Low Frequency Plasma Wave Dispersion and Propagation in Hall Thrusters*†

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Low-frequency oscillations (<250 kHz) in a coaxial Hall thruster with a boron nitride channel are investigated using three axially and azimuthally separated Langmuir probes. Dispersion maps depicting both the azimuthal and axial component wavenumbers, taken at three discharge voltages corresponding to the ionization, negative resistance and saturation branches of the thruster I-V curve, depict features that are qualitatively consistent with those described in the prior literature. These measurements reflect the first detailed study of the propagation of these waves, revealing their complex behavior throughout the channel, varying dramatically with changing voltage. A direct assignment to particular theoretical modes (briefly discussed at the end of the paper) has yet to be made, with the exception of the so-called "circuit" oscillation, which is clearly identified at the higher voltages studied.

Introduction

Closed electron-drift Hall discharges are low-pressure (~0.01 – 0.1 Pa), weakly collisional, magnetized plasma sources that generate a relatively high velocity ion beam suitable for use in space propulsion applications. A particular class of Hall plasma thrusters, the so-called “Stationary Plasma Thruster,” has an annular (co-axial) discharge chamber (usually called “channel”) made of a dielectric material, and has been used in a number of space missions in the former Soviet Union [1]. A variation of this Hall thruster that has a 100 mm diameter outer channel dimension – the SPT-100, has a high specific impulse (1100 – 2000 sec), operates at moderately high thrust levels (60 – 150 mN), and has an exceptionally high thrust efficiency (40-60%). Because of this performance, this particular plasma source is being aggressively developed for use in station keeping applications on western satellites. Clusters of these thrusters, or variations of similar Hall thrusters operating at higher powers, are even being considered as primary propulsion options for orbit transfer.

In a typical co-axial geometry Hall discharge, the plasma is sustained in imposed orthogonal electric and magnetic fields. The discharge electrons are magnetized whereas the more massive propellant ions (usually xenon) are not. Consequently, the electrostatic fields established by the retarded electron flow accelerate the ions to high velocities, typically 60-90% of the discharge voltage (~200–400V). The maximum acceleration occurs in the region between the magnetic poles, where the magnetic field is also a maximum. In a co-axial geometry, the electrons are constrained to move in the closed, azimuthal $\mathbf{E} \times \mathbf{B}$ drift, with cross-field diffusion providing the necessary current to sustain the discharge. An annular ceramic channel confines the electron flow towards the anode, located at its base.

It is widely known that the Hall discharge plasmas exhibit a rich spectrum of fluctuations in plasma properties [2]. While it is not yet known if and how

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these fluctuations can impact the performance of a Hall thruster, it is believed that fluctuations in the bulk plasma properties are partly responsible for anomalous electron transport across the imposed magnetic field [3]. While some studies characterizing the presence and origin of these fluctuations and their possible control were published in the mid 60’s to mid 70’s [3]-[9], they have received increased attention recently [10]-[14], as there is a growing need to extend and enhance the performance of these thrusters for a broader range of space missions.

![Radial magnetic field simulation.](image)

Figure 1 - Radial magnetic field simulation.

Our research presented here is motivated by the possibility of enhancing Hall thruster operation by the active control of these fluctuations, or by the passive suppression of the fluctuations in regions of the discharge channel where a reduction in electron current is desired. To do so, it is important that the nature of these fluctuations is well understood. In this paper, we present an experimental study of relatively low-frequency (< 400 kHz) wave propagation in the channel and slightly downstream of a laboratory Hall thruster. Multiple Langmuir probes immersed in the plasma and operated in saturated ion current collection mode are used to capture the growth (or decay) and propagation direction of disturbances in plasma density. Fourier decomposition analysis are used for qualitative wave analysis [15], and a wavelet analysis (see Appendix) of the signals on each probe are used to characterize the dispersion of these waves, in some cases, believed to be associated with the ionization process. The particular thruster studied here is similar to one that we have studied previously, only we have replaced the alumina channel with one fabricated out of boron nitride. Distinguishing features are seen in the dispersion plots, resulting in the possible identification of many distinctly different wave modes, depending on axial position and discharge voltage. Without a full characterization of the quasi-steady state plasma properties within this discharge, it is premature to conclusively assign these modes to particular physical phenomenon, however, we do speculate on the origin of some of these disturbances based on our previous measurements of the same discharge operating with an alumina channel.

