Optical Emission of a Hall Thruster Plume in Space Condition\textsuperscript{*†}

Georgie Karabadzhak, Flur Gabdullin, Anatoli Korsun, Yuri Plastinin and Ekaterina Tverdokhlebova
Centrak Research Institute of Machine-Building (TSNIIMASH)
Pionerskaya street, 4, Korolev, Moscow region, Russia, 141070
Phone: +7-095-513-5057
E-mail: sot@tse.ru
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A semi-empirical method for modeling the emission of a xenon Hall thruster plume in visible wavelength region under space vacuum condition is considered. The method overcomes the complex issues related to creation of adequate collisional-radiative model for the radiation modeling. Radiation intensity measured in the plume of a thruster with anode layer in laboratory test facility is analyzed from the standpoint of different processes contributing the xenon particle level population. The analysis allowed to separate the radiation excitation processes taking place in space from those related purely to residual background gas in the vacuum chamber. As the result a method for extrapolation of the radiation measured in the laboratory to the space condition is suggested. The method is illustrated on computation of the radiation maps of D-80 thruster plume in space. The radiation intensity of the space plume appeared to be less than in vacuum chamber with residual pressure $p \approx 10^{-4}$ Torr. Also, the radiation intensity of the Hall thruster vacuum plume is shown to be dependent on the angle between the thruster axis and geomagnetic field direction. Maximum of radiation intensity integral over visible region of spectrum is predicted to be order of $10^{-2}$ W/m$^2$ sr for the D-80 thruster.

Introduction

Plume properties of the Hall effect thrusters have been discussed widely in the electric propulsion community with regard to the plume impact on the satellite systems. Most of the discussion is aimed to consideration of the contamination effect, spattering of sensitive surfaces and radio-wave communication interference. From the other hand new surveillance, optical remote sensing and communication space technologies are intensively developing nowadays. Some of these high sensitive devises may happen to operate in the simultaneously with a Hall thruster changing the satellite orbit and/or position. So, the question arises how big interference to the performance of a high sensitive optical sensor one should expect from an operating Hall effect thruster.

No experimental data on the optical emission of a Hall effect thruster plume in space have been reported so far. Most of the plume emission measurements were made in a laboratory vacuum chambers. Typically, these measurements provide scanty and not fully adequate information on the plume radiation due to limited dimensions of the test chamber and finite value of the residual pressure in there. From the other side, purely theoretical modeling of the radiation processes in the plume can not be accomplished now, because of complexity of nature of the processes involved. In this paper an attempt has been made to consider the plume radiation basing on the combination of the computational modeling of the plume parameters (Ne, N$^+$) and experimental measurements of the radiation in the near field of the D-80 Hall thruster with anode layer (TAL).

Computation of the radiation is based on a collisional-radiative model. The model assumes that excitation of any level comes from collisions between particles in the plume. The level excitation is followed by its radiative decay. No other mechanism for de-excitation is considered. This model should be valid for conditions of rarified TAL plume plasma. It is commonly used for computation the Hall effect thruster radiation. Most complex part of this model is related to the level excitation/de-excitation kinetics.

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Information on corresponding rate constants and cross-sections is very poor so far. So, for prediction of the plume radiation in space we use an experimental data on the TAL radiation obtained in vacuum chamber, not directly, however, but extrapolating them to the space condition. The extrapolation itself is based on very general statements of the collisional-radiative model.

In the first section of the paper the D-80 plume properties (particle number densities in most) are modeled. Next part considers the plume radiation in detail. And some examples of computation of integral visible radiation ($\lambda=400-700$ nm) along particular lines of sight in the D-80 space plume are presented in the last part of the paper.

