Methods of Controlling Low-Frequency Oscillation in a Hall Thruster *†

Takeshi Furukawa, Takeshi Miyasaka and Toshi Fujiwara
Department of Aerospace Engineering, Nagoya University
Furo, Chikusa-ku Nagoya 464-8603, Japan.
E-mail: furukawa@momo.nuae.nagoya-u.ac.jp

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Some serious problems still remain for improvement of thruster performance, although Hall thruster is a promising propulsion device for space missions. A problem that should specifically be solved is the discharge current oscillation phenomenon observed under standard operational conditions at high-voltage mode of DC regime. In particular, the large-amplitude and low-frequency discharge current oscillation in 20kHz range is speculated as a main cause for deterioration of propulsion efficiency and operational stability among various oscillation phenomena. Possibility of controlling the amplitude of 20kHz-range oscillation by changing ionization-zone length was examined in our previous work, based on experimental and theoretical analyses. [1] In this paper, the physical mechanisms of ionization process in acceleration channel are studied, using an unsteady one-dimensional numerical analysis; the methods of controlling oscillation amplitude are concretely discussed from the viewpoint of ionization-zone length. The numerical code developed in the present study would be an effective tool for designing a best Hall thruster.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>B</td>
<td>magnetic field</td>
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<tr>
<td>D_e</td>
<td>diffusion coefficient</td>
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<td>D_t</td>
<td>thermal-diffusion coefficient</td>
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<td>E</td>
<td>electric field</td>
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<td>e</td>
<td>electric discharge</td>
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<td>I_T</td>
<td>total current</td>
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<td>k</td>
<td>Boltzmann constant</td>
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<td>L_c</td>
<td>acceleration channel length</td>
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<tr>
<td>L_i</td>
<td>ionization-zone length</td>
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<tr>
<td>m</td>
<td>particle mass</td>
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<td>\dot{m}</td>
<td>mass flow rate of propellant (Xe)</td>
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<td>n</td>
<td>number density</td>
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<tr>
<td>r_i</td>
<td>inner radius of acceleration channel</td>
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<tr>
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<td>outer radius of acceleration channel</td>
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<tr>
<td>S</td>
<td>cross-section of channel</td>
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<tr>
<td>T</td>
<td>temperature</td>
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<td>V_{exit}</td>
<td>voltage at channel exit</td>
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<tr>
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<td>\nu</td>
<td>velocity</td>
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<tr>
<td>z</td>
<td>distance from anode</td>
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<tr>
<td>\beta</td>
<td>excitation parameter</td>
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<td>flux</td>
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<td>\nu_P</td>
<td>ion-production rate per unit volume</td>
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<tr>
<td>\sigma</td>
<td>ionization cross-section</td>
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Subscript

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<tr>
<td>e</td>
<td>electron</td>
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<td>i</td>
<td>ion</td>
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<tr>
<td>ion</td>
<td>electron-neutral ionization collision</td>
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<td>n</td>
<td>neutral</td>
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<td>\langle \rangle</td>
<td>average</td>
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1. Introduction

1.1 Physics on Hall Thruster

Hall thruster is a promising propulsion device for space missions capable of reduction of mass flow rate of propellant due to high specific impulse (1000-3000s). It has been used chiefly as a thruster for satellite station keeping and orbit correction, more than one hundred times up to date. [2] Moreover, it has possibility of achieving the thrust density identical to

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the electromagnetic-acceleration-type thrusters under no restriction of space charge limited current rule; the acceleration part is quasi-neutral.[3] High-power Hall thrusters of 50-100kW class have been developed to acquire a high performance (for example, SPT290 giving about 1500mN), and is expected to shorten the flight time in Mars mission etc. in the future. [4] Development of Hall thruster originates in Russia where their research results overwhelmed other countries. Many of Hall thrusters manufactured in Russia are called Stationary Plasma Thruster (SPT) that are characterized in having a comparatively long acceleration channel and a distribution of magnetic field lines concentrated around channel exit. Recently, studies of SPT on the re-evaluation of performance for verifying possibility of practical use and on the measurement of beam characteristics and thruster abrasion during continuous operation have been actively performed in Europe and US. Also, in case where the thruster is loaded on a satellite, there could be serious problems of insulator wall abrasion and deposition of thruster material over main body of satellite; they have been studied from the viewpoint of plasma in channel and in plume. [5] Necessity of physics-related research on plasma characteristics in Hall thrusters has originated from such backgrounds. However, coupling between electric and magnetic fields, and among electron, ion and neutral transports are rather complicated in Hall thrusters where the plasma properties are far from clear understanding. [2]