**Experiment**

**Stanford Hall Thruster (SHT)**

The Hall discharge plasma source used in this study is a laboratory version of a nominally low-power (< 1kW) Hall thruster, and is described in more detail in previous papers [16]-[17]. This particular source is intended to be used as a test bed for studying the discharge physics and not to serve as an operational prototype plasma accelerator, although the principal design is similar to that of thrusters used in practice. The source consists of an annular ceramic channel, 90 mm in diameter, 11 mm in width, and 80 mm in length. A magnetic circuit consisting of four outer coils, one inner coil, and three iron plates provides a magnetic field (mostly radial in direction) peaked 5 mm upstream of the exit of the discharge channel. Details of the two-dimensional magnetic field distribution for various magnet currents can be found in [19]. Figure 1 shows the axial variation in the radial component of the magnetic field at the mean radius of the channel, as computed with a 2-D axisymmetric finite-element model, for two values (125mA and 200mA) of the magnetic coil current. Experimental measurements of the magnetic field distribution given in [19] indicate that the radial component of the magnetic field drops off by approximately 15% at the inner and outer walls of the acceleration channel. A hollow stainless steel ring with 32 holes of 0.5 mm diameter serves both as the anode and the propellant (gas) input of the discharge. A commercial hollow cathode (Ion Tech HCN-252) is used to neutralize the resulting ion beam and provide the necessary electron current to sustain the discharge. The cathode is mounted such that its exit aperture is approximately 2 cm downstream of the plasma source exit.
Vacuum chamber
All experiments reported here were performed in a 1 m diameter by 1.5 m long stainless steel vacuum chamber. Flanged elbows (0.5 m in diameter) at each end of the vacuum chamber supported two 0.5 m diameter diffusion pumps, operated without baffles, for maximum pumping speed (approximately 6000 l/s). This pumping plant provided an operating pressure of $10^{-4}$ torr, as indicated by an ionization gauge (uncorrected for xenon). Separate direct-current (DC) power supplies powered the anode, cathode heater, cathode keeper, and magnet coils. For the measurements reported on here, the xenon flow rate at the anode was 2 mg/s. An additional 0.3 mg/s of xenon flow was necessary for running the hollow cathode, the body of which was kept at vacuum chamber ground potential. The discharge voltage and current were monitored with digital multimeters. Discharge current oscillations were measured with a powered differential amplifier (Tektronix P5200) placed across a $4 \Omega$ series resistor and recorded by a high-speed PC-based 4-channel digital oscilloscope (National Instruments PCI-5102), with a 20MHz maximum sampling rate and 12 bit maximum vertical resolution.

Discharge Characterization
In all of our previous reported experimental results [16]-[20], the SHT channel walls were made of alumina ($\text{Al}_2\text{O}_3$). The time-averaged plasma properties and plasma fluctuations in this thruster with alumina walls are well documented [16]. In this paper, we present our first experimental study of the discharge operating with boron-nitride (BN) walls.

Current-voltage (I-V) characteristics of the SHT operating with both BN and $\text{Al}_2\text{O}_3$ walls are shown in Figure 2. With alumina walls, the discharge current increases continuously with applied voltage, whereas with boron-nitride walls, the I-V curve is more characteristic of that of modern Hall thrusters, first increasing rapidly (“ionization branch”), then decreasing between 100V - 150V, only to plateau (or increase slightly) between 150V and 200V (so called “current saturation” branch). In the ionization branch, the rapid increase in discharge current with increased voltage is due to the increased mean electron energy, resulting in a higher ionization rate and higher ionization fraction (avalanche ionization). At higher voltage, the ionization fraction has reached a maximum value (nearly 100% propellant utilization, i.e., the conversion of neutral xenon to ionized xenon), and the discharge current either remains constant or increases slightly due to changes in electron mobility. At present, there is no satisfactory explanation given so far in the Hall thruster literature about the appearance of the negative resistance region in the I-V curve, located here at approximately 100V for the discharge with BN walls, even though it seems to be a general feature of modern Hall thruster operation.

The difference in the I-V characteristics for the two wall materials (all else equal) suggests that plasma-wall interactions play an important role in the sustaining of this discharge. The importance of channel walls in Hall thrusters was recognized in the 1960's: The existence of the so-called “near-wall conductivity” arising from electron-wall scattering was first proposed by Morozov to explain the observed anomalous electron conductivity in Hall thrusters [21]. Significant in this near-wall conductivity was the secondary electron emission properties of the wall material, expressed in terms of the secondary electron emission coefficient, $\sigma_{se}$. It is believed that wall materials with high $\sigma_{se}$ give rise to high cross-field electron conductivity, since a large fraction of the incident (hot) electrons are returned to the plasma as cold electrons with new guiding center drifts along the direction of the electric field. The interaction between the plasma electrons and the wall

‡ Note that although this negative resistance is not seen for the case of alumina walls, there is a distinct inflection in the I-V characteristic at this voltage.
is not well understood, requiring a detailed model of the sheath structure. However, the continuing increase in current with voltage (above 150V) in the case of alumina walls, compared to that of boron nitride walls, is consistent with the fact that \( \sigma_{se} (\text{Al}_2\text{O}_3) > \sigma_{se} (\text{BN}) \). Below 150 V, the lower discharge current in the alumina case indicates that other physical processes must be considered.