**Modeling of the TAL plume particle number densities**

The Hall effect thruster plume flow utilizing xenon propellant represents a beam of energetic xenon ions (hundreds of eV) compensated by electrons with much lower energies 1-10 eV. Expansion of such a flow in space is controlled by both self-consistent electric field inside the flow and outer geomagnetic field $B$. A model for description of the parameters of the plume flow (SSM model) has been developed in TSNIIMASH [1,2]. The model accounts for self-consistent electromagnetic fields and generation of secondary plasma in the primary flow. In this model the effect of geomagnetic field is characterized through magnetic interaction parameter $S$ (Ampere parameter), which value is equal to the ratio of transverse electromagnetic forces to pressure gradient:

$$S = \frac{\sigma B^2 \pi a^3}{mN}$$

Figure 1, Figure 2 and Figure 3 depict D-80 plume patterns as predicted by TSNIIMASH model in terms of lines of equal number densities $n_e [cm^{-3}]$, at altitude $h=1200$ km.

At the initial stage of expansion, while $S<1$, the gas kinetic pressure and electrostatic forces are higher than the electromagnetic ones. The magnetic field is being displaced partially out of the volume of plasma and has no effect on plume parameters yet. The nature of the plume expansion exhibits an inertial character. It expands isothermally. The plasma is quasi-neutral here, $N_e = N_i$ and thermally-conductive; the temperature weakly falls along the plume axis from about 4 eV down to 2 eV [3]. In radial direction the temperature is approximately constant due to the high thermal conductivity of the electrons. Figure 1 illustrates the electron number density distribution in this case.

Parameter $S$ increases while the plume expands, and at some distance $x = x_B$ the inequality $S \geq 1$ becomes valid and dynamic pressure becomes comparable to electromagnetic force. According to the SSM model [1] magnetic field deforms the plume configuration here. Plasma expansion across the magnetic field decelerates to the speed of cross diffusion of plasma in a magnetic field $B$. Speed of expansion along the field $B$ becomes equal to the speed of a 1-D inertial expansion. As a result the difference of the speeds affects the configuration of the plasma plume. This effect depends on pitch-angle $\beta$ ($\beta$ is the angle between the vector of the flow velocity $U$ and the magnetic induction vector $B$). In particular, when $\beta=0^\circ$ the lines of equal number density are elongated over the magnetic field line and the longitudinal dimension of the plume becomes much bigger than the transverse one. Electron temperature stays almost constant in the whole plasma plume, being around $T_e \approx 2$ eV (see Ref. [3,4]).

Figure 2 shows the transition of the plume flow from conical shape to a diffuse flow along a geomagnetic field line. Further downstream the flow keeps cylindrical shape even at the long distances from the thruster.

Region of the plume where $S>1$ is most lengthy one. At the altitude $h=1200$ km ionospheric background practically doesn’t influence the plume parameters in this region. The plume represents here a large-scaled but localized ionospheric non-uniformity. The plume shape is controlled by the geomagnetic field. Guided by the geomagnetic line shape the plume curvature becomes noticeable at the spatial scales bigger than 4-5 km (see Figure 3).
Figure 1 Distribution of the electron density $n_e [cm^{-3}]$ in the near field zone.

Figure 2 Change of the plume configuration in transient zone.
Figure 3 Distribution of the electron number density $n_e [cm^{-3}]$ in the far field zone.

**Computation of the plume radiation**

The following processes may potentially contribute to the plume radiation at an orbital condition:

1. Collision of the plume particles between each other,
2. Scattering of the sun radiation on the plume particles in the day time,
3. Plume luminosity under the solar wind proton and electron impact,
4. Plume luminosity under VUV solar radiation.

Even rough estimate of the intensity of these processes leads in the conclusion that first processes prevails over the rest. Its contribution to the total space plume radiation is substantially higher in all cases of interest. Therefore it is discussed in detail in this paper. Other processes have been simply neglected.

Radiation properties of a Hall thruster utilizing xenon propellant were studied in Ref. [5,6]. According to that studies radiation in visible region of spectrum (400-700 nm) is almost completely contributed by the XeI and XeII lines with dominating of XeII emissions. The radiation intensity distribution among the emissive lines was found to be highly non-equilibrium. That means, the excited level population can not be considered as a Boltzman’s one. Instead, kinetic equation allowing for all possible processes of the level population/de-population should be solved for each level. Indeed, this approach is extremely complex and can’t be used in a full scale. Therefore, a semi-empirical method basing on the radiation measurements in a vacuum chamber is applied in this paper for computing the space plume radiation.