1.2 Low-Frequency Oscillation
Some serious problems still remain for improvement of Hall thruster performance. A typical problem that should specifically be solved is discharge current oscillation phenomenon observed under standard operational conditions at high-voltage mode of DC regime; it has been regarded as an unavoidable problem of Hall thruster. Since the oscillation affects electron conductivity and anomalous diffusion in acceleration channel, it appears to be a cause of reducing durability and unstable operation etc. In particular, since a long-time stable operation is required in a space mission, it is an urgent item for development of thruster design to probe the physical mechanisms of discharge current oscillation phenomenon. [1,2,6-12]

The present research focuses its efforts on the large-amplitude and low-frequency discharge current oscillation in 20kHz-range (20kHz-range oscillation) which is speculated as a main cause for deterioration of propulsive efficiency and operational instability among various oscillation phenomena observed in Hall thruster. In order to propose a technique to control the amplitude, the relationship between amplitude and ionization-zone length is analytically targeted by using an unsteady one-dimensional numerical analysis, where optimization of operating conditions and best design of device would be provided after clarifying oscillation phenomenon and plasma states in an acceleration channel.

Since Hall thruster has been developed in Russia as mentioned above, experimental and theoretical analyses have played major roles. In contrast, the numerical approaches performed in recent years are predicted to be important in the future of Hall thruster research. The numerical analyses particularly related to oscillation phenomena have been performed by Martinez-Sanchez, [6,8,13] Boef [2,9,14] and Arakawa [3,15] et al. Development of numerical codes which are capable of simulating the characteristics of real thrusters enables us to properly understand the plasma parameters in acceleration channel, where experimentally accurate measurements are usually difficult, and to discover a better thruster design and prompt evaluation of performance. According to most papers, however, the calculated results do not give quantitative or not even qualitative agreements with experimental results due to imperfect models. Based on such situations, an attempt is given here to construct a numerical code containing the factors that have not been considered until now, which hopefully can help explain the oscillation phenomenon as exactly as possible: (1) Introducing diffusion terms into the electron momentum and energy equations, (2) including a time-derivative term in the electron energy equation, and (3) utilizing a 3rd-order-accurate scheme for space integration.

2. Unsteady 1-Dimensional Numerical Analysis
2.1 Numerical Model
Past works indicated that the 20kHz-range oscillation was caused by periodical ionization mechanism [1,2,8] where ionization process proceeded from anode toward exit in axial direction of acceleration channel. Thus, a simplified unsteady one-dimensional model for 20kHz-range oscillation can be constructed. The concept of this analytical model is, in a word, based on the acceleration part shown in Fig.1 [5, 21, 22], and a set of flowfield equations are composed of (1) mass conservation equations for ion and neutral,
(2) momentum equation for electron, (3) momentum equation for ion, and (4) energy balance equation for electron.

\[ \Gamma_i - \Gamma_e = \frac{V_d}{1} \]

Anode

\[ B \]

Cathode

\[ L_c \]

exit

\[ E \]

\[ r_1 \]

\[ r_2 \]

Channel wall

Fig. 1. Schematic of acceleration part of Hall thruster for 1-D numerical analysis: \( \Gamma_n, \Gamma_i \) and \( \Gamma_e \) are axial fluxes of neutral, ion and electron, \( L_c \) is acceleration channel length, \( r_1 \) and \( r_2 \) are inner and outer radii of acceleration channel, \( E \) is axial electric field, \( B \) is radial magnetic field, \( V_d \) is discharge voltage.

2.2 Assumptions

The following assumptions are made:
1) Radial magnetic field is nearly vertical in axial direction of acceleration channel.
2) Ion is generated by electron-neutral inelastic collision, moving under electro-magnetic force.
3) Ion is accelerated only by axial electric field where a part of generated ions is lost by recombination with electrons on channel wall.
4) Electron is emitted from cathode, moving toward anode by the actions of electric field and diffusions.
5) Electron distribution function is Maxwellian.
6) Plasma is quasi-neutral, where electron number density is calculated from ion mass conservation equation.
7) Neutral species flow at a constant velocity.