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Moving downwards in coil current, we see that between 300 mA - 130 mA, the discharge current is almost constant (~1.9A), the oscillation amplitude is low (~0.4A peak-to-peak) and although the oscillation power spectrum is relatively diffuse and weak (a somewhat irregular signal), a characteristic peak is seen to exist over this range. The frequency of this peak increases from about 5 kHz (at the highest magnet current studied) to about 10 kHz, at a magnet current of 130 mA. From 130 mA down to 70 mA, the mean current rises sharply to 2.6 A, the oscillations become very strong (peak-to-peak as high as twice the mean value) and regular, are nearly sinusoidal in shape, and the power spectra exhibit a sharp frequency that peaks at approximately 14 kHz. Decreasing the current from 130 mA to 40 mA, the mean value in the discharge current increases even further (to 3.9 A), the oscillations become pulse-like, and the peak frequency decreases to 10 kHz. At 30 mA, the mean discharge current jumps to 6A, the oscillations suddenly diminish (0.4A peak to peak) and the characteristic frequency falls to 5kHz. A similar discharge behavior of Hall thrusters was reported by Tilinin in the 1970's [9].

A second series of measurement were carried out at constant magnetic coil current (200 mA) while varying the applied voltage. The results are shown in Figure 4. It is apparent that in the vicinity of 140 V to 150 V, we see a very low oscillation amplitude (about 0.4A for a mean current of 1.8A), consistent with the data of Figure 3. The reason for this quiescent window of operation is not clear. A slight departure from this operating voltage leads to the emergence of strong and well-defined oscillations, believed to be

![Figure 3 - Top: Mean discharge current, maximum amplitude, and standard deviation vs. coil current. Bottom: Power spectrum of the discharge current.](image-url)

A study of the discharge current oscillations was conducted with the SHT lined with the boron-nitride channel. The sampling rate of the PC acquisition board used to record discharge current was set to 2MHz, and each record was 100,000 samples long. In the first series of measurements, the variable parameter was the current through the magnet windings, with the applied discharge voltage set at 150 V. Within the range of magnet current values investigated (30 mA – 300 mA), the magnetic field variation within the channel is expected to retain the general shape shown in Figure 1, with the peak in proportion to the current, since the saturation of the iron cores is found to be significant only beyond magnet currents of 500 mA. The resulting discharge current mean values, oscillation amplitudes, and power spectra, up to 100 kHz, are shown in Figure 3 below.
attributed to the “breathing” mode, which increases in frequency with increasing discharge voltage. This mode corresponds to a bulk oscillation of the plasma, which has been given much attention in the Hall thruster literature during the past few years [11]-[14], due to its strong impact on the external power supply.

Figure 4 - Top: Mean discharge current, maximum amplitude, and standard deviation vs. discharge voltage. Bottom: Power spectrum of the discharge current.

On the discharge current traces, the transition from the quiescent regime to the strong oscillation mode can be observed as the appearance of bursts of strong, regular oscillations, which eventually persist for longer times, and become more frequent as the discharge voltage increases. The impact of this “breathing” mode on the ion stream is illustrated in Figure 5, which depicts simultaneous records of both the discharge current and of the plasma plume floating potential, the latter measured with a Langmuir probe located 20 cm downstream of the channel exit and 10 cm off the thruster axis. The discharge voltage is 180V in this example. The phase shift $\Delta \phi \approx 20\mu s$ between the peaks is approximately the same as the time of flight $t_i$ of the ions from the discharge to the probe determined from:

$$t_i = \frac{L}{v_i} = \frac{L}{\sqrt{\frac{eU}{2m_{Xe}}}} \approx 30\mu s$$  \hspace{1cm} (1)

where $L$ is the distance between the probe and the discharge, $v_i$ is the ion velocity, $e$ the electron charge, $m_{Xe}$ the mass of a xenon ions and $U$ the accelerating potential difference, estimated as about 100V from previous laser induced fluorescence measurements [17]. This simple diagnostic serves to illustrate that when in the breathing mode, ion bursts are concomitant with bursts in discharge current.

Figure 5 - Oscillations of the discharge current and of the floating potential in the plasma plume.

It is noteworthy that at the lower discharge voltages, with operation in the ionization branch of the I-V characteristics, the spectrum associated with the current fluctuations (see Figure 4) broadens somewhat. We do not believe that this low voltage mode is associated with the breathing instability, since we do not expect the channel to be starved of neutral xenon as the ionization fraction is presumably low (a further discussion of this is presented later). A second, higher frequency mode also appears, superimposed on this low frequency mode, with a spectrum that extends...
well beyond 100 kHz. As discussed below, this broadband activity might be associated with a number of possible instabilities, some driven by thermal runaway ionization models, others by gradients in plasma properties.

