'/r)^2 \right]^{-1/2} \, M' \]  \hspace{1cm} (8d)
Solar-electric membrane

Although speculative, the concept of a flexible, tensile solar array (here for short simply referred to as “membrane”) suggests several directions for research, particularly in the field of vapor or chemically deposited thin films. To judge the effort likely to be needed, some data are presented for comparison.

The calculation in the next Section assumes a solar to electric conversion efficiency of 20%. For much heavier solar arrays, this is within the current state of the art of mono-crystalline semiconductors. At present, amorphous silicon (which can be deposited in very thin layers), triple layered (three n and three p layers), is at best about 3% efficient \(^{(2)}\).

For most of the calculation in the next Section, a mass-to-surface ratio of 135 gram/m\(^2\) is assumed. In Table 1, several other such data are given for comparison.

<table>
<thead>
<tr>
<th>mass-to-surface ratio</th>
<th>Hubble Space Telescope solar array (Si on kapton sheets, at the blanket level, omitting all structural supports)</th>
<th>905 gram/m(^2)</th>
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<tbody>
<tr>
<td>Proposed Space Power Satellite (NASA) (omitting structural supports)</td>
<td></td>
<td>200</td>
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<tr>
<td>kapton, 7.6 μm thick (smallest thickness commercially available)</td>
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<td>aluminum conductor at 20 °C, (calculated for a solar array with I_{s,max} = 81 A, voltage drop = 0.01 V)</td>
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<td>0.3</td>
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<td></td>
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<tr>
<td>kapton supported mono-crystalline film</td>
<td></td>
<td>135</td>
</tr>
<tr>
<td>kapton supported amorphous film</td>
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<td>13.5</td>
</tr>
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Table 1: Comparison of some mass-to-surface ratios
Results of a Sample Calculation

A small deep space probe with a final mass of 125 kg is considered. This final mass is made up of the payload, the solar-electric membrane, and the ion engines, together with their propellant feed system and power conditioner. The vehicle is assumed to start out from the minimum velocity required for Earth escape, i.e. with an initial velocity, $v_0 = 29.8$ km/s in heliocentric coordinates, corresponding to the Earth orbital velocity. The propellant loading is 500 kg. (If, instead, the vehicle were started from LEO, the initial propellant load would be approximately 850 kg).

Performance data are summarized in Table 2. As noted before, the mass flow rate is controlled to decrease as the vehicle's distance from the sun increases. To simplify the calculation, it is assumed that the mass flow rate goes to zero at infinite distance. But, exhausting the propellant at a finite, but large, distance, say at 3.0 AU, does not appreciably affect the results).

The escape velocity at an infinite distance from the sun is 60.4 km/s, far higher than what can be achieved by multiple planetary flybys.

Fig.3 shows the calculated trajectory and the velocity at several points along it. In Fig.4, the velocity is graphed as a function of the distance from the sun. It is seen to increase to a maximum of 61.6 km/s before slightly decreasing to the asymptotic value of 60.4 km/s = 12.7 AU/year. This velocity greatly exceeds the one of Voyager 1, launched in 1977 and currently the man made object with the largest distance from the Earth and Sun. (The Voyager 1 velocity is 3.63 AU/year as of January 1, 2001).

The elapsed time after the initial condition at 1 AU is shown in Fig.8.

Deployment

The deployment of large, flat membranes in a micro gravity environment is a major problem in engineering design.

Several configurations, have been proposed, such as origami tessellations. Here, we briefly discuss the deployment of a rectangular membrane that is originally stored in a rolled-up condition. Fig.6, by example, shows the deployment from the payload bay of the Space Shuttle.
mass:
- payload, ion engines, power conditioning, etc.  \( m_{\text{net}} = 100 \text{ kg} \)  \( = \) \( \text{same} \)
- solar-electric membrane \( m_e = 25 \text{ kg} \) \( = \) \( \text{same} \)
- propellant (Xenon) \( m_x = 500 \text{ kg} \) \( = 0 \)
- vehicle, total \( M = 625 \text{ kg} \) \( = 125 \text{ kg} \)

ion engines:  \( \eta_i = 0.85 \)
- beam velocity relative to vehicle \( u_i = 30.10^3 \text{ m/s} \) \( = \) \( \text{same} \)
- specific impulse \( I_{\text{sp}} = 3060 \text{ s} \) \( = \) \( \text{same} \)
- accelerating potential \( V_i = 620 \text{ V} \) \( = \) \( \text{same} \)
- current (total, all engines) \( I_i = 69 \text{ A} \) \( = 0 \)
- mass flow rate (total, all engines) \( m^* = 95 \text{ mg/s} \) \( = 0 \)
- thrust (total, all engines) \( F = 2.80 \text{ N} \) \( = 0 \)