2.3 Spatial Distribution of Magnetic Field

Axial distribution of radial magnetic field used in the analysis is shown in Fig. 2. Plotted are the measured values (for different coil currents) at radial middle of acceleration channel of Nagoya University Hall thruster II, which is so-called Japan-Type Hall Thruster originally designed by Komurasaki [3,16,17]. Its special aspect is that the magnetic field lines distribute nearly flat during acceleration. The analytical expression for this magnetic field profile (coil current 4A) can be approximated as

\[ B(z) = -\frac{1}{64} \times 10^4 \times (z - 0.008)^2 + 0.062. \]  

(1)

Fig. 2. Axial distribution of radial magnetic field considered in Calculation. The plots correspond to the values measured for different coil currents at radial middle of acceleration channel of Nagoya University Hall thruster II.

2.4 Fundamental Equations

a) Conservation equations

In an unsteady state, the conservation equations for ion and neutral species with production/loss terms of plasma are given by Eqs. (2) and (3):

\[ \frac{\partial n_i}{\partial t} + \frac{\partial \Gamma_i}{\partial z} = n_e (\nu_p - \nu_L), \]  

(2)

\[ \frac{\partial n_n}{\partial t} + \frac{\partial \Gamma_n}{\partial z} = -n_e (\nu_p - \nu_L). \]  

(3)

Here, \( \nu_p = n_e \langle \sigma e \rangle_{ion} \) , whereas \( \langle \sigma e \rangle_{ion} \) indicates the following ionization rate \([18]\) , and is plotted for Xe versus electron temperature in Fig. 3:

\[ \langle \sigma e \rangle_{ion} = \sigma \left[ \frac{8kT_e}{\pi n_e} \right] \left[ 1 + \frac{eV_i}{kT_e} \right] \exp \left( -\frac{eV_i}{kT_e} \right), \]  

(4)

where, \( \nu_L = \left[ \frac{2}{(r_2 - r_1)} \right] \sqrt{\frac{kT_e}{m_e}} \exp (-0.5) \]  

(5)

Fig. 3. Dependency of ionization rate for Xe on electron temperature.

b) Electron momentum equation

The electron flows toward anode across radial magnetic field lines is given by the momentum equation, taking account of electric field and divergences of electron temperature and density.
\[ \Gamma_e = -\mu_e n_e E - D_e \frac{\partial n_e}{\partial z} - D_e^T \frac{1}{T_e^e} \frac{\partial T_e}{\partial z}. \quad (6) \]

Here, the diffusion in acceleration channel of Hall thruster, where magnetic flux density is high (Tesla-order), is much greater than the level expected from classical diffusion theory, so that a Bohm-type diffusion coefficient that is a half-empirical formula is adopted as abnormal diffusion states [7]:
\[ D_e = \frac{kT_e^e}{16\varepsilon B}. \quad (7) \]

The thermal-diffusion coefficient is expressed by Eq.(8), using a density-diffusion coefficient:
\[ D_e^T = T_e \frac{\partial D_e}{\partial T_e}. \quad (8) \]

c) Ion momentum equation

Ion is not significantly affected by magnetic field and is accelerated in a quasi-collisionless manner only by axial electric field. The velocity of ion generated by ionization collision between electron and neutral is assumed to be equal to the neutral species velocity. Thus, ion motion can be described by the following momentum equation:
\[ \frac{\partial \Gamma_i}{\partial t} + \frac{\partial n_i v_i^2}{\partial z} = \frac{n_i e E}{m_i} + n_e (v_p - v_L) v_i n_i. \quad (9) \]

d) Electron energy balance equation

Energy balance equation for electron can be written as [5]
\[ \frac{3}{2} \frac{\partial (kT_e n_e)}{\partial t} + \frac{5}{2} \frac{\partial (kT_e \Gamma_e)}{\partial z} = -\zeta \varepsilon T_e - (1 + \beta) n_e v_i e_i \]
\[ \left( \frac{2}{1 - \delta} + \frac{1 - 2\delta}{1 - \delta} \right) n_i v_i kT_e, \quad (10) \]

where \( \beta = 2.0 \quad (19) \), \( \delta(T_e) = \alpha kT_e/e_i \quad (11) \) ( \( \alpha = 0.141 \) and \( \beta = 0.576 \) for Boron Nitride and Xe) [6], \( \zeta \) is the ratio between the energy given to thermal electron and the input power, which is equal to 1.0 if electron distribution function is Maxwellian, and
\[ \chi = \ln(1 - \delta) \cdot \left\{ e^{0.5} / 4 \right\} \sqrt{8m_i / m_e}. \quad (12) \]

e) Numerical method

These fundamental equations of flowfield are solved by a method suitable for this unsteady analysis as follows: The 3rd-order upwind scheme (Kawamura-Kuwahara method), which has a feature suppressing numerical oscillation and yet not necessitating too many lattice points, is used for spatial integration of both ion and neutral species conservation equations. The energy balance equation for electron is integrated using the 4th-order Runge-Kutta-Gill method for time-integration where accumulation of truncation error is suppressed.