Generally, any xenon propelled thruster plume in a vacuum chamber contains electrons, xenon ions (including multiply charged ions) and non-utilized xenon neutral atoms. Collisions between these particles may lead to their excitation followed by radiative decay. In current consideration we assume that no other de-excitation mechanism can exist in the rarified plume plasma. Such collisional-radiative model is proved to be valid in the hall thruster plume and used to apply commonly for the radiation prediction [6,7,8,9,10]. Numerous RPA measurements (see ref [11] for example) showed very small content of highly ionized xenon particles ($Z>2$) in the plume.
So, only collisions between neutral atoms $N_0$, electrons $N_e$, first and second xenon ions $N^+$, $N^{2+}$ are considered in this papers. Also, recombination reactions are neglected in comparison with excitation, because of relatively high energies of the colliding particles. Under these assumptions a set of reactions resulting potentially in excitation of xenon may be written as:

1. $e + Xe \rightarrow (Xe^+) + 2e$
2. $e + Xe^+ \rightarrow (Xe^+) + e$
3. $Xe^+ + Xe \rightarrow (Xe^+) + Xe$
4. $Xe^+ + Xe^{2+} \rightarrow (Xe^+) + Xe^{+2}$
5. $Xe^{+2} + Xe \rightarrow (Xe^+) + Xe^+$

In this set of equations only the processes of excitation of xenon first ion are listed. First reason for this is that XeII lines should dominate in the visible region of TAL radiation spectrum in space. Neutral xenon atoms diverge from the space plume very rapidly and don’t take part in generation of radiation. From the other hand, XeI lines may hardly be produced by collision of two charged particles. Intensive XeIII lines are located in the UV spectral region.

Excitation cross-sections for processes 1, 3 and 5 were reported recently [10,12] for a variety of spectral lines in visible and NIR spectral region. The cross-section for excitation of the ionic spectral lines in visible region in reactions 1,3 appeared to be much smaller in comparison with reaction 5. Evaluation of the reaction rates for typical electron and ion energies in the plume ($T_e=2-3$ eV, $E_{ion}=300$ eV) results in conclusion, that reactions 1,3 may be excluded from system (1), as having much smaller rate than reaction 5.

Specific radiation intensity of any XeII line may be expressed then as:

$$J_{i-j} = \frac{h \nu \cdot A_{i-j}}{\sum_k A_{i-k}} \left[ R_2 N_e N^+ + R_4 N^{+2} N^+ + R_5 N^{+2} N_0 \right]$$

where $R$ – corresponding line excitation rates in processes 2,4,5 and $A$ – spectroscopic constant that doesn’t depend on the plume properties.

Radiation measured in a vacuum chamber may potentially be generated by all three reactions 2,4,5, whereas radiation in space should not be resulted from the collisions involving neutral xenon atoms (reaction 5). As already has been mentioned. Neutral xenon atoms content in space falls very fast downstream from the thruster, due to the high divergence of the atomic flow. So, contribution of the neutral atoms to the radiation measured along any line of sight in the space plume is negligibly small in all cases of interest. That means, radiation measured in the vacuum chamber along particular line of sight in the plume differs from the radiation measured in space by direct contribution from reaction 5.

In addition, presence of the background gas in the vacuum chamber may affect the radiation indirectly through generation of secondary plasma in the charge exchange reactions. The process of secondary plasma generation was considered in [13]. Relative content of secondary plasma ions in some vacuum chambers may reach more than 50% value of total charge accumulated in the plume. Collisions between the plume flow particles (primary particles) and the secondary plasma particles lead in the radiation as well. From the other hand, amount of the secondary plasma in space is negligibly small, because of small neutral xenon number densities [13]. So, one should pick out these processes too when extrapolating a laboratory measurement to the space condition. A simple way on how to do this is described below.