**Langmuir probes**

Plasma density fluctuations inside the channel and near the exit plane were detected by three azimuthally-placed low-impedance Langmuir probes biased negatively with respect to the plasma potential to collect the ion saturation current, with the current density given as [22]:

\[
J_{is} = 0.61 e n_e \sqrt{\frac{kT_e}{m_i}}
\]  

(2)

Here, \(T_e\) is the electron temperature, \(n_e\) is the electron number density, \(m_i\) is the xenon ion mass, \(e\) is the electron charge, and \(k\) is the Boltzmann constant. In Equation (2), it is assumed that the distant plasma is quasi-neutral (\(n_e = n_i\)), and that the ions enter the collisionless sheath at the Bohm velocity. While the fluctuations in collected current can be a result of both fluctuations in electron density and temperature, in prior studies, it has been shown that these low frequency disturbances are largely isothermal [3],[7].

The Langmuir probe design was presented in previous papers [23]. The exposed part of the probe consisted of a 0.254 mm diameter, 5 mm long tungsten wire. The probe base consisted of an alumina tube directly surrounding the tungsten wire (0.508 mm ID, 1.27 mm OD), followed by a stainless steel tube (1.48 mm ID, 1.58 mm OD), connected to the braided ground of a 50 \(\Omega\) co-axial cable. The inner tungsten wire was connected to the center pin of the co-axial cable. This outer stainless steel tubing served as a shield to isolate the extended probe base from disturbances other than at the tip of the probe. The stainless steel tubing was then surrounded by another alumina tube (1.6 mm ID, 2.3 mm OD) and the entire inside air spaces were potted with an alumina paste.

The probe body length was minimized so as to reduce its overall capacitance, and for the studies reported on here, was approximately 10 cm in length. The frequency response of the probe was found to be excellent (unity gain and no measurable phase shift, within the measuring device accuracy) up to 1 MHz, with a 3 dB cutoff at 4 MHz. A coaxial transmission line feed-through provided the transfer of the probe signal through the vacuum chamber to the 4-channel digital oscilloscope card, operating at a 1 MHz sample rate for the measurement reported on here. The recorded signals were 100,000 samples long. The probes were terminated with 50 \(\Omega\) at the input of the oscilloscope card. In this way, the probe tips were negatively biased (close to system ground) with respect to the plasma, which is predominantly at high positive potential, since the anode is maintained at a minimum of 83V, and since the near exit plane potential is always greater than 40V due to the cathode fall and finite electron temperature.

![Figure 6 - Schematic of the experimental arrangement showing the location and orientation of the Langmuir probes.](image)
the propagating plasma instabilities. In our previous experiments [23], the probes were oriented such that the exposed center wire axis was parallel to the axial coordinate of the thruster. Here, the exposed parts of the probes were bent and aligned to be parallel to the radial coordinate in order to prevent shadowing effects. For such an orientation, the assumption regarding the Bohm flux to the probe is strictly not true, since the ions can have a substantial directed velocity (as much as 4 or 5 times the Bohm value). A comparison to prior measurements taken with axially-oriented probes confirm that the results are not very sensitive to probe orientation, and that current collection is still in the ion saturation regime.

Probe translation along the axial direction (the direction of the annular channel) was provided by a translation stage driven by a Slo-Syn stepper motor, powered and controlled by a Compumotor model SX Microstepping Drive/Indexer System. For this study, the axial locations of probe #1 and #2 varied between a distance of approximately 60 mm from the anode (z = -60 mm) and 30 mm beyond the exit plane (z = 30 mm), with the z = 0 reference taken to be the exit plane of the discharge, and negative positions implying that the probe locations are inside the discharge channel. For any given position z, the location of probe #3 was z-6 mm. Before starting the thruster, the probes were moved far away from it (z=+150). During thruster operation, the probes were moved towards the anode and stopped at every measuring position, starting at z=-30mm. Data collection for each axial scan took approximately 30 minutes.

Results

For all the measurements presented here, the mass flow rate of xenon injected at the anode is 2mg/s, and the magnets current is 200mA. Three points on the I-V curve were chosen for a detailed analysis of plasma fluctuations (see Figure 4): 83V (ionization branch), 110V (negative resistance branch), and 150V (current saturation branch, quiescent regime).

A primary concern when implementing an intrusive plasma diagnostics, such as an electrostatic probe, is the possible perturbation induced by the presence of the probes themselves. When operating with alumina walls, the discharge was found to be relatively insensitive to the insertion of the electrostatic probes, as determined qualitatively by monitoring the discharge current when the probes were scanned axially along the channel. When operating with boron nitride walls, perturbations on the discharge were apparent when the three probes were positioned upstream of the exit plane. Oscillograms of the three probe signals and the discharge current are shown in Figure 7, for the case where the probes were located outside of the thruster channel (z = +20 mm) and for the case where the probes where 20 mm upstream of the exit of the channel (z = -20 mm). In the former

![Figure 7 - Simultaneous oscillograms of the discharge current and of the probe signals, U_d = 150V. Left: z = -20mm, mean discharge current 2.23A. Right: z = +20mm, mean discharge current 1.87A. Remark: the left and right vertical scales are the same for the discharge current, but not for the probes.](image-url)
Figure 8 - Mean values, oscillations of the discharge current, probe #1. Error bars indicate the oscillation standard deviation.