f) Calculation of electric field

From the conservation equation for ion Eq.(2) and the conservation equation for electron \( i \rightarrow e \) in Eq.(2) under electrically quasi-neutral assumption, current conservation equation is obtained as follows:
\[ \frac{\partial (\Gamma_i - \Gamma_e)}{\partial z} = \frac{1}{eS} \frac{\partial I}{\partial z} = 0. \quad (13) \]

This equation indicates that the current depends only on time and not on space. Therefore, the electric field for arbitrary current is acquired from Eq.(6):
\[ E(z,t) = \frac{1}{\mu_e (z,t) n_e (z,t)} \left\{ \frac{I_{\Gamma}(t)}{eS} - n_i (z,t) v_i (z,t) \right\}. \quad (14) \]

Thus, the voltage is given by integrating the electric field as follows:
\[ V_c = -\int_0^L E(z,t) \, dz, \quad (15) \]
\[ V_d = V_c + V_{exit}. \quad (16) \]

In order to determine the correct current for a given voltage \( V_d \), the current is altered and calculation of Eqs.(14), (15) and (16) is repeated until the calculated voltage \( V_c + V_{exit} \) agrees with \( V_d \).

g) Boundary conditions

Since the results of calculation are strongly dependent on boundary conditions, the boundary values as reasonable as possible are adopted. According to the experiment, in other words, the ion density at anode is set to nonzero \( 5.0 \times 10^{16} \text{ m}^{-3} \), while the electron temperature at channel exit is considered to be 3.5eV (one tenth of the voltage still remaining at exit of channel \( \approx 35 \text{V} \)); this looks valid from the viewpoint that the electron temperature in Hall thruster is nearly one tenth of the voltage [5]). In the meantime, note that the equilibrium discharge current after a quasi-steady state is reached is independent of initial conditions.

3. Results and Discussion

3.1 Effect of Diffusion Terms

In the present study, we introduce the diffusion terms (density-diffusion and thermal-diffusion) into the electron momentum and energy balance equations, which is the first attempt among such unsteady one-dimensional numerical analyses. First, by introducing the density-diffusion term, the amplitude of discharge current oscillation in 20kHz-range obtained by
calculation (Fig.4) closely approaches the one observed experimentally, as shown in Fig.5.

Fig.4. Time evolution of discharge current predicted by Calculation (a) without diffusion term, and (b) with diffusion term, for \( V_d = 200 \text{V}, \quad \dot{m} = 1.38 \text{mg/s}, \quad B = 0.062 \text{T}, \quad L_c = 16 \text{mm} \) and \( T_A = 1000 \text{K} \).

In comparison with the case of no diffusion term, in other words, the amplitude decreases down to approximately a half. The reason appears to be that the spatial dispersion of plasma is promoted by density-diffusion where the local ionization is relaxed, and as a result, the rise of plasma density at time of ionization is moderated. Second, even if the thermal-diffusion term is included, little difference is seen in amplitude. This result seems to be valid because the thermal-diffusion term is smaller than the other terms by orders of magnitude. Accordingly, it is concluded that the density-diffusion besides electric field plays an important role in the motion of electron in acceleration channel; thus, inclusion of diffusion terms is absolutely necessary in the study of such oscillation phenomena.

Fig.5. Time evolution of discharge current obtained experimentally for the same conditions as in Calculation (except for neutral species temperature).

3.2 Definition of Ionization-zone Length

To discuss the relationship between amplitude and ionization-zone length in an unsteady one-dimensional numerical analysis, a new parameter “equilibrium length of ionization-zone” is introduced.

Fig.6. Axial distribution of neutral species density in acceleration channel at a certain time, where definition of ionization-zone length is given.