Consider the plume of a Hall thruster operating at $U=300$ V in a vacuum chamber. At each point the plume contains certain amount of primary ions with energy $E\sim300$ eV and secondary plasma ions with $E\sim0.03$ eV. So, that:

$$N^+ = N^+_1 + N^+_2$$

In addition, a portion of doubly ionized xenon particles with energy $E\sim600$ eV is presented there. We do not consider slow $Xe^{2+}$ ions because their number density is very small even at a high background pressure ($p\sim0.5$ mTorr). These positive ions are compensated by electrons which we divide onto “primary” (that is compensating primary ions) and “secondary” (compensating secondary ions).

$$N_e = N_{e1} + N_{e2}$$

This subdivision doesn’t make a physical sense, indeed. The primary and secondary electrons are indistinguishable. However, such subdivision is useful for further consideration.
Number densities of all plume components right at the plume/discharge boundary do not depend on the background pressure in the region $p<0.5$ mTorr. This fact was proved through direct measurement of the discharge radiation spectra. The spectra appeared to be practically independent on the background pressure variation, provided that other operating parameters were staying constant. Downstream from the thruster exit the primary ions experience charge exchange collisions with the background xenon neutral atoms and generate the secondary plasma. According to the model discussed in ref. [13], number density of the secondary particles at the point $x$ downstream from the thruster should be proportional to the neutral atom number density:

$$N^+_2 = k_1 N_0$$

$$N_{e2} = k_2 N_0$$

Number density of the primary ions should also change due to collisions with neutrals:

$$N^+_1(x) = N^+_1(0)(1 - \exp(N_0\sigma x)),$$

where $\sigma$ is the charge exchange cross-section. However, at distances $x<250$ mm where radiation measurements were performed the primary beam depletion appeared to be relatively small (less than 20%). So, we neglected it. Under these assumptions expression (2) for radiation may be re-written in the form of:

$$J_{i \rightarrow j} = A \left( R N^+_1 + R N^+_2 + R N^+_3 N_0 \right) =$$

$$\left( R_2 (N_{e1} + k_2 N_0)(N^+_1 + k_1 N_0) + R_4 N^+_2 (N^+_1 + k_1 N_0) + R_5 N^+_2 N_0 \right).$$

In turn, this expression is easily reduced to a binomial:

$$J_{i \rightarrow j} = A (a N^+_0 + b N_0 + R_2 N_{e1} N^+_1 + R_4 N^+_2 + R_5 N^+_3 N_0) \cdot (4)$$

where $a$ and $b$ are some constants independent on $N_0$.

Relation 4 provides a key to exclusion of the background gas impact onto radiation measurement in vacuum chamber. Measuring the radiation intensity versus neutral atom number density and extrapolating the data to $N_0=0$ condition one should obtain a radiation inherent to the space condition.

$$J_{i \rightarrow j} = A \cdot (R N^+_1 + R N^+_2 N_0) \ (5)$$

Figure 4 shows dependence of the radiation intensity measured along the line of sight crossing the thruster axis at the distance $x=225$ mm from the thruster versus residual Xe atom number density in the test chamber. Formula for binomial extrapolation is also presented in the figure. According to the extrapolation the radiation intensity at zero pressure condition should be of about 55% of intensity measured at $N_0=4\cdot10^{12} \text{ cm}^{-3}$ and about 78% of radiation measured at $N_0=2\cdot10^{12} \text{ cm}^{-3}$. Integral visible radiation intensity value at zero pressure condition derived from these measurements should equal:

$$J_{225} = \int_{400 \text{ nm}}^{700 \text{ nm}} d\lambda \int_{i \rightarrow j} J_{i \rightarrow j} dz = 2.26 \cdot 10^{-2} \text{ W m}^{-2} \text{ sr}^{-1} \ (6)$$
Figure 4 Pressure dependence of D-80 visible radiation intensity measured in TSNIIMASH test facility.