Figure 9 - Power spectra (log scales) of the discharge current and of probe #1. For every probe position, the spectrum is normalized the total energy of the signal.
case, it is apparent that the probe signal has structure that is unlike that seen in the discharge current, and furthermore, there is little evidence of the presence of the strong breathing mode instability. However, as seen in Figure 7, the insertion of the probes to a position of \( z = -20 \text{ mm} \) appears to “activate” the breathing mode instability, which is then detected by the probes themselves. It seems then that at a discharge voltage of 150V, the breathing mode detected within the channel is not intrinsic, i.e., it is present only because of the perturbing effects of the probes.

Figure 8 provides yet another view of possible perturbing effects that the probes have on the discharge. The left panels in the figure show the change in the mean value (circles), the average amplitude (bars), and the maximum and minimum values (dashed lines) in the discharge current as the probes are positioned within the channel. The panels on the right are the same values for the signal detected by probe #1 (probes #2 and #3 are qualitatively similar). It is apparent that at the lower voltages, the probes have a small effect on the discharge, however, at 150V, the oscillation amplitude is clearly increased when the probes enter the channel. It is noteworthy that when the probes draw the greatest amount of ion current (between positions of \( z = -40 \text{ mm} \) and \( z = -20 \text{ mm} \)), the amplitude of the breathing mode oscillations decrease slightly.

Spectral maps of the amplitude of the plasma density disturbances detected by the electrostatic probes for various positions within the channel (and at the three discharge voltages) are shown in Figure 9. Also shown to the left of these maps is the spectral decomposition in the discharge current verses the probe position. In all three discharge voltage cases, the discharge current is dominated by strong, low frequency components. As mentioned above, the low frequency components seen at low discharge voltage are not likely to be associated with the breathing mode. However, the oscillograms seen at 110V, and certainly at 150V when the probes are inserted in the channel are clearly associated with this breathing mode instability. The frequency of the breathing mode increases with increased discharge voltage - a typical behavior for this instability, which is a characteristic feature in the circuit current of all Hall thrusters. It is apparent from this figure, as we have seen from the previous two figures, that the probes are somewhat non-perturbing for the 83V and 110V discharge conditions. However, an inspection of the 150V case again reveals that the nature of the discharge current oscillations change suddenly when the probes enter the channel (actually, at about \( z = -10 \text{ mm} \), where the plasma density increases rapidly). It is clear then that dispersion maps generated for the 150V case (for \( z < -10 \text{ mm} \)) should be interpreted with caution, since the discharge appears to be operating in a mode largely affected by the probes themselves. The probes also detect a rather broadband disturbance extending to 100 kHz. The propagation behavior of these higher frequency modes will be discussed later in the paper. At lower discharge voltages (83V), a relatively strong mode appears at positions downstream of \( z = +20 \text{ mm} \), approximately over the region where both the plasma density and/or magnetic field is high. This mode is easily distinguished from the lower frequency mode. This mode also spans a relatively narrow frequency range, about \( 15 - 25 \text{ kHz} \). Further upstream of the exit plane, the low voltage case also seems to have relatively strong activity extending to approximately 75 kHz, and the emergence of another sharply defined disturbance at 20 kHz, close to the anode.

Figure 10 shows the \( f-k_\theta \) (azimuthal component) dispersion maps derived from the wavelet analysis, for the 150V, 110V, and 83V case (top to bottom), for four locations (\( z = -60 \text{ mm} \), -30 mm, 0 mm, and 30 mm). Dispersion maps for a finer range of axial position (\( z = -10 \text{ mm} \), -15 mm, -20 mm, and -25 mm) are given in Figure 11. To aid in the interpretation of these maps, contour diagrams of the azimuthal \( (k_\theta) \) and axial \( (k_z) \) component wavenumbers (versus frequency and axial position) are given in Figure 12. These contour plots were obtained by extracting the strongest peak of the dispersion spectrum for every frequency and probe position. The azimuthal wavenumber component is determined directly from the signals detected by probes #1 and #2. However, because of the orientation of the three probes (no two probes lie on the same azimuthal position separated by just the axial spacing), the axial component was determined from the signals on all three probes. In interpreting the combined axial and azimuthal dispersion data, an assumption is made that any disturbance extracted from the \( k_\theta \) measurement
Figure 10: Frequency-azimuthal wavenumber spectra (log scales). Top to bottom: $U_d = 150V, 110V, 83V$. Left to right: $z = -60mm, -30mm, 0, +30mm$.