Fig.6 shows the spatial distribution of neutral species density in acceleration channel at a certain time (corresponding to Point ➁ in Fig.11). Since ionization arises immediately behind anode in our thruster, as shown in Fig.6, the position of anode is considered to be the start position of ionization. In addition, the position where 95% of initial neutral species supplied from anode has been consumed by ionization is assumed to be the ionization completion position. In consequence, the ionization-zone length \( L_i \) is defined by the distance between the position of anode and the position where neutral species density decreases down to 5% of neutral species density injected from anode. Then, the ionization-zone length turns out to be a function of time, as shown in Fig.7, so that the equilibrium length of ionization-zone \( L_{i,\text{eq}} \) is defined as the average of maximum and minimum ionization-zone lengths, as calculated by Eq.(17):

\[
L_{i,\text{eq}} = \frac{L_{i,\text{max}} + L_{i,\text{min}}}{2}. \quad (17)
\]

Fig.7. Time evolution of ionization-zone length and definition of equilibrium length of ionization-zone.

3.3 Dependency on Voltage

Dependency of both frequency and amplitude of 20kHz-range oscillation on discharge voltage, which is the most important operating parameter of Hall thruster, is studied. With regard to frequency, both calculated and experimental results increase with
discharge voltage after the onset of 20kHz-range oscillation regime, as shown in Fig. 8.

In Section 3.3, our calculation gave a qualitative agreement with experiments, but not quantitative. A main reason for disagreement can be (1) the wrong assignment of neutral species temperature, besides (2) the present simplified model, or (3) simply the experimental errors. In the present analysis, the neutral species temperature is set to 1000K\textsuperscript{12,13} without rigorous theoretical background; it is usually difficult to accurately measure the temperature of neutral species incoming to ionization-zone, and the actual temperature of neutral species would be higher than the standard value, since neutral sonic-inlet in ionization zone is heated by anode, electron, channel wall etc.\textsuperscript{1} If the neutral species temperature is set higher, the frequency lowers and the amplitude gets smaller. As a result, the calculated results approach the experiments.

In this manner, although it is unavoidable to set the neutral species temperature as a boundary condition in calculation, the oscillation in 20kHz-range turns out to be sensitive to the temperature of neutral species incoming to ionization-zone. When the dependency of amplitude on neutral species temperature is calculated, the amplitude decreases interestingly with the neutral species temperature, as shown in Fig. 10. Now, interpreting this result in a practical way, the amplitude may be decreased if the neutral species temperature incoming to ionization-zone be raised. Namely, the fact that the sound speed of neutral species increases with the neutral temperature moves the ionizing completion position toward the direction of channel exit, so that the ionization-zone length increases; consequently, the rapid rise of plasma density at time of ionization is suppressed and the amplitude decreases. Thus, the neutral species...
temperature can be changed as a method of controlling the amplitude of 20kHz-range oscillation.

3.5 Thruster Design in terms of Ionization-zone Length

In this section, the time evolution of plasma and electromagnetic field in periodic ionization process which seems to be a cause of 20kHz-range oscillation is studied, and the theoretically best design of Hall thruster is discussed, in particular, from the viewpoint of ionization-zone length.

Fig.11. Time evolution of discharge current and ionization-zone length, for \( V_d = 200 \) V, \( m = 1.38 \) mg/s, \( B = 0.062 \) T, \( L_c = 16 \) mm and \( T_n = 1000 \) K.

Fig.11 shows the time evolution of discharge current and ionization-zone length. Figs.13.1-13.4 are the spatial distribution of electron temperature and plasma density in acceleration channel at the feature points ①-④ of discharge current oscillation shown in Fig.11. The spatial distribution of neutral species density and electric field at Point ① in Fig.11 is represented in Fig.12.

Fig.12. Spatial distribution of neutral species density and electric field at Point ① in Fig.10.

First of all, as seen in Figs.13.1 and 12, the position that neutral species density comes down to 5% of initial neutral species density injected from anode \( z_{n_m, \text{min}} \), which corresponds to the ionization completion position as defined in Section 3.2, is nearly in agreement with both the position of maximum plasma density \( z_{n_p, \text{max}} \) and the position of maximum electron temperature \( z_{eT, \text{max}} \). The turning point where the increasing tendency changes to the decreasing one viewing from channel exit, which is caused by consuming, for ionization, the energy that the electrons moving toward anode have acquired from electric field. This relationship \( (z_{n_m, \text{min}} \approx z_{n_p, \text{max}} \approx z_{eT, \text{max}}) \) is recognized for Points ②-④, as shown in Figs.13.2-13.4 (although the relevant figures are not shown in this paper). Accordingly, based on the prediction that the amplitude decreases by extending the ionization-zone length, it is preferable to suppress the amplitude to place the position \( z_{n_p, \text{max}} \approx z_{eT, \text{max}} \) closest to an exit side.