Radiation $J_{225}$ is that one should expect to see in space along a particular line of sight crossing the thruster axis at $x=225$ mm. For practical application extrapolation of this value to any chosen line of sight in the plume would be useful. In the space condition energy of colliding ions doesn’t vary down to very long distances from the thruster. The electron temperature in the TAL plume also stays approximately constant starting at 20-30 cm from the thruster [3]. That means the line excitation rates in formula 5 may be considered as constants in the far field of the plume. Assuming also that content of the first and second ions relative to electrons is constant in the far field the radiation along any line of sight in the plume may be expressed as:

$$J = \int_{400nm}^{700nm} d\lambda \int J_{i \rightarrow j} dz = C \cdot \int N_e^2 d$$  \hspace{1cm} (7)

In particular, for the line of sight crossing the thruster axis at $x=225$ mm this expression gives:

$$C \cdot \int N_e^2 d = 2.26 \cdot 10^{-2} \frac{W}{cm^2sr}$$  \hspace{1cm} (8)

The integral in relation (8) was computed with basing on the plume model described in previous section. This allowed us to use this relation for determination of the constant scaling constant $C=5.38 \cdot 10^{-34} W \cdot m^3/sr$. Having this constant determined from experiment the problem of computation of the radiation intensity in the space plume is reduced to computation of integral $\int N_e^2 d$ in expression (7). The integral, in turn, is computed basing on the plume model developed earlier [1,2].

**Results of computation of the space plume radiation**

Method for computation of a Hall thruster plume radiation intensity at the space condition was applied for modeling of the D-80 TAL radiation maps in an imaginary experiment. A presumptive optical radiation sensor (OS) was placed a meter away from the thruster. The radiation intensity was computed along various lines of sight starting at the sensor position and crossing the plume. Sketch of the computation geometry is given in Figure 5.
In this figure the line of sight of the OS is determined by angles $\theta_z$ (analog of the pitch angle) and $\theta_y$ (yaw angle). The magnetic field lines are directed along the X axis. The radiation intensity computations were performed for D-80 TAL operating at $U=300 \, V$ and $I=4.3 \, A$ on two different orbits.

Figure 6 shows the radiation map computed for $h=1200 \, km$ orbit. The radiation map exhibits two maximums. One at $\theta_z=0$, $\theta_y=0$ corresponds to the line of sight crossing the TAL axis at the very near field, where the plasma is relatively dense. Note, that presented method for computation of the radiation intensity may be not valid in the very near field zone.

Another maximum is located at $\theta_z=90^\circ$, $\theta_y=0$. This line of sight corresponds to the geomagnetic field direction. Charged particles are trapped by the magnetic field and plume is elongated in this direction. So, the radiation maximum is simply explained by the long integration path inside the plume. This effect should be taken into account during integration of the optical payloads onboard a satellite with an operating Hall thruster.

For comparison Figure 7 shows the radiation map computed for GEO. At this orbit influence of the geomagnetic field onto the TAL plume is much smaller and radiation map has more symmetric character.
Figure 6 D-80 radiation intensity map computed for orbit h=1200 km (legend in W/m$^2$ sr).

Figure 7 D-80 radiation intensity map computed for GEO (legend in W/m$^2$ sr).
Conclusions

Study of the processes responsible for generation of radiation in a Hall thruster plume in a vacuum test chamber and in space has shown these processes can be separated. Contribution of the radiation excitation processes related to the residual gas in the vacuum chamber may be removed from the total radiation intensity value by a simple extrapolation of the experimental data to zero pressure condition. This provides a method of extrapolation of the radiation data obtained in the test chamber to the space condition.

A simple approach has been suggested for computation of the far field plume radiation basing on the near field measurements. This approach should be useful for engineering estimates of the radiation intensity in the far field zone of a Hall thruster plume in space.

In future the method for computation of a Hall effect thruster plume radiation is thought to be improved, primarily, through implementation of a new excitation cross-section data obtained in single collision beam experiments.

References