Figure 11 - Frequency-azimuthal wavenumber spectra (log scales). Top to bottom: $U_d = 150V, 110V, 83V$. Left to right: $z = -25mm, -20mm, -15mm, -10mm$. 
corresponded to the same disturbance extracted from the \( k_z \) measurement at the same position and the same frequency.

It is apparent that the dispersion characteristics of the disturbances detected by the probes are strongly dependent on location within the channel. In trying to summarize our observations, we shall first draw attention to the maps associated with the lowest discharge voltage case (83V), which appeared to have the least perturbing affects on the discharge. Note that the color scales vary from map to map, to accentuate the features for any single position. Outside of the discharge \((z = 30 \text{ mm})\), between 0 and 50 kHz, there are two loci of points representing strong disturbances, at very low values of \( k_\theta \). The locus of points at very low frequency lies very near the \( k_\theta = 0 \) line, with no apparent azimuthal propagation direction. While it is tempting to attribute this mode to the breathing mode, as mentioned above, the appearance of the breathing instability is suspect at these low voltages. The assignment of this disturbance is still questionable. Because its behavior does not change substantially with position within the channel, this disturbance may be the so-called “spoke instability” often seen when there is incomplete ionization in the channel [3]. The locus of points at higher frequency, i.e., the second strong mode seen in the maps of Figure 10, seems to favor positive wave propagation direction \((i.e., k_\theta > 0)\), with a mean value of \( k_\theta \approx 5 \text{ m}^{-1} \). This value is consistent with an \( m = 1 \) \((k_\theta = 3 \text{ m}^{-1})\) or \( m = 2 \) \((k_\theta = 6 \text{ m}^{-1})\) azimuthal mode, perhaps attributable to an ionization instability driven by axial gradients in plasma density. Further upstream, this mode drifts closer towards the axis \((k_\theta = 0 \text{ at } z = 0)\), and then, by close examination of the 83V panels in Figure 11, seems to have drifted towards negative values of \( k_\theta \) with a centroid of about \( k_\theta \approx -5 \text{ m}^{-1} \) at \( z = -20 \text{ mm} \) (2 cm upstream of the exit plane). In almost all cases, this disturbance appears as a resonant azimuthal mode (locus is untilted in the dispersion map), although the precise mode number is difficult to define. An examination of the wavenumber maps shown in Figure 12 also shows that the azimuthal wavenumber passes from positive to negative values, while the axial component wavenumber is close to zero (within error) with the exception of the region in the vicinity of the exit plane \((z = 0 \text{ mm})\), where \( k_x = 10 - 20 \text{ m}^{-1} \). At 83V, the higher frequency disturbances \((50 - 200 \text{ kHz})\) all seem to be centered about \( k_x = 0 \text{ m}^{-1} \), with activity very near the Nyquist limit (the vertical borders of the maps, located at approximately 62 \text{ m}^{-1} \). Although the data in Figure 13 suggests that near the exit plane, the disturbances may be tilted to propagate towards the anode and with a component in the electron drift direction, the breadth of the disturbances in wavenumber space makes this difficult to conclude with certainty. An examination of the dispersion plot of Figure 13 at the distance nearest the anode \((z = -60 \text{ mm})\) does suggest very clearly that the higher frequency components for this near-anode disturbance is a tilted wave propagating in the opposite direction of the \( \mathbf{E} \times \mathbf{B} \) drift, and towards the exit (or cathode). This phase velocity of this disturbance is estimated to be about 1% of the drift velocity, and this disturbance may be related to the fluctuations seen in the tail of the electron energy distribution (or electron temperature) at similar locations within the channel, in previous studies in our laboratory [16].

We now move to discuss the results for the slightly higher discharge voltage, 110V (in the negative resistance region of the I-V characteristic). At this voltage, there is clear evidence of the existence of the breathing mode at nearly all axial positions. Also, an examination of the dispersion maps of Figures 10 and 11 shows that downstream of the position \( z = -10 \text{ mm} \), where the gradient in the inhomogeneity parameter is negative, i.e., where:

\[
\frac{B_z}{n_e} \frac{\partial}{\partial z} \left( \frac{n_e}{B_z} \right) < 0
\]

there is a persistence of very strong, broadband disturbances extending to beyond 250 kHz. From Figure 10, we see that these waves are of low azimuthal wavenumber propagating in the direction of the \( \mathbf{E} \times \mathbf{B} \) drift. An inspection of Figure 12 suggests that near \( z = 0 \text{ mm} \), they may develop a strong axial component wavenumber as well (and so they may be tilted). A curious thing happens to these waves upstream of the \( z = -10 \text{ location} \): the axial component nearly disappears, and the azimuthal component first becomes negative (at \( z = -15 \text{ mm} \)), then, reverses its direction (at \( z = -20, 25 \text{ mm} \), see Figure 11). We do not yet have a clear explanation for this unusual
behavior for this instability, but the location of the
reversal (about $z = -20$ mm), may correspond to the
location in either the peak of the inhomogeneity
parameter, or to the peak in plasma density. It is also
noteworthy that from inspection of the dispersion map
in Figure 11, at $z = -25$ mm, the frequency varies
nearly linearly with $k_\theta$, suggesting a constant
azimuthal phase velocity of about $8 - 10$ km/s.