Thus, an idea satisfying this requisite in thruster design can be examined theoretically. Again from Figs.13.1 and 12, the position \( z_{n_p, \text{max}} \approx z_{eT, \text{max}} \) nearly agrees with the position of maximum electric field \( z_{E, \text{max}} \). Moreover, this position \( z_{E, \text{max}} \) is nearly equal to the position of maximum magnetic field \( z_{B, \text{max}} \). Namely, the fact that the electric field reaches maximum at position \( z_{B, \text{max}} \) enhances the electron diffusion due to current continuity, minimizing the electron restriction by magnetic field. Therefore, in the acceleration channel of constant length (long enough to complete ionization in terms of propellant efficiency), placing the maximum position of coil volumes number \( z_{c_m, \text{max}} \) closest to an exit side, which corresponds to the maximum position of magnetic field \( z_{B, \text{max}} \), leads to extension of ionization-zone.
length and causes decrease of amplitude as a result. In this manner, placing the maximum positions of coil volumes number max, \( n_{cz} \) closest to an exit side helps decrease the loss of plasma on the wall behind ionizing position and increase the amount of ion beam extraction, which is desirable from the viewpoint of acceleration efficiency.

In summary, according to the above-mentioned qualitative discussions, it is desirable for a better thruster design that the position of maximum magnetic field \( z_{B,\text{max}} \) should be placed closest to an exit side from the viewpoint of controlling the amplitude and raising the acceleration efficiency; it is necessary that the acceleration channel is of minimum length just enough to complete ionization in view of propellant efficiency.

### 3.6 Influence of Excitation parameter or Electron distribution function

Qualitative disagreements between calculations and experiments are discussed about in this section. The frequencies and the amplitudes obtained by calculations are larger than the ones observed experimentally, as shown in Fig.8 and Fig.9. Main reasons for the disagreements seem to be based on (1) the wrong value of excitation parameter \( \beta \), or (2) the distribution function of electrons. In this analysis, \( \beta \) is set to 2.0 according to the papers \cite{7,19}. If \( \beta \) is set larger than 2.0, the frequency lower and the amplitude gets smaller, as shown in Fig.14. As a results, the calculated results approach the experiments.

![Fig.14. Dependency of calculated amplitude and frequency on \( \beta \) for \( dV=200V, m=1.38mg/s, B=0.062T, L_c=16mm \) and \( T_n=1000K \).](image)

Also, if the value of \( \zeta \) is less than 1.0, calculations close to experiments (Fig.15). As another reason, it can be considered that the potential still remaining at exit of acceleration channel \( V_{\text{exit}} \) is fixed with regularity (35V): It turns out experimentally that \( V_{\text{exit}} \) increases in connection with discharge voltage \cite{20}. Therefore, the changing width of discharge voltage (180V-220V) in calculation may be taken larger than the one of actuality.

![Fig.15. Dependency of calculated amplitude and frequency on \( \zeta \) for the same conditions as in Fig.14](image)

### 4. Conclusion

As a step to the main goal of proposing a method to control the amplitude of 20kHz-range oscillation, an unsteady one-dimensional numerical analysis is performed in the axial direction of acceleration channel, giving the following findings:

- The amplitude of 20kHz-range oscillation can be controlled by changing ionization-zone length.
- By introducing the density-diffusion term into electron momentum and electron energy balance equations, the calculated oscillation behaviors approaches the one observed experimentally.
- A possibility is suggested that raising the temperature of neutral species incoming to ionization-zone enables us to decrease the amplitude.
- A numerical code estimating more exactly the oscillation phenomenon and physical quantities in plasma has been developed, where its usefulness is verified through comparison with experimental results.
- The time evolution of plasma and electromagnetic field during periodic ionization process is examined, and it is found out that the position of maximum magnetic field should be placed closest to an exit side.

In the near feature, the studies based on the present numerical code of which the effectiveness is verified here are planned. For example, (1) the dependency of amplitude on magnetic field strength, distribution of magnetic field and mass flow rate of propellant, (2) estimation of performance (acceleration efficiency,
energy efficiency and propellant efficiency), and (3) development of an analytical model that makes it possible to capture more practically the plasma phenomena in a narrow acceleration part, where experimental examination is limited because of difficulty in detailed measurements.

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References