solar-electric membrane:
- output power \( P_e = 50 \text{ kW} \) \( = 0 \)

area:
- gallium arsenide, \( \eta_e = 0.20 \) \( A_e = 184 \text{ m}^2 \) \( = \) \( \text{same} \)
- amorphous Si, \( \eta_e = 0.02 \) \( A_e = 1840 \text{ m}^2 \) \( = \) \( \text{same} \)

mass per unit area:
- gallium arsenide \( 135 \text{ gram/m}^2 \) \( = \) \( \text{same} \)
- amorphous Si \( 13.5 \text{ gram/m}^2 \) \( = \) \( \text{same} \)

acceleration: \( \text{max.} = 0.64 \times 10^{-3} \text{ } g_0 \) \( = 0.49 \times 10^{-3} \text{ } g_0 \) \( = 0 \)

velocity:
- heliocentric velocity of vehicle \( v = 29.8 \text{ km/s} \) \( = 60.4 \text{ km/s} \)
- radial component \( v_r = 0 \) \( = 60.4 \text{ km/s} \)

<table>
<thead>
<tr>
<th></th>
<th>initial</th>
<th>final</th>
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<td>( m_{\text{net}} )</td>
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</tr>
<tr>
<td>( m_e )</td>
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</tr>
<tr>
<td>( m_x )</td>
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</tr>
<tr>
<td>( M )</td>
<td>625 kg</td>
<td>125 kg</td>
</tr>
<tr>
<td>( u_i )</td>
<td>( 30.10^3 ) m/s</td>
<td>same</td>
</tr>
<tr>
<td>( I_{\text{sp}} )</td>
<td>3060 s</td>
<td>same</td>
</tr>
<tr>
<td>( V_i )</td>
<td>620 V</td>
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<td>( I_i )</td>
<td>69 A</td>
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</tr>
<tr>
<td>( m^* )</td>
<td>95 mg/s</td>
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</tr>
<tr>
<td>( F )</td>
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</tr>
<tr>
<td>( P_e )</td>
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<td>( A_e )</td>
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<td>( A_e )</td>
<td>1840 m(^2)</td>
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<td>( \eta_e )</td>
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<tr>
<td>( \eta_e )</td>
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<tr>
<td>( v )</td>
<td>29.8 km/s</td>
<td>60.4 km/s</td>
</tr>
<tr>
<td>( v_r )</td>
<td>0</td>
<td>60.4 km/s</td>
</tr>
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</table>

Table 2: Calculated performance data for a Flying Carpet deep-space probe. Membrane mass ratio = 135 gram/m\(^2\)
Orbiter. As indicated in the figure, ion engines are located at each corner of the membrane, one of them being an integral part of the spacecraft and providing most of the thrust.

The spacecraft is attached to one of the two corners that are on the innermost winding of the roll. In unrolling, the spacecraft therefore rotates with the spindle. During deployment, the roll is unwound by a motor which is a part of the support structure in the payload bay. At the same time, the membrane is pulled and stabilized by the two ion engines already off the roll.

Stability and control pose major theoretical and experimental problems. Their analysis remains to be done.

Conclusions

Given the development of very light weight photoelectric membranes with mass ratios of 135 gram/m² or less, it appears to be possible to develop deep space probes that are characterized by very large escape velocities from the solar system, with velocities greatly exceeding what is possible with planetary flybys. Such probes should make it possible to explore the interstellar medium with travel times less than ten years.

In the sample calculation presented in this paper, the final velocity of the space probe in heliocentric coordinates is 60.4 km/s. Much higher velocities yet could probably be achieved by a solar swing-by at a distance from the sun of 0.5 AU or less, as is schematically indicated in Fig.(7).

References:


(2) Courtesy Siemens Solar GmbH, Munich
The "Flying Carpet" Concept

Fig. 1: Spacecraft and photoelectric membrane, deployed

initial mass = 625 kg
final mass = 125 kg
Fig. 2: Schematic trajectory and acceleration components of the vehicle.
At $r = \infty$, $v = 60.4 \text{ km/s}$.

Earth Orbit

Fig. 3: Computed trajectory and velocity in heliocentric coordinates
Fig. 4: Velocity in heliocentric coordinates as a function of distance from the sun. $k = \text{mass of the membrane per unit area}$
Fig. 5: Elapsed time as a function of distance from the sun. $k = $ mass of the membrane per unit area
Fig. 6: Proposed deployment of the membrane and spacecraft in a microgravity environment. The deployment is assisted by the thrust of the two outer ion engines.

Fig. 7: Schematic of solar swing-by with enhanced solar-electric power input.