At the highest discharge velocity studied (150V), there
appears to be only one strong mode at $z = 0$, 30 mm,
that extends over a very wide range of frequencies
(well beyond the limit shown). The dispersion maps
show that downstream of the exit plane (where the
probes are least perturbing), $k_\theta$ favors positive values,
with a very strange shape at $z = 30$ mm ($k_\theta$ approaches
zero for low and very high frequencies). The data in
Figure 12 suggests that these waves are largely
azimuthal in their propagation direction. Just upstream
of the exit plane ($z = -10$ mm, say see Figure 11), this
mode seems to suddenly tilt in f-$k_\theta$ space. Note that
the appearance of high frequency activity at negative
wavenumbers is clearly due to aliasing beyond the
Nyquist limit. This is the only case where a branch of
the f-$k_\theta$ spectra going beyond the spatial Nyquist limit
is revealed. In Figure 12, the aliasing effect was
compensated for by artificially “unfolding” the
corresponding branch into the positive azimuthal
wavenumber region. Figure 13 shows that this mode is
strongly dependent on the direction of the magnetic
field, i.e., it reverses its direction when the direction of
the radial component to the magnetic field is reversed.
In essence, it preserves its propagation direction to be
along the direction of the azimuthal $E\times B$ drift. The
dispersion map at $z = -20$ mm seems to hint at the
existence of two possible modes (one with a very high
azimuthal phase velocity, the other with a low
azimuthal phase velocity), although the interpretation
of any of the maps for upstream locations beyond $z = -10$ mm is difficult because of the perturbing affects of
the probes. The data of Figure 13 strongly supports
the possibility that this mode is related to the gradient
in the magnetic field, or the inhomogeneity parameter.

Figure 12 – Strongest branches of the dispersion spectra. Top to bottom: $U_d = 150V$,
$110V$, $83V$. Left: wavenumber azimuthal component (positive means in the $E\times B$
direction). Right: wavenumber axial component.
Discussion

The present understanding of plasma oscillations in Hall thrusters is mostly qualitative. Recently, Choueiri gave an overview of past research aimed at understanding oscillations in Hall thruster plasmas, comparing experimental observations of transient behavior to various theories derived from fluid models [2]. Perhaps the only mode that we identify that is undeniably assigned is that of the so-called “breathing mode” (introduced also as the “circuit” and “loop” instability in the Russian literature). We see evidence of this mode at both 110V, and at 150V, although in the later case, it is activated by the electrostatic probes. This mode arises through a competition between the fast removal of ions (by the electric field) and the slow supply of neutrals in the channel, coupled through the ionization process. Fife et al. compared it to a simple “predator-prey” model and estimated the frequency $f$ to be:

$$f = \frac{1}{L} \sqrt{\frac{V_i V_n}{V}}$$

(4)

where $L$ is the length of the zone in which the oscillations occur, and $V_i$ and $V_n$ are the bulk velocity of the ions and of the neutrals, respectively [12]. Table 1, below, computes this frequency based on estimated plasma flow properties. These frequencies are in good agreement with those seen in our channel at the higher discharge voltages. This mode is also expected to be homogeneous within the channel (hence the description as “breathing”), as we see in our experiments, in agreement with more recent simulations [13].

<table>
<thead>
<tr>
<th>$U_d$ (V)</th>
<th>$L$ (m)</th>
<th>$V_i$ (m/s)</th>
<th>$V_n$ (m/s)</th>
<th>$f$ (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 V</td>
<td>0.08</td>
<td>3000</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>200 V</td>
<td>0.08</td>
<td>8000</td>
<td>200</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 1: “breathing mode” frequency

As mentioned above, there have been other low frequency disturbances attributed to ionization, described as early as the 1960’s, as axially tilted, azimuthally propagating “spokes” [3]. These spokes were found to have one end anchored to the anode, and rotated in the direction of the $\mathbf{E} \times \mathbf{B}$ drift with frequencies ranging from 5 to 25 kHz. The phase velocity of this disturbance was found to be a few km/s, and this mode was prominent mainly in the ionization branch of the thruster I-V curve. Although this instability may contribute to the anomalous electron conductivity and may play a role in the appearance of the negative resistance branch of the I-V characteristic, its physics remains poorly understood, and it cannot be clearly identified from the analysis of Figure 10 through Figure 12, although it may be responsible for the very low frequency disturbance that persists throughout the channel at the lowest discharge voltage case studied (83V).

Several other kinds of disturbances can propagate over the frequency range that we have examined here. In Choueiri’s review paper [2], these categories of instabilities were described as azimuthal “drift magnetosonic” waves, “cross-field ionization” waves, axially propagating “transit-time” waves, and waves associated to weakly ionized inhomogeneous plasmas\textsuperscript{1}. Using the corresponding instability criteria given in [2] and estimating the plasma properties from our previous measurements in the same Hall discharge but operating with an alumina channel, we verified that all these instabilities can be excited for a wide

\textsuperscript{1}Although the propellant utilization in a Hall thruster is very high, the ionized fraction is actually low (approximately 10% at maximum), due to the considerable difference between the ion and neutral velocities.
range a thruster operation. For example, in Figure 14 we provide a contour plot of the predicted wavenumbers for ionization instabilities driven by strong plasma density gradients in weakly ionized plasmas [24]. Note that the corresponding wavenumbers and frequencies fall within the range of those observed in our experiments.

While the reader is referred to the review of Choueiri for a broad introduction to the types of instabilities that may be responsible for the fluctuations seen in these discharges, there is the possibility of yet another type of ionization wave, which is not discussed in the Hall thruster literature. This class of waves, which was extensively discussed by Franklin [25], was first observed in non-magnetized low-pressure gas discharges as propagating “striations” detected by electrostatic probes. The stability analysis of a plasma column conducted in Ref. [25], shows that these ionization waves are driven primarily by the dependence of the ionization frequency $\nu_i$ on the mean electron energy $T_e$ – perhaps the most “classical” types of ionization instabilities. The resulting dispersion relation is given as:

$$K = \frac{10W}{4 + 9W^2}$$ (5)

Here $K$ and $W$ are dimensionless variables related to the axial wavenumber and frequency:

$$K = 2\pi \frac{T_{e0}}{E_0} k ; W = 2\pi \left( T_{e0} \frac{d\nu_i}{dT_e} \right)^{-1} f$$ (6)

Here, $T_{e0}$ and $E_0$ are the steady-state electric field and mean electron energy (in eV), respectively.

Figure 14 – Theoretical dispersion plots of density-gradient ion waves for the Stanford Hall Thruster with alumina walls. Top to bottom: $U_d = 200V$, 160V, 100V.

Figure 15 – Theoretical dispersion plots of non-isothermal ionization waves for the Stanford Hall Thruster with alumina walls. Top to bottom: $U_d = 200V$, 160V, 100V.
Figure 15 shows the calculated dispersion maps resulting from these types of “classical” ionization instabilities. Near the channel exit, these waves are predicted to propagate in the direction of the electric field. While the predicted wavenumbers correspond to the ranges seen in our experiments, it is noted, however, that the direction of propagation is in contradiction to what we find based on the maps shown in Figure 12.

Summary

To summarize, we have reported on our continued studies aimed at characterizing the plasma fluctuations seen in Hall thrusters. In this particular study, measurements of plasma disturbances are detected with three axially and azimuthally separated Langmuir probes, in a Hall discharge with a boron nitride channel. Dispersion maps taken at three discharge voltages, characterizing fluctuations below 250 kHz, depict features that are qualitatively consistent with those described in the prior literature. These measurements reflect the first detailed study of the propagation of these waves, although a direct assignment to particular modes has not yet been made. Future work will be aimed at providing a theoretical/numerical basis for the understanding of the origin of these waves.

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Appendix

The wavelet analysis method that we used in this study is described elsewhere (see e.g. [26]). It was applied with much success in an investigation of turbulence in the Earth's magnetospheric foreshock[27].

The basic assumption is that the considered space-time varying scalar or vector field $B(x,t)$ can be decomposed into a superposition of plane waves:

$$B(x,t) = \int \int B(k,\omega) \exp(-i(\omega t - k \cdot x)) dk d\omega \quad (7)$$

(for simplicity and without loss of generality, the field is supposed to have a zero mean).

The quantity that describes the wave dispersion is the joint frequency-wavenumber spectrum, $S(k,\omega)$:

$$S(k,\omega) = \left\langle B^*(k,\omega)B(k,\omega) \right\rangle \quad (8)$$

which is usually evaluated from the Fourier transform of $B(x,t)$, i.e., $B(k,\omega)$. When studying nonlinear phenomena, the assumption of a superposition of plane waves is often too restrictive, and plane wave packets are used instead, which correspond to replacing the Fourier transform by the wavelet transform [27]:

$$B(x,a,\tau) = \int B(x,t) \frac{1}{\sqrt{a}} h^\prime \left( \frac{t-\tau}{a} \right) dt \quad (9)$$

where $h(t)$ is the analyzing wavelet. Here, we used the Morlet wavelets:

$$h(t) = \pi^{-1/4} \sigma^{-1/2} \exp(2\pi it) \exp(-\frac{t^2}{2\sigma^2}) \quad (10)$$

because of their good time-frequency resolution.

References


